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Soft Generalized Closed Sets in Soft Čech Closure Space

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Abstract

In this paper, we introduce soft generalized closed sets in soft $\check{C}$ech closure spaces which are defined over an initial universe with a fixed set of parameters. Also we investigate the behavior relative to union, intersection and soft subspaces of soft g-closed sets as well as soft g-open sets. In the study of soft g-closed sets, we introduce a new separation axiom, namely $T_{\frac{1}{2}}$-space which lie between T_0 and T_1.

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1. Introduction

Closed sets are fundamental objects in a topological space. For example, one can define the topology on a set by using either the axioms for the closed sets or the Kuratowski closure axioms. In 1970, N. Levine [7] initiated the study of generalized closed sets in topological space in order to extend some important properties of closed sets to a larger family of sets. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets.

E. Čech [2] introduced the concept of closure spaces. In Čech's approach the operator satisfies idempotent condition among Kuratowski axioms. This condition need not hold for every set A of X. When this condition is also true, the operator becomes topological closure operator. Thus, the concept of closure space is the generalisation of a topological space. In 1999, D. Molodtsov [8] introduced the notion of soft set to deal with problems of incomplete information. Later, he applied this theory to several directions [9] and [10]. Generalized closed sets in Čech closed spaces were introduced by Chawalit Boonpok [5]. R. Gowri and G. Jegadeesan [6] introduced and studied the concept of lower separation axioms in soft Čech closure spaces.

In this paper, we introduce soft generalized closed sets in soft Čech closure spaces which are defined over an initial universe with a fixed set of parameters. Also we investigate the behavior relative to union, intersection and soft subspaces of soft g-closed sets as well as soft g-open sets. In the study of soft g-closed sets, we introduce a new separation axiom, namely $T_{\frac{1}{2}}$-space which lie between T_0 and T_1.

2. Preliminaries

In this section, we recall the basic definitions of soft Čech closure spaces.

Definition 2.1. Let X be an initial universe set, A be a set of parameters. Then the function $k : P(X_{F_A}) \to P(X_{F_A})$ defined from a soft power set $P(X_{F_A})$ to itself over X is called Čech Closure operator if it satisfies the following axioms:

(C1) $k(\emptyset_A) = \emptyset_A$.

(C2) $F_A \subseteq k(F_A)$.

(C3) $k(F_A \cup G_A) = k(F_A) \cup k(G_A)$.

Then (X, k, A) or (F_A, k) is called a soft Čech closure space.

Definition 2.2. A soft subset $U_A$of a soft $\check{C}$ech closure space (F_A, k) is said to be soft k-closed (soft closed) if $k(U_A) = U_A$.

Definition 2.3. A soft subset U_A of a soft $\check{C}$ech closure space (F_A, k) is said to be soft k-open (soft open) if $k(U_A^C) = U_A^C$.

Definition 2.4. A soft set $Int(U_A)$ with respect to the closure operator k is defined as $Int(U_A) = F_A - k(F_A - U_A) = [k(U_A^C)]^C$. Here $U_A^C = F_A - U_A$.

Definition 2.5. A soft subset U_A in a soft $\check{C}$ech closure space (F_A, k) is called Soft neighbourhood of e_F if $e_F \in Int(U_A)$.

Definition 2.6. If (F_A, k) be a soft $\check{C}$ech closure space, then the associate soft topology on F_A is $\tau = \left\{U_A^C : k(U_A) = U_A\right\}$.

Definition 2.7. Let (F_A, k) be a soft Čech closure space. A soft Čech closure space (G_A, k^*) is called a soft subspace of (F_A, k) if $G_A \subseteq F_A$ and $k^*(U_A) = k(U_A) \cap G_A$, for each soft subset $U_A \subseteq G_A$.

3. Soft generalized closed sets

In this section, we introduce and characterize a new class of soft generalized closed sets in Soft Čech closure spaces.

Definition 3.1. Let (F_A, k) be a soft Čech closure space. A soft subset $U_A \subseteq F_A$ is called a soft generalized closed set, briefly a soft g-closed set if $k(U_A) \subseteq G_A$ whenever G_A is an soft open subset of (F_A, k) with $U_A \subseteq G_A$. A soft subset $U_A \subseteq F_A$ is called a soft generalized open set, briefly a soft g-open set, if its complement is soft g-closed.

