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On quasi-reduced rings

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Abstract

We in this note introduce the concept of quasi-reduced rings which is a generalization of reduced rings. We study the basic structure of quasi-reduced rings and construct suitable examples to the situations raised naturally in the process.

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1. Quasi-reduced rings

Throughout this paper all rings are associative with identity unless otherwise stated. Given a ring R we use $J(R)$, $N_*(R)$, and $N(R)$ to represent the Jacobson radical, the prime radical (i.e., lower nilradical), and the set of all nilpotent elements in R, respectively; and $r_R(-)$ ($l_R(-)$) is used for the right (left) annihilator over R, i.e., $r_R(S) = \{a \in R \mid sa = 0 \text{ for all } s \in S\}$ ($l_R(S) = \{b \in R \mid bs = 0 \text{ for all } s \in S\}$), where $S \subseteq R$ or S is a subset of a right (left) R-module. If $S = \{a\}$ then we write $r_R(a)$ ($l_R(a)$) in place of $r_R(\{a\})$ ($l_R(\{a\})$). $a \in R$ is said to be right (left) regular if $r_R(a) = 0$ ($l_R(a) = 0$). $a \in R$ is called a left (right) zero-divisor if $r_R(a) \neq 0$ ($l_R(a) \neq 0$). A *zero-divisor* means an element that is neither right nor left regular. Denote the n by n full (resp., upper triangular) matrix ring over R by $Mat_n(R)$ (resp., $U_n(R)$). $D_n(R)$ (resp., $N_n(R)$) denotes the subring $\{A = (a_{ij}) \in U_n(R) \mid a_{11} = \cdots = a_{nn}\}$ (resp., $\{A = (a_{ij}) \in U_n(R) \mid a_{11} = \cdots = a_{nn} = 0\}$) of $U_n(R)$. Use E_{ij} for the matrix with (i, j)-entry 1 and elsewhere 0. $R[x]$ denotes the polynomial ring with an indeterminate x over R. Let $\mathbb{Z}$ ($\mathbb{Z}_n$) denote the ring of integers (modulo n).

It is well-known that in a commutative ring the set of nilpotent elements coincides with the prime radical. This property is also possessed by certain noncommutative rings, which are called 2-*primal*, as in Birkenmeier et al. [3, Proposition 3.2.1]. Shin

[8, Proposition 1.11] proved that given a ring R, $N_*(R) = N(R)$ if and only if every minimal prime ideal P of R is completely prime (i.e., R/P is a domain). Hirano [4] used the term *N-ring* for the concept of 2-primal.

A well-known property between commutative and 2-primal is the *insertion-of-factors-property* (or simply *IFP*) due to Bell [2]. A right (or left) ideal I of a ring R (possibly without identity) is said to have the *IFP* if $ab \in I$ implies $aRb \subseteq I$ for $a, b \in R$. A ring R is called *IFP* if the zero ideal of R has the IFP. Shin [8] used the term *SI* for the IFP. Narbonne use [7] *semicommutative* for the IFP. Shin proved that IFP rings are 2-primal [8, Theorem 1.5]. A ring (possibly without identity) is usually called *reduced* if it has no nonzero nilpotent elements. It is trivial to check that reduced rings are IFP, whence the IFP condition is also between reduced and 2-primal. It is trivial that subrings of IFP rings are also IFP. Note that a ring R is 2-primal if and only if $R/N_*(R)$ is a reduced ring. We use these facts freely in this note.

We in this note introduce another generalization of IFP rings that is different from the 2-primal condition. We shall call a ring R (possibly without identity) *quasi-reduced* if we have $aRa = 0$ whenever $a^2 = 0$ for $a \in R$.

A ring (possibly without identity) is usually called *Abelian* if every idempotent is central. It is easily checked that IFP rings are Abelian. IFP rings are clearly quasi-reduced but the converse need not hold by the following.

Example 1.1. Let F be a domain and $R = U_2(F)$. Let $A = (a_{ij})$ and $B = (b_{ij}) \in R$ be nonzero matrices such that $AB = 0$. Then $A, B \in N_2(F)$, and this yields $ARB = 0$. Thus R is quasi-reduced, but not IFP because R is non-Abelian.

When given rings are semiprime the preceding conditions are equivalent.

Proposition 1.2. Suppose that a ring R is semiprime. Then the following conditions are equivalent:

(1) R is reduced;

(2) R is IFP;

(3) R is 2-primal;

(4) R is quasi-reduced.

Proof. Recall that a ring R is 2-primal if and only if $R/N_*(R)$ is reduced. So it suffices to show (4) $\Rightarrow$ (1) because (1) $\Rightarrow$ (2), (2) $\Rightarrow$ (3), and (1) $\Rightarrow$ (4) are obvious. If $a^2 = 0$, then $aRa = 0$ by assumption. Since R is semiprime, $a = 0$ and so R is reduced. ■

The following is a basic property of quasi-reduced rings.

