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Stability of Relative Efficiency in DEA of Life Insurers and Takaful Operators

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Abstract

It is essential to test the stability of the efficient frontier and the reliability of efficiency scores obtained from DEA. This becomes more significant if the results will be used to make management decisions or to enhance the efficiency of the firm. Therefore, this study will perform several tests to ensure the stability of the relative efficiency obtained from the DEA. These tests are demonstrated on DEA efficiency scores of risk and investment management function of life insurers and takaful operators. Several stability tests performed in this study on the illustrative data show a stable efficient frontier. The test also indicated that the efficiency score is reliable in discriminating between efficient and inefficient decision making units (DMUs). Therefore it can be concluded that the DEA model used is appropriate in furnishing a comprehensive guide towards the best practices that other firms might adopt and worst practices that other firms should avoid. In turn, the managerial decision-making can be made with more confidence.

Keywords: Stability, Data Envelopment Analysis (DEA), efficiency, risk management, investment management, takaful

Introduction

Data envelopment analysis (DEA) is the non-parametric approach which is completely data-based frontier efficiency methodology in which it does not require specific functional form and no accommodation of noise or error term. Chronologically, DEA arose from Farrell's (1957) measurement of relative technical and allocative efficiency which is strongly influenced by Koopmans (1951) and Debreu (1951) which has been formulated in a mathematical programming framework first introduced by Charnes, Cooper and Rhodes (1978). Later, due to the vulnerabilities in the existing DEA models and adaptability to the problem in hand, the DEA has been enhanced by Banker, Charnes and Cooper (1984) and other researchers such as Byrnes, Fare and Grosskopf (1984) and, Thiry and Tulkens (1992). The centre attention of DEA is largely on the technological aspects of production correspondences, thus it can be applied to calculate technical and scale efficiency without requiring estimates of input and output prices. On the other hand, if the data on input prices are available, cost efficiency also can be measured by using DEA (Aly, Grabowsky, Pasurka and Rangan, 1990; Ferrier and Lovell, 1990). Cummins and Weiss (2000) write, “Intuitively, the method involves searching for a convex combination of firms in the industry that dominate a given firm”. They further explained that these firms form the given firm’s reference set, and if the reference set comprises only of the firm itself, it is said self-efficient and has efficiency score equal to 1. Conversely, if other firms instituted the dominant set, then the firm’s efficiency is less than 1 which is deviated from the frontier and considered as inefficient.

The frontier efficiency methodologies seemed very important and this new benchmarking techniques measured the firm performance relative to best practice frontiers derived from firms in the industry or branches within financial firms (Berger and Humphrey, 1997; Cummins, 1999). The advantage of such measures, as compared to financial ratio analysis, is their ability to summarize firm performance in a single statistic that controls for differences among firms using a sophisticated multidimensional framework (Cummins, 1999). Moreover, all economic hypotheses (related to insurers) about such matters as economies of scope and scale, distribution systems, organizational forms and the effect of merger and acquisitions will not be convincing unless they applied the frontier-based performance measures (Cummins and Weiss, 2000). With these platforms, several attempts have been made to study the efficiency of various sectors in economy by adopting this non-parametric methodologies including agricultural and industrial as well as service sectors.

Indeed, there is no doubt that the application of DEA has been extensively used in efficiency studies since it was first developed by Charnes et al. (1978), however, if the results obtained were to be used for making a management decision or would like to improve the performance of firms alleged to be less efficient, precautions should be taken. Avkiran (2007) clarified that the stability and reliability (integrity and sensitivity are also used interchangeably) analysis is very important to strengthen the results of DEA. These analyses had to be performed in order to minimize the possibility of making managerial decisions based on inaccurate and unreliable findings. Recognizing the importance of the stability and reliability test of the efficient frontier and efficient decision making units (DMUs) derived from DEA, yet most of the efficiency studies still overlooked to perform the test. Therefore, on these grounds, this study recognized several stability and reliability tests in strengthening the results of DEA. This study contributes in terms of providing step-by-step procedures for performing stability analysis. In addition, this study adds to the growing literature on the stability analysis of DEA results.

The remainder of this study proceeds as follows: in section 2 discusses the relevant literature reviews, section 3 illustrates the methodology; section 4 describes the data and variables used; 5 discusses the experimental results; and finally, the conclusion is provided in section 6, followed by some useful references.

