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Linear Sweep Algorithm for Vehicle Routing Problem with Simultaneous Pickup and Delivery between Two Depots with Several Nodes

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Abstract

Distribution companies face great challenges to organize their fleet efficiently. In this work, a study of capacitated vehicle routing problem with simultaneous pickup and delivery between two depots with several nodes spread in between is taken up. The purpose of this study is to develop a heuristic procedure for the above mentioned problem and analyse the results. The problem consists of two depots with 25 nodes spread in between them. A heuristic method called linear sweep algorithm have been developed to solve the problem. The objectives of this work were to minimize the number of vehicles used and maximize the utilization of vehicle capacity. The proposed heuristic was able to achieve both the objectives and hence found to be useful for solving the above problem in different scenarios.

Keywords: capacitated vehicle routing problem, simultaneous pickup and delivery, capacity utilization, delivery priority, cluster and route, linear sweep algorithm

Introduction

Physical distribution is one of the key functions in logistics industry. It is a very costly function, especially for the distribution industries and is called as vehicle routing problem (VRP) in the operation research literature. This problem was firstly proposed by Dantzig and Ramser in 1959 as a generalized problem of travelling salesman problem and from then many hundreds of research works have been published on several variants of VRP. VRPs are found to be useful in many real world applications like goods distribution, mail delivery, school bus routing, garbage collection, private travel operations, courier service applications, newspaper distribution, etc,.

Even though VRP is classified into many variants, the most basic and important variants of VRP are capacitated VRP (CVRP), VRP with time windows (VRPTW), VRP with pickup and delivery (VRPPD) and Open VRP (OVRP). The problem taken up in this work is that of a private logistics service provider, who has several depots across the region with customers spread across all the depots. CVRP with simultaneous pickup and delivery between two depots with several nodes spread in between them is considered.

The remaining part of this paper is organized as follows: section 2 comprises a brief note on the variants of VRP with a review of the published work on CVRP and OVRP presented in section 3. Problem taken up in this work and the mathematical model developed are explained in section 4. The various assumptions made in this work are also listed. The following section 5 discusses the heuristic procedure developed for solving the problem and the results obtained are given in section 6. Finally the conclusion arrived at and the scope for future work, are presented.

VRP and its variants

In this section, the basic variants of VRP such as the classical VRP, VRPTW, CVRP, VRPPD and OVRP are described. Even though there are further sub divisions of VRP, they are all either combinations or modifications of the above variants and hence the description is limited to only the above said basic variants. The classical VRP is defined by Laporte [1] as follows:

Let G = (V, A) be a graph where V = {1,2,...,n) is a set of vertices representing cities with the depot located at vertex 1 and A is a set of arcs. With every arc (i, j): $i \neq j$ is associated a non-negative distance matrix $C = (c_{ij})$. c_{ij} can be interpreted as travel cost or travel time. When C is symmetrical, it is often convenient to replace A by a set of undirected edges. In classical VRP the customers are known in advance and also the driving time between the customers and the service times at each customer are used to be known. The VRP consists of designing a set of least cost vehicle routes in such a way that each city in V is visited exactly once by only one vehicle and all vehicle routes start and end at the depot with some more side constraints being satisfied.

Capacitated VRP:

The CVRP can be described as follows: Let G = (V, E) an undirected graph is given where V = {0,1,…….n} is the set of n+1 vertices and E is the set of edges. Vertex 0 represents the depot and the remaining of the vertex set V corresponds to n customers.

A non-negative cost d_{ij} is associated with each edge $\{i,j\} \in E$. The q_i units are supplied of from depot 0 (assuming $q_0 = 0$). A set of m identical vehicles of capacity Q is stationed at depot 0 and must be used to supply the customers. A route is defined as a least cost simple cycle of graph G passing through depot 0 and such that the total demand of vertices visited does not exceed vehicle capacity.