Proposition 1.3.

(1) The class of quasi-reduced rings is closed under subrings.

(2) The class of quasi-reduced rings is closed under direct sums and direct products.

Proof.

(1) Let R be a quasi-reduced ring, and S be a subring of R. If $a^2 = 0$ for $a \in S$, then $aRa =$ and so $aSa = 0$. Thus S is quasi-reduced.

(2) It suffices to argue about the case of direct products by help of (1). Let $R = \prod_{i \in I} R_i$ be the direct product of quasi-reduced rings R_i. Let $a = (a_i)$ be in R such that $a^2 = 0$. Then $a_i a_i = 0$ for all $i \in I$. Since R_i is quasi-reduced, $a_i R_i a_i = 0$ and so we have $aRa = 0$. ■

Remark 1.4. The ideals, subrings and direct sums in Proposition 1.3 are considered as rings possibly without identity.

It is natural to ask whether factor rings of quasi-reduced rings are quasi-reduced. But the answer is negative as the following shows.

Example 1.5. Let R be the ring of quaternions with integer coefficients. Then R is a domain, so quasi-reduced. However, for any odd prime integer q, the ring R/qR is isomorphic to the 2 by 2 matrix ring over the field $\mathbb{Z}_q$ of integers modulo q, by the argument in [6, Exercise 2A]. Thus R/qR cannot be quasi-reduced because semiprime quasi-reduced rings are reduced by Proposition 1.2, noting that $Mat_2(\mathbb{Z}_q)$ is semiprime.

2. Examples of quasi-reduced Rings

In this section we observe several useful examples of quasi-reduced rings.

Concerning polynomial rings over some kinds of rings related to quasi-reduced rings, we have the following useful results:

(1) A ring R is reduced if and only if $R[x]$ is reduced obviously.

(2) A ring R is 2-primal if and only if $R[x]$ is 2-primal by [3, Proposition 2.6].

Based on the preceding results one may conjecture that a ring R is quasi-reduced if and only if $R[x]$ is quasi-reduced. However the following example provides a counterexample.

Example 2.1. We apply the construction and argument in [5, Example 2]. Let $A = \mathbb{Z}_2\langle a_0, a_1, a_2, c\rangle$ be the free algebra with noncommuting indeterminates a_0, a_1, a_2, c over $\mathbb{Z}_2$, and B be the subalgebra of A consist of all polynomials of zero constant term. Let I be the ideal of A generated by

$$a_0a_0,\ a_1a_2 + a_2a_1,\ a_0a_1 + a_1a_0,\ a_0a_2 + a_1a_1 + a_2a_0,\ a_2a_2,$$
$$a_0ra_0,\ a_2ra_2,\ (a_0 + a_1 + a_2)r(a_0 + a_1 + a_2),\ \text{ and } r_1r_2r_3r_4,$$

where $r \in A$ and $r_1, r_2, r_3, r_4 \in B$.

Let next $R = A/I$. Notice that $(a_0 + a_1x + a_2x^2)(a_0 + a_1x + a_2x^2) \in I[x]$, but $(a_0 + a_1x + a_2x^2)c(a_0 + a_1x + a_2x^2) \notin I[x]$ because $a_0ca_1 + a_1ca_0 \notin I$; hence $R[x]$ is not quasi-reduced.

We will show that R is quasi-reduced. Each product of indeterminates a_0, a_1, a_2, c is called a *monomial* and we say that α is a monomial of degree n if it is a product of exactly n generators. Let H_n be the set of all linear combinations of monomials of degree n over $\mathbb{Z}_2$. Observe that H_n is finite for any n and that the ideal I of R is homogeneous (i.e., if $\sum_{i=1}^{s} r_i \in I$ with $r_i \in H_i$ then every r_i is in I).

Claim 1. If $f_1^2 \in I$ for $f_1 \in H_1$ then $f_1rf_1 \in I$ for any $r \in A$.

Proof. By the definition of I we obtain the following cases: $(f_1 = a_0)$, $(f_1 = a_2)$, or $(f_1 = a_0 + a_1 + a_2)$. So we complete the proof, using the definition of I again. ■

Claim 2. If $f^2 \in I$ for $f \in B$ then $frf \in I$ for all $r \in B$.