Literature review

The issue of stability or sensitivity of the efficient frontier and efficient DMUs obtained from DEA has long been debated by previous researchers. A comprehensive study of the recent developments on stability and sensitivity analysis which is limited to data perturbation can be obtained in the study by Cooper et al. (2001). Based on this study, among the first research to highlight this issue is Charnes et al. (1985) when they found that the sensitivity analysis used in the linear program is not compatible with DEA. Therefore, they proposed a new algorithm to test the sensitivity of the data variation of Charnes-Cooper-Rhodes (CCR) model in DEA. Their study was restricted to changes in a single output for a particular DMU. In addition, their study also provided a "locate ranges of variation" of efficient DMU (Charnes et al., 1985, p. 140) – that is on how much variation is possible but still maintain the attained efficiency rating. Similarly, appropriate conditions preserving efficiency are determined in Charnes and Neralic (1990). In contrast to Charnes et al.(1985), Charnes and Neralic (1990) studied the sensitivity of the DEA additive model. However, both studies dealt with "data changes via updating the inverse of the optimal basis matrix" (Zhu, 1996, p. 451).

Sensitivity analysis is further extended to variations in both inputs and outputs for a particular DMU and the relevant studies are Charnes et al. (1992, 1996) and Zhu (1996). Charnes et al. (1992, 1996) had introduced the concept of metric that allows variation in all inputs and outputs for one DMU. On the other hand, Zhu (1996) focused on upward and downward variations of inputs and outputs of "an efficienct DMU preserving efficiency" (p. 451). According to his study, the DMU is absolutely efficient when it immunes to a certain increase in input, or a certain decrease in an output.Super-efficiency method was developed as a sensitivity analysis technique by Charnes et al. (1992), Rousseau and Semple (1995) and Charnes et al. (1996) in a case of simultaneous changes in all inputs and outputs for a particular DMU. In this method the test DMU is removed from the reference set (Andersen and Petersen, 1993).

Later, the concept and methods of sensitivity analysis was developed further by allowing the simultaneous variation in all inputs and outputs for all DMUs. This assumption is claimed to be more realistic since each DMU is possibly exposed to data changes/errors. By using the super-efficiency-based approach, Seiford and Zhu (1998) assumed the same data changes in the test DMU and the remaining DMUs in order to determine the sufficient conditions preserving extreme-efficiency DMU's efficiency. Note that Thompson et al. (1994) employed Strong Complementary Slackness Condition (SCSC) multipliers in order to examine the stability of CCR model by allowing greater data variations (in opposite directions and same percentages) compared to Seiford and Zhu (1998). Comparing both approaches, super-efficiency-based approach "may generate a larger stability region than the SCSC method does" (Zhu, 2001, p. 444). Zhu (2001), however, applied various super-efficiency models to analyse DEA efficiency classification. He claimed that the results from this approach are stable and unique.

Another type of DEA sensitivity test is dealt with model changes, variable selection (omitting an input or output) or changes to the number of DMUs, e.g., Ahn and

Seiford (1993), Smith (1997), Simar and Wilson (1998) and Ramanathan (2003). The sensitivity analysis which involves the selection of variables and changes in the number of DMU has been adopted by Avkiran (2007). Additionally, Avkiran (2007) also conducted an integrity analysis to verify the classification ability of efficiency scores. She applied the stability and integrity analysis to SBM model in DEA, which has not been done by previous researchers. Tyagi et al. (2009) performed the sensitivity analysis as suggested by Avkiran (2007) in assessing the performance of academic performance in India. In contrast to previous study, Jahanshahloo et al. (2011) focused on the sensitivity analysis of inefficient DMUs. They managed to develop an efficient region for inefficient DMUs that is known as “Necessary Change Region”.

Methodology

Stability Test:

The stability of a frontier refers to the ability to produce the same efficient DMUs despite to data perturbations in the form of addition or deletion of the variables or units (Avkiran, 2006). Several independent tests will be performed for this purpose and will be divided as follows:

Degrees of freedom:

The DEA methodology also is not spared from dealing with the problem of degree of freedom as other statistics-oriented methodologies. In this model, degree of freedom will increase with the number of DMUs and decrease with the number of inputs and outputs. A few rules of thumb have been proposed in previous studies to select the number of DMUs, inputs and outputs in order to satisfy the condition of degree of freedom. Cooper, Li, Seiford and Zu (2004) suggested that the value of $n \geq \max\{m \times s, 3(m+s)\}$, is sufficient for DEA to be used, where n is the number of DMUs, m is the number of inputs and s is the number of outputs. These guidelines were in line with what has been agreed by DEA convention where the minimum number of DMUs is greater than 3 times the number of inputs plus outputs (Barros, Nektarios and Assaf, 2010). In addition, Taluri (2000) stated that in adopting DEA as a technique to measure efficiency, for 5 inputs and 5 outputs, at least 25 or so units will appear efficient and, thus, the data set needs to be greater than 25 for any discrimination.