VRP with Time Windows:
The VRPTW is a generalization of the well-known VRP. It can be viewed as a combined vehicle routing and scheduling problem which often arises in many real world applications. It is to optimize the use of a fleet of vehicles that must make a number of stops to serve a set of customers, and to specify which customers should be served by each vehicle and in what order, to minimize cost, subject to vehicle capacity and service time restrictions. The problem involves assignment of vehicles to trips such that the assignment cost and the corresponding routing cost are minimal.

VRP with Pickup and Delivery:
The problems that need to be solved in real-life situations are usually much more complicated than the classical VRP. One important complication is that goods not only need to be brought from the depot to the customers, but also must be picked up at a number of customers and brought back to the depot. VRPPD may be further classified as VRP with backhauls, where the problem is sometimes divided onto two independent CVRPs; one for the delivery (linehaul) customers and one for the pickup (backhaul) customers, such that some vehicles would be designated to linehaul customers and others to backhaul customers, VRP with delivery in forward direction and pick up in reverse direction and VRP with simultaneous pickup and delivery.

Open VRP:
In cases where the vehicles are not required to return to the central depot after deliveries have been satisfied is referred to as the open vehicle routing problem (OVRP). It can be a slight variant of the standard CVRP model by simply removing the return arc to the central depot. The vehicle after servicing all the customers assigned to it need not return to the central depot. Therefore, the goal of OVRP is to design a set of Hamiltonian paths for satisfying customer demands.

Literature Review
The most basic and widely studied transportation model is the capacitated vehicle routing problem (CVRP) which as Li et al [2] suggest, is easy to state and difficult to solve. CVRP model consists of customer population with deterministic demand central depot with homogeneous fleet of vehicles. Aim of CVRP is to design a set of Hamiltonian cycles (routes) starting and terminating at the central depot, such that the demand of customers is totally satisfied with (i) each customer visited only by a single vehicle (ii) total demand of customers assigned to a vehicle does not exceed the capacity of the vehicle and (iii) overall travel cost of the designed route set is minimized.
There are wide variety of research works published in the VRP with pickup and delivery. An early survey of the VRP and its variants was published by Laporte [1], which served as a guide for many more research works that followed. A detailed survey of the VRP has been published by Parragh et al [3] and a survey of models and

algorithms has been published by Yeun et al [4]. A more recent survey on new mathematical models for the generalized VRP has been published by Pop et al [5].

A tabu search algorithm for solving VRP SPD was proposed by Montane and Galvao [6]. The algorithm was used to solve a set of 87 test problems with 50 to 400 clients. A cluster and search heuristic to solve the VRPPD was proposed by Ganesh and Narendran [7] to solve a problem in the collection of blood samples and distribution to hospitals from a blood bank. They proposed a multi-phase heuristic that clusters nodes based on proximity, orients them along a route using shrink-wrap algorithm and allots vehicles using generalized assignment procedure. Then they employed genetic algorithm for an intensive final search. In VRPPD, a multi depot VRP has been solved by Laporte et al [8], where intermediate replenishment of vehicles were carried out along the route. This method utilized a heuristic method that combined adaptive memory technique and tabu search technique.

Gajpal and Abad [9] used multi-ant colony system (MACS) to solve VRP with backhauls in which there are linehaul as well as backhaul customers. Zachariadis et al [10] developed an adaptive memory methodology to solve VRP with simultaneous pickups and deliveries. For a similar problem, Subramanian et al [11] presented a parallel heuristic for solving VRPSPD which was found to give better results for some benchmark problems from the already available literature. Sombuntham and Kachitvichayanukul [12] proposed a particle swarm optimization algorithm with multiple social learning structures for solving the practical case of multi-depot vehicle routing problem with simultaneous pickup and delivery and time window.