Example 3.2. Let the initial universe set $X = \{u_1, u_2\}$ and $E = \{x_1, x_2, x_3\}$ be the parameters. Let $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then $P(X_{F_A})$ are

$$
\begin{aligned}
&F_{1A} = \{(x_1, \{u_1\})\}, \quad F_{2A} = \{(x_1, \{u_2\})\},\\
&F_{3A} = \{(x_1, \{u_1, u_2\})\}, \quad F_{4A} = \{(x_2, \{u_1\})\},\\
&F_{5A} = \{(x_2, \{u_2\})\}, \quad F_{6A} = \{(x_2, \{u_1, u_2\})\},\\
&F_{7A} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, \quad F_{8A} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\},\\
&F_{9A} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, \quad F_{10A} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\},\\
&F_{11A} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}, \quad F_{12A} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\},\\
&F_{13A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}, \quad F_{14A} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\},\\
&F_{15A} = F_A, \quad F_{16A} = \emptyset_A.
\end{aligned}
$$

An operator $k : P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$$
\begin{aligned}
&k(F_{1A}) = F_{1A}, k(F_{2A}) = F_{12A}, k(F_{4A}) = F_{4A},\\
&k(F_{5A}) = F_{14}, k(F_{7A}) = F_{7A}, k(F_{8A}) = F_{14A},\\
&k(F_{9A}) = F_{12A}, k(F_{3A}) = k(F_{6A}) = k(F_{10A}) = k(F_{11A})\\
&\qquad = k(F_{12A}) = k(F_{13A}) = k(F_{14A}) = k(F_A) = F_A, k(\emptyset_A) = \emptyset_A.
\end{aligned}
$$

Here, the soft g-closed sets are $\emptyset_A$, F_{1A}, F_{4A}, F_{7A}, F_{8A}, F_{9A}, F_{11A}, F_{13A}, F_A.

Remark 3.3. Every soft closed sets are soft g-closed, but the converse is not true as shown in the following example.

Example 3.4. In the example 3.2, F_{8A}, F_{9A}, F_{11A}, F_{13A} are soft g-closed, but not soft closed.

Theorem 3.5. Let (F_A, k) be a soft Čech closure space. If U_A and V_A are soft g-closed subsets of (F_A, k), then $U_A \cup V_A$ is soft g-closed.

Proof. Let G_A be a soft open subset of (F_A, k) such that $U_A \cup V_A \subseteq G_A$. Then $U_A \subseteq G_A$ and $V_A \subseteq G_A$. Since, U_A and V_A are soft g-closed, $k[U_A] \subseteq G_A$ and $k[V_A] \subseteq G_A$. Then, $k[U_A \cup V_A] = k[U_A] \cup k[V_A] \subseteq G_A$. Therefore, $U_A \cup V_A$ is soft g-closed. ■

Result 3.6. The following example shows that, the intersection of two soft g-closed sets need not be a soft g-closed set.

Example 3.7. Let us consider the soft subsets of F_A that are given in example 3.2. An operator $k : P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$$\begin{aligned} k(F_{1A}) &= k(F_{7A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, k(F_{2A}) = F_{10A}, \\ k(F_{4A}) &= k(F_{5A}) = k(F_{6A}) = F_{6A}, k(F_{3A}) = k(F_{13A}) = k(F_{14A}) \\ &= k(F_A) = F_A, k(F_{9A}) = k(F_{10A}) = k(F_{12A}) = F_{12A}, \\ k(\emptyset_A) &= \emptyset_A. \end{aligned}$$

Here, take the soft g-closed sets, $U_A = F_{7A}$ and $V_A = F_{8A}$. Then $U_A \cap V_A = F_{7A} \cap F_{8A} = F_{1A}$, which is not a soft g-closed set.

Theorem 3.8. Let (F_A, k) be a soft Čech closure space. If U_A is soft g-closed and V_A is soft closed in F_A, then $U_A \cap V_A$ is soft g-closed.

Proof. Let G_A be a soft open subset of (F_A, k) such that $U_A \cap V_A \subseteq G_A$. Then $U_A \subseteq G_A \cup V_A^c$ and $k[U_A] \subseteq G_A \cup V_A^c$. That is, $U_A \subseteq G_A \cup (F_A - V_A)$ and $k[U_A] \subseteq G_A \cup (F_A - V_A)$. Then, $k[U_A] \cap V_A \subseteq G_A$. Since, V_A is soft closed. Therefore, $k[U_A \cap V_A] \subseteq G_A$. Hence, $U_A \cap V_A$ is soft g-closed. ■

Theorem 3.9. Let (H_A, k^*) be a soft closed subspace of (F_A, k). If V_A is a soft g-closed subset of (H_A, k^*), then V_A is a soft g-closed subset of (F_A, k).