Removing of inputs and outputs:

Initially, this test is focusing on removing the input/output independently and examining its impact on efficient frontier membership (Avkiran, 2006). The input/output that will be removed is selected randomly. The removed input/output will be returned to the sample before the next removal in order to have the same degree of freedom. Thus, at one time, there are two sets of samples that will be compared for the efficient frontier membership-a set that contains complete inputs/outputs which is referred by Avkiran (2006, p. 228) as "full-complement" sample and a set that contains reduced inputs/outputs. The efficient frontier is said to be stable if no new

DMUs are classified as efficient on the frontier from the reduced input/output set– meaning that the full-complement set and the reduced input/output set, both maintain the same efficient DMUs, although at a different number.

The second focus of this test is to examine the effect of removing the input/output on rankings. If the deletion of input/output has the impact on ranking, differently stated that, a ranking produced by the reduced input/output set is different from the ranking produced by the full-complement set, thus, it is most likely that the efficient frontier is not stable. The Spearman's rank correlation is applied to measure the degree of correspondence between the ranks from these 2 sets (Avkiran, 2006). The null hypothesis states that the ranks based on the full-complement set is different from the ranks based on the reduced input/output set in which it is said that there is no correlation. It is highly expected to find support for the alternative hypothesis.

Removing of DMUs:

This procedure is based on a study by Avkiran (2006) in which the top one-third efficient insurers in each year of both activities are deleted. Again, this test is focusing on removing the DMUs and examining its impact on efficient frontier membership. In addition, this approach is to moderate the disruption to the efficiency frontier where the number of efficient firms may vary. The efficient frontier is considered stable if and only if any insurer that is listed as efficient from truncated samples also efficient in the full-complement samples.

Integrity Test:

The integrity (reliability) test is performed to ensure that the efficiency score obtained can discriminate between the efficient and inefficient DMUs. One method used is to examine the efficient DMUs of those listed by the DEA analysis should have output to input ratio higher than inefficient DMUs (Avkiran, 2006). As commonly known, output to input ratio is used measure of efficiency (Cooper, Seiford and Tone, 2007).

Data

DMU :

According to Thanassoulis (2001), DMU has a control over the process it used to transform its resources into outcomes. The DMU observed in this study are the risk and investment management function of insurers/takaful operators. The illustrative data used in this study consists of 20 players in the insurance and takaful industry that consistently present for the period 2003 to 2007. These include 7 conventional life insurers, 9 conventional composite insurers and 4 takaful operators. The selection of the firms is restricted to direct insurers (composite and life) operated in Malaysia and takaful operators. Moreover, data for this study is limited to life and family takaful business as well as investment-linked business. For the composite insurers, which offer general and life products, the data is segregated between the two lines of business and can be obtained from the companies' financial report.

The efficient frontier and efficiency score tested refer to the efficiency of the risk and investment management function. The efficiency scores are obtained from slack based measure (SBM) model. The SBM model is a variant of the additive DEA model, which was first presented by Tone (2001). As in the additive model, the SBM differs from the CCR and BCC model as it combines both orientations in a single model, i.e. input-oriented model and output-oriented model. SBM focuses on maximizing the non-zero slacks in the optimal objective. The slacks give the estimate of input excess and output shortfalls that could be improved without worsening any other input and output. SBM is conducted separately in order to derive the efficient frontier of risk and investment management for each year. The mathematical form of SBM is given in Appendix A.

Selection of inputs and outputs:

The input and output variables must be related to the function of both risk and investment management.In terms of inputs, it seems that the inputs that are commonly used in previous studies such as labour, business services and material, and financial capital may be less appropriate because these inputs are more applicable if the insurer itself is the observed DMU. However, in terms of output, it is likely that value-added or intermediation approach can still be applied since the outcomes of both risk and investment management should be consistent with the outcomes of the insurers/takaful operators as a whole.

Risk management inputs:

Insurers/takaful operators assumed various kind of actuarial and financial risk. They must be very careful about their risk profiles and address them in their management control framework because it associates with the performance improvement (Doff, 2007). Hence, an efficient risk management is seen as an important requirement in reducing the exposure to risk by handling the amount of risk accepted in a better way. This would imply that the resources or input of risk management is the risk itself (Ren, 2007). Babbel and Santomero (1999) viewed that there were no risk classification schemes to be just right and accurate, but most of the risks affecting the insurer had been added to the lists. In the context of this study, the combination of risk classification between Doff (2007) and the study by Ren (2007) will be applied which ultimately considers 3 type of risks that are very significant to insurers/takaful operators namely investment risk, underwriting risk and leverage.