Zachariadis and Kiranoudis [13] proposed a local search metaheuristic algorithm for VRP with simultaneous pickups and deliveries. Kanthavel and Prasad [14] focused on maximum utilisation of loading capacity and determined the optimum set of vehicle routes for CVRP by a nested particle swarm optimization (NPSO) technique. The algorithm was implemented as master PSO and slave PSO for the identification of candidate list and route sequence in nested form to optimize the model. This NPSO algorithm developed the vehicle schedule without any local optimization technique. Pop et al [15] published a work proposing two new models of the generalized vehicle routing problem based on integer programming. The first model was called the node formulation and the second one was called flow formulation. Also it was shown that under specific circumstances, the proposed models reduced to the well known routing problems.

In all the research works stated above, the nodes or clients are supposed to be scattered around a single depot. The vehicle starts the trip from the depot and ends the trip again at the same depot. But in case of parcel service operators, they operate vehicles between two depots and try to serve the customers in between those two depots by designing various routes covering all the customers or nodes. Hence the existing model and algorithms may not exactly fit for solving this type of problem. A cluster and route approach with delivery first priority was developed [16], which produced encouraging results in terms of vehicle capacity utilization. But the distance travelled by the vehicles was higher and hence this work is proposed to minimize the distance.

Problem Definition

Our interest in the OVRP is motivated both by its theoretical and practical importance. In theoretical terms, the OVRP is a NP-hard combinatorial optimization problem: solving the OVRP to optimality implies that the best Hamiltonian path is obtained for each cluster of customers assigned to a vehicle. Since finding the best Hamiltonian path for a customer set is NP-hard, so is the OVRP [17]. From the commercial perspective, real-world distribution activities for any parcel service operations fit into the OVRP framework.

In case of logistics service providers, vehicles are operated between two central depots at opposite ends. Vehicles travel through the nodes, called as collection centres located between the two depots as shown in fig.1.

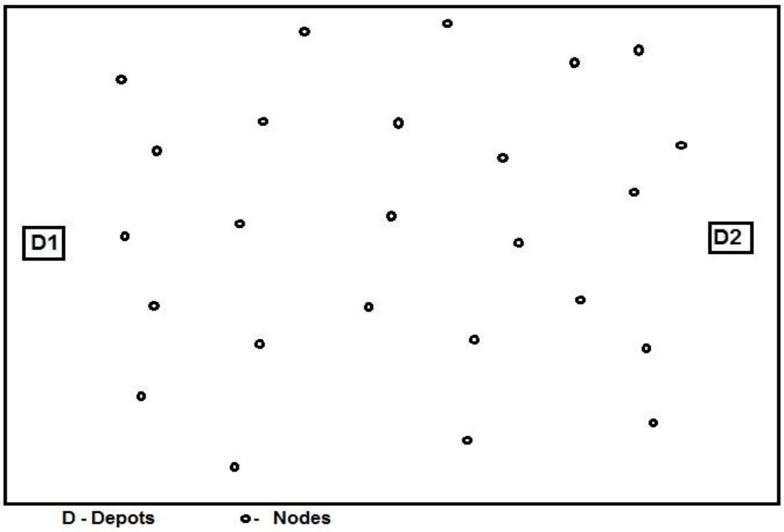

Figure 1: Illustration of nodes distributed between two depots located at opposite ends

Here each vehicle starts from the starting depot with goods assigned to various nodes. At each node, delivery is completed and goods for the second depot are picked up and leaves for the next node assigned. It travels through various nodes assigned to it and from the last node assigned moves to the second depot. In terms of VRP objective, most researchers assume that the cost for hiring an additional vehicle far surpasses any travel cost savings achieved by this additional route [18]. Therefore their primary target is to minimize the number of vehicles and secondarily minimize the total distance travelled.

In graph theoretic terms, OVRP is defined on a graph G = (V, A), where $V = \{v_0, v_1, \ldots v_n\}$ is the vertex set and $A = \{ (v_i, v_j) : v_i, v_j \in V, i \neq j, j \neq 0 \}$ is the arc set. Vertex v_0 and v_n represents the starting and ending depots. The remaining (n-1) vertices of $V \backslash \{v_0, v_n\}$ represent the node set or customer set. With each customer vertex is associated a non-negative known demand q_i, whereas with each (v_i, v_j) ∈ A is associated a cost C_{ij} which corresponds to the cost or travel time or distance for travelling from v_i to v_j.