Proof. Let G_A be a soft open subset of (F_A, k) such that $V_A \subseteq G_A$. Then, $V_A \subseteq G_A \cap H_A$. Since, V_A is soft g-closed and $G_A \cap H_A$ is soft open in (H_A, k^*). Therefore, $k[V_A] \cap H_A = k^*[V_A] \subseteq G_A$. But H_A is a soft closed subset of (F_A, k) and $k[V_A] \subseteq G_A$. Hence, V_A is a soft g-closed subset of (F_A, k). ■

Theorem 3.10. Let (F_A, k) be a soft Čech closure space and let $U_A \subseteq F_A$. If U_A is both soft open and soft g-closed, then U_A is soft closed.

Proof. It is obvious. ■

Theorem 3.11. Let (F_A, k) be a Čech closure space and let $U_A \subseteq F_A$. If U_A is soft g-closed, then $k[U_A] - U_A$ has no non-empty soft closed subset.

Proof. Suppose that U_A is soft g-closed. Let V_A be a soft closed subset of $k[U_A] - U_A$. Then $V_A \subseteq k[U_A] \cap (F_A - U_A)$ and so $U_A \subseteq F_A - V_A$. Consequently, $V_A \subseteq F_A - k[U_A]$. Since, $V_A \subseteq k[U_A]$, $V_A \subseteq k[U_A] \cap (F_A - k[U_A]) = \emptyset_A$. Thus $V_A = \emptyset_A$. Therefore, $k[U_A] - U_A$ contains no nonempty soft closed set. ■

Remark 3.12. The converse of the above theorem 3.11 is not true as shown in the following example.

Example 3.13. Let us consider the soft subsets of F_A that are given in example 3.2. An operator $k : P(X_{F_A}) \to P(X_{F_A})$ is defined from soft power set $P(X_{F_A})$ to itself over X as follows.

$$\begin{aligned} &k(F_{1A}) = k(F_{5A}) = F_{8A}, k(F_{3A}) = k(F_{9A}) = k(F_{13A}) = F_{13A}, \\ &k(F_{2A}) = F_{3A}, k(F_{4A}) = F_{4A}, k(F_{6A}) = k(F_{8A}) = k(F_{11A}) = F_{11A}, \\ &k(F_{7A}) = F_{7A}, k(F_{10A}) = F_{14A}, k(F_{12A}) = k(F_{14A}) = k(F_A) \\ &\qquad = F_A, k(\emptyset_A) = \emptyset_A. \end{aligned}$$

Here, take $U_A = F_{2A}$. Then, $k[U_A] - U_A = k[F_{2A}] - F_{2A} = F_{1A}$, which does not contain non-empty soft closed subset. But, F_{2A} is not soft g-closed.

Corollary 3.14. Let (F_A, k) be a soft Čech closure space and let U_A be a soft g-closed subset of (F_A, k). Then U_A is soft closed if and only if $k[U_A] - U_A$ is soft closed.

Proof. Let U_A be a soft g-closed subset of (F_A, k). If U_A is soft closed, then $k[U_A] - U_A = \emptyset_A$. Since, $\emptyset_A$ is soft closed. Therefore, $k[U_A] - U_A$ is also soft closed. Conversely, Suppose that $k[U_A] - U_A$ is soft closed. Since, U_A is soft g-closed, by theorem 3.11, $k[U_A] - U_A = \emptyset_A$. This implies, $k[U_A] = U_A$. Hence, U_A is soft closed. ■

Theorem 3.15. Let (F_A, k) be a soft Čech closure space. A soft set $U_A \subseteq F_A$ is soft g-open if and only if $V_A \subseteq F_A - k[F_A - U_A]$ whenever V_A is soft closed and $V_A \subseteq U_A$.