Risk management outputs:

In determining the output, it is very important to know the services that are relevant to risk management. With a knowledge that risk management is a key function of insurers/takaful operators, the outcomes to be achieved in the risk management must be able to describe the overall outcome of insurers/takaful operators. Perhaps the value-added approach is more suitable to be applied in determining the output of risk management. Based on this approach, the selected output must be able to describe 3 main services provided by insurers/takaful operators namely risk pooling/bearing, real financial services and intermediation. Following prior researches with value-added approach such as Cummins and Zi (1997), Eling and Luhnen (2010) and Laverty and Grace (2010), net incurred benefits plus addition to reserve is treated as outputs for the risk management in this study. The inputs and outputs of risk management are summarized in Table 1.

Table 1: Inputs and output of risk management

Input	Output
Investment risk Underwriting risk Leverage	Net incurred benefit plus reserves

Investment management inputs:

The successful operation of the insurer and its relationship with customers is significantly depending on the investment management (Black and Skipper, 2000). Insurer will use their technical provisions (reserves) and equity capital for investment purposes. However, reserves are the largest source of investment funds in which it sometimes reaches more than 80% (Black and Skipper, 2000). Thus, the first input of investment management is what is known as net actuarial reserves. Continuing on the same notes, the final input for investment management performance analysis is total investment assets. Insurance firms place their investment in a variety of instruments including equity and debt issues or bonds, mortgages, loans, government securities and real estates. However, because of the unique nature of life insurance firm's operation and the resulting risk profiles, the majority of life insurance firms' assets comprise fixed-income investments (Black and Skipper, 2000) which include Government securities, corporate bonds, mortgages and private loans.

Investment management outputs:

It seems that investment management functions fulfil the pure intermediaries' function of insurers. Therefore, the choice of outputs for investment management function is following the intermediation approach. Based on the work by Brockett et al. (2005) together with Wu et al. (2007) and Ren (2007) as well as Yang (2006), the objectives or targets of the intermediation functions of insurers/takaful operators that is solvency and profitability can be treated as the output variables. Thus, in this study, the solvency measurement is represented by the solvency score obtained from Total Financial Index (Hsiao, 2005), while profitability is represented by rate of return on investments (Brockett et al., 2005; 2004). The inputs and outputs for investment management are described in Table 2.

Table 2: Inputs and outputs of investment management

Input	Output
Actuarial reserves Total investment assets	Solvency score Profitability

Results

Outlier inspection:

Since outliers have a major impact on other DMU scores that are derived from the efficient frontier technique such as DEA (Avkiran, 2007), one must examine the presence of outliers before the analysis can be preceded. He suggested that the problem of outliers has to be resolved so that DEA results obtained are reliable. One proposed method for outlier detection is to use super-efficiency analysis. The super efficiency model has been introduced by Andersen and Petersen (1993) which allows the DMU to achieve the efficiency scores greater than one. Therefore, the efficient DMUs are no longer tied with the efficiency score equal to one, but in fact can be ranked among the efficient units. With this advantage, the super-efficiency model has been used for several purposes such as "sensitivity testing, identification of outliers, and as a method of circumventing the bounded-range problem...", (Coelli et al., 2005, p. 201). To identify the outliers and DMUs which are not belonged to the data set, Hartman et al. (2001) claimed that DMUs with super-efficiency scores of 2 or 3 above were scrutinized as potential outliers because it is considered as having an excessive impact on the efficient frontier. However, Avkiran (2007) imposed more stringent rules where firm with efficiency score of 2 or above were treated as potential outliers. Therefore, based on this background, this study will adopt a super-efficiency method for identifying outliers.

Table 3 is the final result of several runs of a SBM of efficiency with super efficiency scores of risk management data. For data years 2003, 2004, 2006 and 2007, it reveals no further outliers after second runs of SBM, while for data year 2005, no further outliers are detected only after the third run. Two insurers/takaful operators fall into this category namely insurer G and S (2003); R and S (2004); F and T (2006); H and P (2007). Meanwhile there are 5 insurers/takaful operators which are considered as outliers in 2005 i.e. insurer/takaful operator B, C, G, R and T. Thing that seems clear is that these outliers have a very small investment and underwriting risk. Thus, it can be concluded that the exposure to investment and underwriting risk of these companies are very low. Because of this, it is desirable to remove these companies from the existing data set in order to maintain sample homogeneity. This leaves the final numbers of insurers/takaful operators to be taken into account for the year 2003-2007 are 18, 18, 15, 18, and 18, respectively.