The cost matrix can be obtained by calculating Euclidean distances between vertex pairs, so that $C_{ij} = C_{ji}$ ($0 < i, j \leq n, i \neq j$). In this work, actual distances between various nodes spread between two depots are considered. The same distance matrix is applied for both the heuristic approaches for consistency in comparing the results. The primary goal of the problem is to design a set of Hamiltonian paths (open routes) so that the size of the path set is minimized (minimization of vehicles), whereas, the secondary objective is to minimize the total cost of the generated paths.

The following constraints must be satisfied: (i) Every path originates from the first depot v0 and ends at the second depot vn (ii) Each customer vertex is assigned to a single path only and (iii) the total demand of the customer set assigned to a single path does not exceed the maximum carrying capacity Q of the vehicles (capacity constraint).The aim of this paper is to present a heuristic solution approach for the defined problem. The approach is modified sweep algorithm called as Linear Sweep Algorithm for assigning nodes to vehicles.

Assumptions made in this work:

i) The total quantity to be picked up from a node and quantity to be delivered to the node are known at the start of the trip.
ii) Homogeneous vehicles (vehicles with equal capacity) are used.
iii) Goods are also considered to be homogeneous.
iv) There is no time window for service at any nodes.
v) Cost of travel has not been considered.
vi) Total distance of travel for any vehicle is not limited.
vii) Goods are transported only between the depots and nodes and not between the nodes.
viii) One node is served by only one vehicle.

Mathematical Model:

Notations:

V: set of vehicles
N: set of nodes
n: no .of nodes
i,j: stop index of vehicle
C: vehicle capacity
P: pickup demand of node i
D: delivery demand of node i
d_{ij}: distance between nodes i and j

Decision Variables:

x_{ijk} = 1, if vehicle k visits j after i.

q_{ik}: the total cumulative load after visiting i

Objective Function:

Min $\sum_{i=0}^{n} \sum_{j=1}^{n+1} \sum_{k=1}^{v} d_{ij}\, x_{ijk}$ - 1

Subject to the following constraints:

Only one vehicle can visit a customer and only once

$\sum_{k=1}^{v} \sum_{i=0}^{n} x_{ijk} = 1$

Where j = 1,2,….n+1 - 2

Only one vehicle can leave a customer and only once

$\sum_{k=1}^{v} \sum_{j=1}^{n+1} x_{ijk} = 1$

Where i = 1,2,….n+1 - 3

Every vehicle leaves the starting depot once

$\sum_{j=1}^{n+1} x_{0jk} = 1$

Where k = 1,2,….,v - 4

Every vehicle enters the ending depot once

$\sum_{i=1}^{n} x_{i,n+1,k} = 1$

Where k = 1,2,….,v - 5

Every vehicle entering a customer node must leave the node

$\sum_{i \neq m}^{n} x_{imk} = \sum_{j \neq m}^{n+1} x_{mjk}$

Where k = 1,2,….v & m = 1,2,….n - 6

Total load after leaving node j should not exceed capacity

$q_{ik} + P_j - D_j \leq Q$

Where k = 1,2,….v & m = 1,2,….n - 7

Sub tour elimination constraint

$\sum_{i \in S} \sum_{j \in S(:i \neq j)} x_{ijk} \geq 1$ S ⊆ N and k = 1,2,….v - 8

All x_{ijk} are binary

$x_{ijk} \in \{0,1\}$ i=0,….n, j = 1,…n+1, k = 1,…v - 9

Linear Sweep Algorithm

In the traditional CVRP, the nodes are surrounding the central depot. The sweep algorithm as illustrated in Fig. 2 is applied as follows:

(i) A radial line with central depot as centre point starts form 0^o and sweeps through nodes surrounding the depot in either clockwise or counter-clockwise direction.