Proof. Suppose that U_A is soft g-open and V_A be a soft closed subset of (F_A, k) such that $V_A \subseteq U_A$. Then $F_A - U_A \subseteq F_A - V_A$. But, $F_A - U_A$ is soft g-closed and $F_A - V_A$ is soft open. This implies that, $k[F_A - U_A] \subseteq F_A - V_A$. Therefore, $V_A \subseteq F_A - k[F_A - U_A]$. Conversely, Let G_A be a soft open subset of (F_A, k) such that $F_A - U_A \subseteq G_A$. Then $F_A - G_A \subseteq U_A$. Since, $F_A - G_A$ is soft closed, $F_A - G_A \subseteq F_A - k[F_A - U_A]$. This implies, $k[F_A - U_A] \subseteq G_A$. Therefore, $F_A - U_A$ is soft g-closed. Hence, U_A is soft g-open. ■

Remark 3.16. The following example shows that, the union of two soft g-open sets need not be a soft g-open.

Example 3.17. In example 3.7, take soft g-open sets $U_A = F_{4A}$ and $V_A = F_{5A}$. Then, $U_A \cup V_A = F_{6A}$, which is not a soft g-open.

Theorem 3.18. Let (F_A, k) be a soft Čech closure space. If U_A is soft g-open and V_A is soft open in F_A, then $U_A \cup V_A$ is soft g-open.

Proof. Let G_A be a soft closed subset of (F_A, k) such that $G_A \subseteq U_A \cup V_A$. Then $F_A-(U_A \cup V_A) \subseteq F_A-G_A$. Hence, $(F_A-U_A)\cap(F_A-V_A) \subseteq F_A-G_A$. By theorem 3.8, $(F_A-U_A)\cap(F_A-V_A)$ is soft g-closed. Therefore, $k[(F_A-U_A)\cap(F_A-V_A)] \subseteq F_A-G_A$. Consequently, $G_A \subseteq F_A - k[(F_A - U_A) \cap (F_A - V_A)] = F_A - k[F_A - (U_A \cup V_A)]$. By theorem 3.15, $U_A \cup V_A$ is soft g-open. ■

Theorem 3.19. Let (F_A, k) be a soft Čech closure space. If U_A and V_A are soft g-open subsets of (F_A, k), then $U_A \cap V_A$ is soft g-open.

Proof. Let G_A be a soft closed subset of (F_A, k) such that $G_A \subseteq U_A \cap V_A$. Then, $F_A - (U_A \cap V_A) \subseteq F_A - G_A$. Consequently, $(F_A - U_A) \cup (F_A - V_A) \subseteq F_A - G_A$. By theorem 3.5, $(F_A-U_A)\cup(F_A-V_A)$ is soft g-closed. Thus, $k[(F_A-U_A)\cup(F_A-V_A)] \subseteq F_A - G_A$. Hence, $G_A \subseteq F_A - k[(F_A - U_A) \cup (F_A - V_A)] = F_A - k[F_A - (U_A \cap V_A)]$. By theorem 3.15, $U_A \cap V_A$ is soft g-open. ■

Theorem 3.20. Let (F_A, k) be a soft Čech closure space. If U_A is a soft g-open subset of F_A, then $G_A = F_A$ whenever G_A is soft open and $(F_A-k[F_A-U_A])\cup(F_A-U_A) \subseteq G_A$.

Proof. Assume that U_A is soft g-open. Let G_A be a soft open subset of (F_A, k) such that $(F_A - k[F_A - U_A]) \cup (F_A - U_A) \subseteq G_A$. Then $(F_A - G_A) \subseteq F_A - ((F_A - k[F_A - U_A]) \cup (F_A - U_A))$. Therefore, $F_A - G_A \subseteq k[F_A - U_A] \cap U_A$ or equivalently, $F_A - G_A \subseteq k[F_A - U_A] - (F_A - U_A)$. But, $F_A - G_A$ is soft closed and $F_A - U_A$ is soft g-closed. By theorem 3.11, $F_A - G_A = \emptyset_A$. Consequently, $F_A = G_A$. ■

Remark 3.21. The converse of the above theorem 3.20 is not true as shown in the following example.

Example 3.22. In example 3.7, take $U_A = F_{11A}$. Then, $(F_A - k[F_A - U_A]) \cup (F_A - U_A) = F_{13A} \subseteq G_A$, whenever G_A is soft open. This implies, $G_A = F_A$, but U_A is not soft g-open.

Theorem 3.23. Let (F_A, k) be a soft Čech closure space and let $U_A \subseteq F_A$. If U_A is a soft g-closed, then $k[U_A] - U_A$ is soft g-open.