Table 3: Outlier inspection-super-efficiency score of insurers/takaful operators for risk management data

2003	Score	2004	Score	2005	Score	2006	Score	2007	Score
G	**148.88**	**S**	**2059.73**	**B**	**570.40**	**T**	**2.41**	**H**	**8.89**
S	**18.95**	**R**	**3.36**	**T**	**417.07**	**F**	**2.94**	**P**	**2.05**
R	1.69	N	1.31	**G**	**9.03**	R	1.67	K	1.11
K	1.48	P	1.30	**R**	**2.63**	E	1.12	D	0.61
P	1.33	L	1.16	**C**	**2.52**	P	1.10	E	0.58
A	0.65	K	1.11	F	1.23	K	1.10	G	0.42
N	0.63	J	0.56	P	1.15	L	1.06	A	0.38

I	0.56	A	0.48	N	1.13	N	1.01	Q	0.37
E	0.56	I	0.42	E	1.11	D	1.01	N	0.37
D	0.51	E	0.39	K	1.06	G	0.86	B	0.37
B	0.16	C	0.38	I	0.85	A	0.61	R	0.34
J	0.41	D	0.37	S	0.81	B	0.60	S	0.34
H	0.38	F	0.36	D	0.46	I	0.56	L	0.33
L	0.37	Q	0.36	L	0.43	S	0.53	J	0.32
F	0.34	H	0.32	H	0.39	J	0.39	I	0.32
M	0.23	G	0.29	J	0.38	M	0.35	F	0.27
Q	0.13	B	0.28	Q	0.34	C	0.34	M	0.26
C	0.08	M	0.23	M	0.30	O	0.26	C	0.20
T	0.07	T	0.22	A	0.30	Q	0.25	O	0.19
O	0.03	O	0.16	O	0.17	H	0.13	T	0.14

Note: Outliers are in bold.

It is rather surprising that no outlier is detected for the investment management data for years 2003-2007. This can be demonstrated by the results reported in Table 4 in which all super-efficiency score indicates a value of less than 2. Accordingly, the final number of insurers/takaful operators for the investment management data is remain the same i.e. 20 companies. Further analyses include SBM, stability and integrity tests will only deal with data sets that do not contain outliers for both the risk and investment management.

Table 4: Outlier inspection-super-efficiency score of insurers/takaful operators for investment management data

2003	Score	2004	Score	2005	Score	2006	Score	2007	Score
O	1.93	O	1.56	M	1.17	M	1.14	Q	1.30
L	1.15	M	1.08	O	1.13	H	1.12	M	1.26
I	1.09	I	1.00	Q	1.07	E	1.12	E	1.14
M	0.89	F	0.69	F	1.03	F	0.90	D	1.00
B	0.67	G	0.68	D	1.02	Q	0.88	K	1.00
F	0.63	Q	0.61	L	1.01	T	0.81	F	0.91
A	0.60	A	0.58	E	0.79	R	0.69	T	0.80
R	0.51	R	0.55	T	0.74	G	0.56	R	0.78
D	0.50	T	0.52	R	0.74	K	0.56	C	0.75
J	0.46	D	0.49	I	0.66	J	0.51	P	0.68
G	0.38	E	0.40	J	0.63	A	0.50	I	0.65
S	0.35	S	0.39	C	0.63	C	0.50	G	0.64
C	0.34	J	0.37	S	0.60	D	0.47	J	0.57
T	0.29	B	0.34	P	0.58	S	0.47	A	0.56
E	0.28	P	0.34	B	0.52	I	0.45	S	0.55
K	0.26	H	0.32	A	0.52	P	0.44	L	0.50
Q	0.25	L	0.29	H	0.52	L	0.41	O	0.48
H	0.22	C	0.27	G	0.52	N	0.37	N	0.47

P	0.17	K	0.27	K	0.50	B	0.37	B	0.37
N	0.16	N	0.19	N	0.42	O	0.33	H	0.28

SBM relative efficiency score:
Generally, the efficiency of risk and investment management is achieved by different insurers/takaful operators for each year from 2003-2007. According to Table 5, for risk management, it is obvious that insurer K is the only insurer that has been on the frontier for the entire years. In addition, the performance of insurer N and P are also encouraging as for achieving efficient risk management for 4 times. Apart from insurers H, J, M, and O, other insurers/takaful operators are enjoying efficient risk management at least for one year.

Likewise, the distribution of insurers/takaful operators that are efficient in terms of investment management efficiency is also not the same throughout the years (see Table 6). In fact, none of the insurers/takaful operators is seen to preserve efficiency for the 5 consecutive years. However, among the insurers/takaful operators that are having efficient investment management, insurer M and O is the most prominent one because they have been on the frontier for 4 and 3 times respectively. Insurers D, E, F, H, I, K, L and Q also present efficient investment management at least once in 5 years time, while another 10 insurers namely insurers A, B, C, G, J, N, P, R, S and T experience inefficient investment management.