Proof. Observe that $f = f_1 + f_2 + f_3 + f_4$ and $r = r_1 + r_2 + r_3 + r_4$ for some $f_1, r_1 \in H_1$, $f_2, r_2 \in H_2$, $f_3, r_3 \in H_3$ and some $f_4, r_4 \in I$. Note that $H_i \subseteq I$ for $i \geq 4$. So $frf = f_1r_1f_1 + h$ for some $h \in I$. $f^2 \in I$ implies $f_1f_1 \in I$ since I is homogeneous; hence $f_1r_1f_1 \in I$ by Claim 1. Consequently $frf \in I$. ■

To see that R is quasi-reduced, we will show that $yry \in I$ for all $r \in A$ if $y^2 \in I$ for $y \in A$. Write $y = \alpha + z$ with $\alpha \in \mathbb{Z}_2$ and $z \in B$. So $\alpha^2 + \alpha z + z\alpha + z^2 = y^2 \in I$; hence $\alpha = 0$ and $z^2 \in I$. Hence $yry = zrz \in I$ for all $r \in A$ by help of Claim 2. Therefore R is a quasi-reduced ring.

But if given rings are semiprime then we have the following equivalence.

Proposition 2.2. Suppose that a ring R is semiprime. Then the following conditions are equivalent:

(1) R is quasi-reduced;

(2) $R[x]$ is quasi-reduced;

(3) R is reduced;

(4) $R[x]$ is reduced.

Proof. By help of Proposition 1.2, using the fact that R is reduced if and only if $R[x]$ is reduced. ■

Given a ring R, an endomorphism σ of R, and a σ-derivation δ of R, the *Ore extension* $R[x;\sigma,\delta]$ of R is the ring obtained by giving the polynomial ring $R[x]$ the new multiplication $xr=\sigma(r)x+\delta(r)$ for all $r\in R$. If $\delta=0$ then we write $R[x;\sigma]$ and call it a *skew* polynomial ring. If $\sigma=1$ then we write $R[x;\delta]$ and call it a *differential* polynomial ring. It is also natural to check quasi-reduced-ness of these two kinds of extensions. But there exist counterexamples to these cases as follows.

Proposition 2.3. There exists a quasi-reduced ring over which the skew polynomial ring need not be quasi-reduced.

Proof. For a division ring D let $R=D\oplus D$, then R is quasi-reduced obviously. Define $\sigma: R\to R$ by $\sigma(s,t)=(t,s)$. Then σ is an automorphism of R. Let $S=R[x;\sigma]$ be the skew polynomial ring over R by σ. We `claim` that S is semiprime. Let I be a nonzero ideal of S. Then we pick a nonzero $f(x)$ in I which is of the smallest degree in I. Say $f(x)=a+bx+\cdots+cx^n$ with $a,b,\ldots,c\in R$ and $c\neq 0$. If n is even, then $f(x)^2=a^2+\cdots+c\sigma^n(c)x^{2n}=a^2+\cdots+c^2x^{2n}\in I^2\subseteq I$ is nonzero because c is nonzero and σ is of order 2. Next if n is odd then $f(x)x=ax+bx^2+\cdots+cx^{n+1}\in I$; hence $[f(x)x]^2\in I^2\subseteq I$ is also nonzero by the same computation. Thus I^2 is nonzero and so S is semiprime. But $N(S)\neq 0$ as can be seen by $((1,0)x)((1,0)x)=0$; hence S is not reduced. Therefore S is not quasi-reduced because semiprime quasi-reduced rings are reduced by Proposition 1.2. ∎

Proposition 2.4. There exists a quasi-reduced ring over which the differential polynomial ring need not be quasi-reduced.

Proof. The proof is essentially due to [1, Example 11]. Let $R=F[x]/(x^2)$ and define $\delta: R\to R$ by $\delta(x+(x^2))=1+(x^2)$, where F is a field of characteristic 2 and $(x^2)=F[x]x^2$. Then R is quasi-reduced since R is commutative. Next let $S=R[x;\delta]$. Then $[x+(x^2)]^2=0$ but $[x+(x^2)]S[x+(x^2)]\neq 0$ so R is not quasi-reduced. ∎

In the following we see an example of quasi-reduced upper triangular matrix rings.

Proposition 2.5. Let R be a reduced ring. Then

$$S=\left\{\begin{pmatrix} a & b & c\\ 0 & d & e\\ 0 & 0 & f\end{pmatrix} \mid a,b,c,d,e,f\in R\right\}=U_3(R)$$

is a quasi-reduced ring.