Proof. Suppose that U_A is soft g-open. Let G_A be a soft closed subset of (F_A, k) such that $G_A \subseteq k[U_A] - U_A$. By theorem 3.11, $G_A = \emptyset_A$ and hence $G_A \subseteq F_A - k[F_A - (k[U_A] - U_A)]$. By theorem 3.15, $k[U_A] - U_A$ is soft g-open. ■

Remark 3.24. The converse of the above theorem 3.23 is not true as shown in the following example.

Example 3.25. In example 3.7, take $U_A = F_{2A}$. Then, $k[U_A] - U_A = F_{5A}$, which is soft g-open. But U_A is not soft g-closed.

4. $T_{\frac{1}{2}}$-soft Čech closure space.

In this section, we introduce $T_{\frac{1}{2}}$-soft Čech closure spaces through soft g-closed sets and investigate some of their properties.

Definition 4.1. A soft Čech closure space (F_A, k) is said to be a $T_{\frac{1}{2}}$-space if every soft g-closed set is soft closed in F_A.

Theorem 4.2. Let (F_A, k) be a soft Čech closure space, then

1. $T_{\frac{1}{2}}$-space $\Rightarrow T_0$-space.

2. T_1-space $\Rightarrow T_{\frac{1}{2}}$-space.

Proof. The proof is obvious. ■

Theorem 4.3. Let (F_A, k) be a soft Čech closure space. Then (F_A, k) is a $T_{\frac{1}{2}}$-space if and only if every singleton soft subset of F_A is either soft closed or soft open.

Proof. Let $(x, u) \in F_A$ and suppose that $\{(x, u)\}$ is not a soft closed subset of (F_A, k). Then $F_A - \{(x, u)\}$ is not a soft open subset of F_A. Then the only soft open subset containing $F_A - \{(x, u)\}$ is F_A. Hence, $F_A - \{(x, u)\}$ is a soft g-closed subset of (F_A, k). Since, (F_A, k) is a $T_{\frac{1}{2}}$-space. This implies, $F_A - \{(x, u)\}$ is a soft closed subset of F_A. Therefore, $\{(x, u)\}$ is a soft open subset of F_A. Conversely, let U_A be a soft g-closed subset of (F_A, k). Suppose that $(x, u) \notin U_A$. Then $\{(x, u)\} \subseteq F_A - U_A$. This implies, $U_A \subseteq F_A - \{(x, u)\}$. If $\{(x, u)\}$ is a soft open subset of F_A, then $F_A - \{(x, u)\}$ is a soft closed subset in F_A. Now, $k[U_A] \subseteq k[F_A - \{(x, u)\}] = F_A - \{(x, u)\}$, thus $(x, u) \notin k[U_A]$. If $\{(x, u)\}$ is a soft closed subset of F_A, then $F_A - \{(x, u)\}$ is a soft open subset in F_A. Since, U_A is soft g-closed. This implies $k[U_A] \subseteq F_A - \{(x, u)\}$. Therefore, $(x, u) \notin k[U_A]$. we have, $k[U_A] \subseteq U_A$ for both the cases. Thus, $k[U_A] = U_A$. Hence, U_A is soft closed subset of (F_A, k). Therefore (F_A, k) is $T_{\frac{1}{2}}$-space. ■

Theorem 4.4. Let (F_A, k) be a $T_{\frac{1}{2}}$-soft *Čech* closure space and if (G_A, k^*) be a soft closed subspace of (F_A, k), then (G_A, k^*) is also $T_{\frac{1}{2}}$-space.

Proof. Let U_A be a soft g-closed subset of (G_A, k^*). Since, $G_A \subseteq F_A$, then U_A is also soft g-closed subset of (F_A, k). Since, (F_A, k) is $T_{\frac{1}{2}}$-space, U_A is a soft closed subset of F_A. This implies, U_A is a soft closed subset of G_A. Therefore, (G_A, k^*) is also $T_{\frac{1}{2}}$-space.

■

5. Conclusion

In this paper, we introduced soft g-closed sets in soft Čech closure spaces which are defined over an initial universe with a fixed set of parameters. Also we investigated the behavior relative to union, intersection and soft subspaces of soft g-closed sets as well as soft g-open sets. Through soft g-closed sets, we introduced a new separation axiom, namely $T_{\frac{1}{2}}$-space which lie between T_0 and T_1.

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