Overall, inefficiencies in risk management are mostly caused by the failure to manage all three inputs at optimum level. In the context of this study, it was found that insurers/takaful operators have dealt with excessive leverage, underwriting and investment risk. In terms of investment management, inefficiency is caused by shortage of the second output that is solvency at the required level. Moreover, the heterogeneity or dispersion of both the risk and investment management efficiency declined during the period of 2003-2007. This is particularly encouraging because it shows that the insurers/takaful operators are converging towards the best practices (Cummins, 1999). However, the decreasing rate is quite slow and this condition is reasonable because there are some insurers that show very low efficiency score of risk and investment management. This indicates that the insurers/takaful operators are most likely not put an enough effort to compete intensively with each other to achieve efficient risk and investment management.

Table 5: SBM results for risk management for individual insurer/takaful operator

2003		2004		2005		2006		2007	
DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score
A	0.6496	A	1.0000	A	0.2960	A	0.6072	A	1.0000
B	0.4815	B	1.0000	D	0.4650	B	0.5993	B	1.0000
C	1.0000	C	0.9334	E	1.0000	C	0.4911	C	0.5040
D	0.5051	D	0.3747	F	1.0000	D	1.0000	D	1.0000
E	0.5842	E	0.3942	H	0.3895	E	1.0000	E	0.7466

F	0.4606	F	0.3727	I	0.8498	G	1.0000	F	0.6337
H	0.3765	G	1.0000	J	0.3824	H	0.1340	G	1.0000
I	1.0000	H	0.3175	K	1.0000	I	0.6005	I	1.0000
J	0.8292	I	0.4200	L	0.4299	J	0.3883	J	0.6015
K	1.0000	J	0.5967	M	0.2994	K	1.0000	K	1.0000
L	0.3739	K	1.0000	N	1.0000	L	1.0000	L	0.5634
M	0.2256	L	1.0000	O	0.1674	M	0.3472	M	0.5315
N	0.6349	M	0.2279	P	1.0000	N	1.0000	N	1.0000
O	0.0312	N	1.0000	Q	0.3416	O	0.2600	O	0.2379
P	1.0000	O	0.1617	S	0.8144	P	1.0000	Q	1.0000
Q	0.1926	P	1.0000			Q	0.2545	R	0.7621
R	1.0000	Q	0.3569			R	1.0000	S	1.0000
T	0.3911	T	1.0000			S	0.7008	T	0.4453

Table 6: SBM results for investment management for individual insurer/takaful operator

2003		2004		2005		2006		2007	
DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score	DMU	Efficiency Score
A	0.5984	A	0.5771	A	0.5213	A	0.4997	A	0.5578
B	0.6718	B	0.3412	B	0.5232	B	0.3669	B	0.3670
C	0.3390	C	0.2733	C	0.6268	C	0.4978	C	0.7488
D	0.5018	D	0.4877	D	1.0000	D	0.4748	D	1.0000
E	0.2789	E	0.4005	E	0.7904	E	1.0000	E	1.0000
F	0.6312	F	0.6899	F	1.0000	F	0.9001	F	0.9064
G	0.3801	G	0.6762	G	0.5198	G	0.5632	G	0.6359
H	0.2205	H	0.3235	H	0.5202	H	1.0000	H	0.2804
I	1.0000	I	1.0000	I	0.6645	I	0.4528	I	0.6469
J	0.4614	J	0.3672	J	0.6329	J	0.5144	J	0.5730
K	0.2615	K	0.2704	K	0.5007	K	0.5570	K	1.0000
L	1.0000	L	0.2919	L	1.0000	L	0.4079	L	0.4967
M	0.8935	M	1.0000	M	1.0000	M	1.0000	M	1.0000
N	0.1576	N	0.1870	N	0.4188	N	0.3691	N	0.4668
O	1.0000	O	1.0000	O	1.0000	O	0.3349	O	0.4820
P	0.1724	P	0.3377	P	0.5781	P	0.4420	P	0.6796
Q	0.2532	Q	0.6091	Q	1.0000	Q	0.8806	Q	1.0000
R	0.5116	R	0.5500	R	0.7408	R	0.6892	R	0.7758
S	0.3490	S	0.3939	S	0.6019	S	0.4692	S	0.5509
T	0.2945	T	0.5160	T	0.7429	T	0.8098	T	0.8046

Stability and integrity test results:

The results from stability and integrity test provide further confidence and support to the use of SBM model and the efficient frontier produced. The results from 3 stability test conducted are shown in Table 7, 8 and 9. In the case of this study, the input and output variables used in both activities i.e. risk management and investment management are (3,1) and (2,2) respectively. Table 7reveals that the number of DMUs involved each year for both activities is consistent with the benchmark by Cooper et al. (2004), DEA convention and Talluri (2000) and therefore the degrees of freedom conditions are satisfied.