(ii) As the first node is encountered, it is assigned to the first vehicle and checked for the capacity constraint. If fit then the node is assigned as visited.

(iii) Then the line is swept again for getting the second node.

(iv) As the second node is assigned, the capacity constraint is applied first and if it passes through, it is assigned to the vehicle and the node is set as visited.

(v) The sweeping continues until the vehicle capacity is full or any constraint is over-ruled.

(vi) Then the procedure starts with vehicle number 2 for the remaining nodes.

(vii) The procedure is repeated until all the nodes are assigned to vehicles.

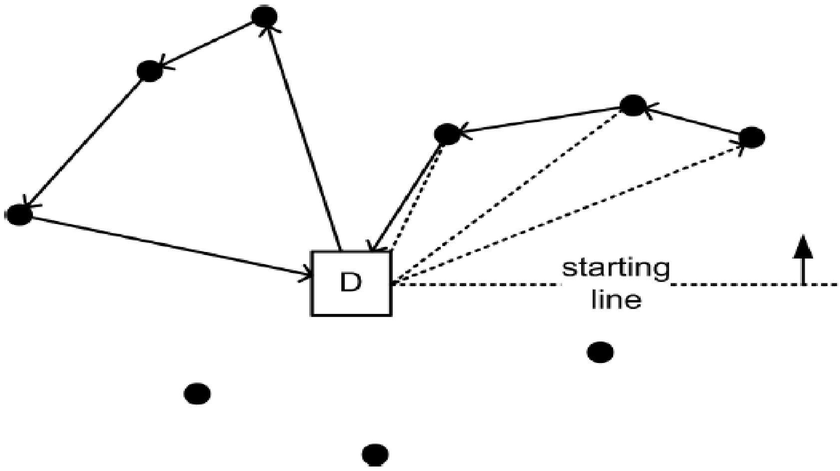

Figure 2: Illustration of Sweep Algorithm [Laporte 2004]

In our work, the sweep algorithm is modified as linear sweep algorithm. The nodes are distributed not surrounding one central depot, but between two depots at opposite ends as shown in fig.1. Hence it is not possible to adopt the same sweep algorithm as described earlier and needs modification. The nodes are arranged in a graph and are assigned with x and y coordinates. The node number is assigned based on the nearness to the first depot, so that the backtracking of vehicle can be avoided by arranging the assigned nodes based on the node number. Now a line sweeps from the bottom most node and assigns the node to first vehicle as illustrated in Fig. 3. The remaining procedure is as per the original sweep algorithm.

Improvement of Local Sweep Algorithm by Skipping Nodes:
In the linear sweep algorithm, the nodes are assigned as per the sorted order and once a constraint is violated, the allocation of nodes for a vehicle is stopped, even if it has capacity to utilize. Hence a improvement to the above algorithm is made by introducing a variable called skip. Here, if when a node is allocated to a vehicle and a constraint is violated, then the node is skipped and the next node is tried for allocation, so that the capacity utilization is maximized. But the node skipping cannot be done all the remaining nodes, because by skipping more nodes in the sorted order, the total distance travelled by the vehicle will drastically increase. Hence in this work it is proposed to set the skip variable to two nodes. For a single vehicle, only two nodes can be skipped for allocation. By this way, the deviation in the distance travelled by the vehicle is minimized.