Proof. $\begin{pmatrix} a & b & c\\ 0 & d & e\\ 0 & 0 & f\end{pmatrix}\begin{pmatrix} a & b & c\\ 0 & d & e\\ 0 & 0 & f\end{pmatrix}=0$ for $\begin{pmatrix} a & b & c\\ 0 & d & e\\ 0 & 0 & f\end{pmatrix}\in S$. Then we have

$$a=d=f=0 \text{ and } be=0$$

by a simple computation. Note that $eb = 0$ because R is reduced. So we have also

$$\begin{pmatrix} 0 & b & c \\ 0 & 0 & e \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x & y & z \\ 0 & u & v \\ 0 & 0 & w \end{pmatrix} \begin{pmatrix} 0 & b & c \\ 0 & 0 & e \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & bue \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

for all $\begin{pmatrix} x & y & z \\ 0 & u & v \\ 0 & 0 & w \end{pmatrix} \in S$, because $(bue)^2 = bu(eb)ue = bu0ue = 0$ (hence $bue = 0$ by the reducedness of R). Thus S is quasi-reduced. ■

Proposition 2.5 provides us the following useful relation among quasi-reduced, reduced, polynomial ring, and upper triangular matrix ring.

Theorem 2.6. For a ring R the following conditions are equivalent:

(1) R is reduced;

(2) $U_3(R)$ is quasi-reduced;

(3) $U_2(R)$ is quasi-reduced;

(4) $U_3(R[x])$ is quasi-reduced;

(5) $U_2(R[x])$ is quasi-reduced;

(6) $R[x]$ is reduced.

Proof. The implication (2) ⇒ (3) follows Proposition 1.3, and (1) ⇒ (2) is shown by Proposition 2.5.

(3) ⇒ (1). Let $U_2(R)$ be quasi-reduced, and assume on the contrary that R is nonreduced. Then $a^2 = 0$ for some nonzero $a \in R$. Consider the matrix

$$M = \begin{pmatrix} a & 1 \\ 0 & -a \end{pmatrix}$$

in $U_2(R)$. Then $M^2 = 0$. However

$$\begin{pmatrix} a & 1 \\ 0 & -a \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & 1 \\ 0 & -a \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -a \end{pmatrix} \begin{pmatrix} a & 1 \\ 0 & -a \end{pmatrix} = \begin{pmatrix} 0 & -a \\ 0 & 0 \end{pmatrix} \neq 0,$$

a contradiction to $U_2(R)$ being quasi-reduced. Thus R is reduced.

The proof for the equivalence among the conditions (4), (5), and (6) is similar, recalling that R is reduced if and only if $R[x]$ is reduced. ■

Based on Proposition 2.5, one may ask whether $U_n(S)$ may be also a quasi-reduced ring for $n \geq 4$ over a reduced ring S. But the following example answers negatively.

Example 2.7. Let S be any ring and $R = U_4(S)$. Take

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

in R. Then $A^2 = 0$. But

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \neq 0,$$

concluding that R is not quasi-reduced.

Based on Proposition 2.5, one may also suspect that $U_3(S)$ is a quasi-reduced ring over a quasi-reduced ring S. But the following example erases the possibility.

Example 2.8. Let T be a reduced ring and let $S = U_2(T)$. Then S is quasi-reduced by Propositions 1.3(1) and 2.5. Let $R = U_3(S) = \left\{ \begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{pmatrix} \mid a_{ij} \in S \right\}$ and take

$$A = \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

in R. Then we have $A^2 = 0$, but

$$A \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

$$\times A = \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \neq 0,$$

concluding that R is not quasi-reduced.

Considering Example 2.8, one may suspect that $U_2(S)$ may be a quasi-reduced ring over a quasi-reduced ring S. But the following example erases the possibility.

Example 2.9. $S = \mathbb{Z}_4$ is clearly a quasi-reduced ring. Let $R = U_2(S)$ and take $A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$. Then $A^2 = 0$. But $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \neq 0$. So R is not quasi-reduced.

We examine next the quasi-reducedness of factor ring case. It is natural to conjecture that R is a quasi-reduced ring if for any nonzero proper ideal I of R, R/I and I are quasi-reduced, where I is considered as a quasi-reduced ring without identity. However we have a negative answer to this situation by the above example. Letting $I = \begin{pmatrix} 0 & \mathbb{Z}_4 \\ 0 & 0 \end{pmatrix}$. Then I is quasi-reduced and so is R/I since R/I is commutative. But R is not quasi-reduced.

It is also natural to conjecture that R is a quasi-reduced ring if for any nonzero proper nil (or nilpotent) ideal I of R, R/I and I are quasi-reduced, where I is considered as a quasi-reduced ring without identity. However we have a negative answer to this situation by the preceding argument since I is nilpotent.

However if we take another condition "I is reduced" then we have an affirmative answer as in the following.

Proposition 2.10. For a ring R suppose that R/I is a quasi-reduced ring for a proper ideal I of R. If I is a reduced ring without identity then R is quasi-reduced.

Proof. Let $a^2 = 0$ with $a \in R$. Then we have $aRa \subseteq I$ because R/I is quasi-reduced. So we get $aRa = 0$ since $(aRa)^2 = 0$ and I is reduced. Therefore R is quasi-reduced. ∎

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