Table 7: Stability test – degree of freedom: Number of DMUs involved[a]

Year	Risk Management(m=3;s=1) $n \geq max\{3,12\} = n \geq 12$	Investment Management (m=2;s=2) $n \geq max\{4,12\} = n \geq 12$
2003	18	20
2004	18	20
2005	15	20
2006	18	20
2007	18	20

Note: [a]$n \geq max\{m \times s, 3(m+s)\}$,

In this study, there are 3 inputs and 1 output for risk management and 2 inputs and 2 outputs for investment management. For risk management, only the removal of input can be done, no removal of output as there is only 1 output involved. Because of this, 2 inputs will be removed independently. However, unlike the investment management activity, the removal of both input and output can be done independently. This study has arbitrary chosen the 'leverage' and 'investment risk' (for risk management activity) and 'total investment' and 'investment return' (for investment management activity) to be removed independently from the full complement set. The SBM-DEA is applied to both sets (full-complement and reduced input/output set) to identify the efficient frontier membership of each activity.Table 8 lists the efficient frontier membership from the full-complement and the reduced input/output set for risk and investment management activities.

Table 8: Stability test: The efficient frontier membership after the removal of input/output

Panel A: Risk Management Activity														
[a]FC03	W/O LEV03	W/O IR03	FC04	W/O LEV04	W/O IR04	FC05	W/O LEV05	W/O IR05	FC06	W/O LEV06	W/O IR06	FC07	W/O LEV07	W/O IR07
R	R	P	T	T	P	P	P	P	R	R	K	S	S	K
P	I	I	A	A	N	N	F	K	P	P	E	A	A	G
K	C		B	B		K		E	N	G	D	B	I	D
C			P			E			L	E		Q	G	
I			N			F			D			D		
			L						E			N		
			K						G			K		

			G						K			G		
												I		
Panel B: Investment Management Activity														
FC03	W/O TI03	W/O IRn03	FC04	W/O TI04	W/O IRn04	FC05	W/O TI05	W/O IRn05	FC06	W/O TI06	W/O IRn06	FC07	W/O TI07	W/O IRn07
T	T	Q	T	T	O	O	O	O	O	O	O	P	P	O
Q	Q		O	O		L	L		K	H	H	O	O	
L			M			D			H	E		K		
J			K						E					
D														
I														

Note: [a]FC stands for full-complement set; Second and third column headings of each activity refer to the reduced input/output without LEV (leverage), IR (investment risk), TI (total investment), and IRn (investment return)

It appears that an insurer that is classified as efficient in the reduced input/output set is also efficient in the full-complement set throughout the study period. Nevertheless, there are wide disparities in the number of efficient insurers between the full-complement and reduced input/output set in which the number of efficient insurers from reduced input/output set is less than full-complement set. This reduction is due “to the rise in degrees of freedom compared to that of the full-complement model and, thus, the sharper discrimination it brings to the analysis regarding efficient and inefficient banks” (Avkiran, 2007, p. 229). From this result, it can be concluded that no new DMUs are classified as efficient on the frontier from the reduced input/output set – thus, it is most likely that the efficient frontier is stable. These observations were further strengthened by the result of the Spearman rank correlation that is applied to the whole sample of 2003-2007 between the full-complement set and the 2 reduced input/output set of both activities. For the risk management activity, when the inputs i.e. leverage and investment risk is removed independently, ranks are significantly correlated at 0.791 and 0.690 (with a two-tailed significance of 0.000) respectively. Meanwhile, for investment management activity, when the input i.e. total investment asset and the output i.e. investment return are removed independently from the full-complement set, Spearman rank correlation coefficients shows the value of 0.794 and 0.713 with a two-tailed significance of 0.000 respectively. Therefore, the null hypotheses are rejected at 1% significance level for both activities. The results obtained indicates that the ranking from the full-complement set and reduced input/output set for the risk and investment activities are the same and hence, it can be said that the efficient frontier is stable.

Results, as shown in Table 9, represents that any insurer that is listed as efficient from truncated samples also efficient in the full-complement samples (true along the sample period for both risk and investment activities) and this situation also prove that the efficient frontier is stable. For instance, for the year 2007, insurer D, F, G, I, K, N and Q are in both samples of risk management activities. The same thing goes for the investment activity in which insurer D, E, K, M and O can be found in both samples.

However, generally, the efficient insurers are differently ranked in both samples and also the proportion of efficient insurers in truncated sample has been increased. Avkiran (2007) said that this situation is due to the reduction in the sample size and the loss of ability to discriminate.