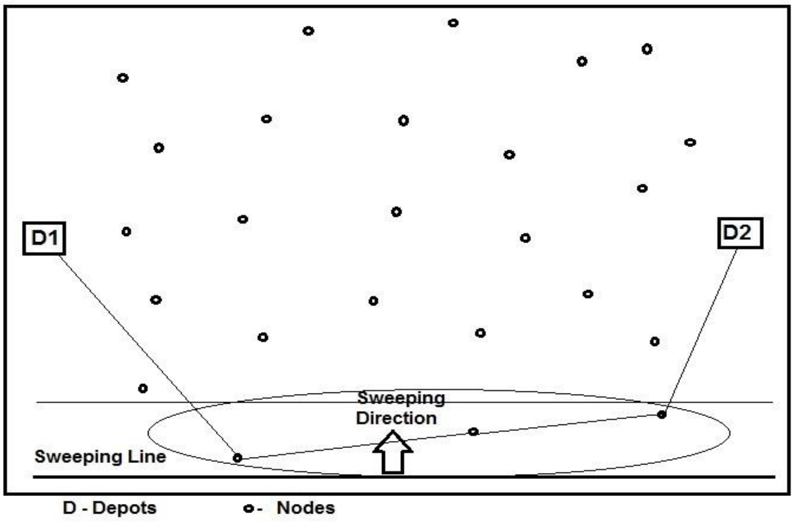

Figure 3: Illustration of Linear Sweep Algorithm

Results and Discussion

The problem taken up in this work consists of 25 nodes spread between two depots. In this route four different random data sets were created hypothetically. The linear sweep algorithm and sweep algorithm with node skipping were applied for the five instances of the route as per the procedure discussed in section 5. The number of vehicles utilized, routes developed for each vehicle, total distance travelled by each vehicle and the average capacity utilization of the vehicle for one instance of the problem using the two procedures is given in table 1 and 2 respectively. A comparision of the total number of vehicles utilized, average distance travelled by the vehicles and the average vehicle capacity utilization in the different instances of the route are given in table 3.

Table 1: Vehicle routes developed for one instance of Route 1 using sweep algorithm

Vehicle number	Route developed	Distance travelled (km)	Vehicle capacity utilized (%)
1	S-8-13-E	404	49.67
2	S-11-12-16-E	417	68.75
3	S-3-6-17-E	346	80.50
4	S-19-22-25-E	381	68.50
5	S-9-23-24-E	363	55.00
6	S-1-4-20-E	394	42.00
7	S-15-21-E	426	41.00
8	S-2-18-E	368	78.33
9	S-7-14-E	368	64.00
10	S-5-10-E	365	62.00

Table 2: Vehicle routes developed using sweep algorithm with node skipping for the same instance

Vehicle number	Route developed	Distance travelled (km)	Vehicle capacity utilized (%)
1	S-8-12-13-E	396	73.25
2	S-11-16-17-E	381	65.75
3	S-3-6-22-E	341	81.50
4	S-19-24-25-E	386	73.00
5	S-1-9-20-23-E	401	59.20
6	S-4-15-21-E	403	55.00
7	S-7-14-E	368	64.00
8	S-2-18-E	368	78.33
9	S-5-10-E	365	62.00

Table 3: Consolidated result of various instances of Route 1

Case No.	No. of vehicles utilized	Average distance travelled (km)	Average vehicle capacity utilized (%)	No. of vehicles utilized	Average distance travelled (km)	Average vehicle capacity utilized (%)
Using sweep algorithm				Using sweep algorithm with node skipping		
C1	10	383.2	60.98	9	378.78	68.00
C2	8	378.38	72.57	8	381	75.12
C3	9	382.22	61.30	9	382.33	61.90
C4	10	386.4	59.35	10	379.90	63.25

Conclusion

Generally parcel service operators adopt fixed route policy for pickup and delivery of goods to their customers which is resulting in loss by way of utilizing more vehicles. Also the vehicles are under-utilized in some routes, whereas some routes require additional vehicles to meet the demand. To solve this problem, a heuristic approach based on linear sweep algorithm has been developed in this work which has successfully maximized the average utilization of vehicles and the distance travelled is also found to be optimum. Also this gives rise to a combination of VRP such as VRP with simultaneous pickup and delivery and Capacitated VRP. The inter-nodal transit is also not considered in this work, which when considered will add complexity to the problem. Also the above heuristics are to be tested for larger data sets of routes having nodes up to 200.

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