Table 9: Stability test: The efficient frontier membership after the top one-third of efficient insurers is deleted

Panel A: Risk Management Activity									
[a]FC03	Trunc03	FC04	Trunc04	FC05	Trunc05	FC06	Trunc06	FC07	Trunc07
R	P	T	P	P	N	R	S	S	Q
P	A	A	B	N	K	P	A	A	N
K	K	B	N	K	F	N	B	B	D
C	C	P	L	E	E	L	N	Q	K
I	J	N	K	F		D	D	D	F
	I	M	G			E	E	N	G
		K				G	G	K	I
		G				K	L	G	
							I	I	
							K		
Panel B: Investment Management Activity									
FC03	Trunc03	FC04	Trunc04	FC05	Trunc05	FC06	Trunc06	FC07	Trunc07
N	T	O	Q	Q	M	M	K	Q	O
L	M	M	A	O	L	H	H	M	M
I	B	I	M	M	F	E	E	K	K
	L		I	L	D			E	F
	I		F	D				D	D
	F			F					E

Note: [a]FC stands for full-complement set; second column headings of each activity refer to the truncated sample.

The integrity test results obtained from this study as shown in Table 10 found that 13 out of 15 output to input ratios are higher for the efficient group of the risk management activities. Meanwhile, for the investment management activity, the efficient insurers/takaful operators exhibit higher values of output to input ratios for all 20 ratios calculated. Thus, these results indicate that the efficient DMUs of those listed by the SBM-DEA have output to input ratio higher than inefficient DMUs (Avkiran, 2007). This implies that SBM-DEA is appropriate as a means to separate between efficient and inefficient DMU.

Table 10: Integrity test of efficiency[a]

	Risk Management Activity				Investment Management Activity				
	#insurer	R1	R2	R3	#insurer	R4	R5	R6	R7
2003									
Eff	5	6.13	2093.09	85.04	3	0.11	0.15	73.12	93.75
Ineff	13	7.78	706.83	24.73	17	0.10	0.08	19.14	15.33
2004									
Eff	8	8.65	5881.58	53.37	3	0.14	0.11	61.95	46.61
Ineff	10	11.33	795.28	18.56	17	0.09	0.08	18.41	15.29
2005									
Eff	5	15.77	4610.72	63.39	6	0.12	0.09	29.22	23.60
Ineff	10	11.00	1106.54	14.03	14	0.09	0.08	15.92	13.44
2006									
Eff	8	15.82	3028.15	18.05	3	0.16	0.12	39.64	29.03
Ineff	10	8.32	1788.68	13.70	17	0.10	0.08	19.81	16.42
2007									
Eff	9	12.62	3306.47	99.74	5	0.11	0.09	31.84	26.22
Ineff	9	9.15	2562.35	15.98	15	0.09	0.07	19.23	16.52

Note: [a] R1:benefit plus reserve/leverage; R2:benefit plus reserve/investment risk; R3:benefit plus reserve/underwriting risk; R4: investment return/total investment asset; R5: investment return/actuarial reserve; R6: solvency/total

In short, all results obtained from the stability and integrity test add credence to the SBM approach and resulting measures which is parallel with Hughes and Yaisawarng (2004).

Conclusions

The stability test is a countercheck technique in strengthening the results obtained from the frontier efficiency methodologies such as DEA. Several tests performed in this study on the illustrative data showed that a stable efficient frontier. In addition, the test also indicated that the efficiency score is reliable in discriminating between efficient and inefficient DMUs. Therefore it can be concluded that the DEA model used (in the case of this study-SBM) is appropriate in furnishing a comprehensive guide towards the best practices that other firms might adopt and worst practices that other firms should avoid. In turn, the managerial decision-making can be made with more confidence.

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Appendix A. Mathematical form of SBM DEA

According to Tone (2001), for each DMU$_j$ $(j = 1, \dots, n)$ and input matric $X = x_{ij} \in R^{mxn}$ used by DMU$_j$ and amount of output matric $Y = y_{ij} \in R^{sxn}$ yielded by DMU$_j$, with the assumption, the data set is positive $X > 0$ and $Y > 0$, the production possibility set for SBM is defined by:

$P = \{(x, y) | x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\}$

where λ is a nonnegative vector in R^n. In an attempt to estimate the efficiency of a DMU (x_o, y_o), the following fractional program (FP) is formulated:

$$(SBM_{FP}) \min_{\lambda_j s_i^- s_r^+} \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}}{1 + \frac{1}{s}\sum_{r=1}^{s} s_r^+ / y_{ro}}$$

subject to

$x_o = X\lambda_j + s^-$

$y_o = Y\lambda_j - s^+$

$0 \leq \lambda, s^-, s^+$