

An n-dimensional analysis for predicting long run behavior of stock market trend using Fuzzy Relational Maps

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Abstract

The crucial role of connecting the available information about the object to be analyzed as well as the relationship between them is a great challenge for academicians, researchers and financial analyst. The analysis of stock market proposed so far is a qualitative trend analysis which is determinable. A system's behavior can be well built by a dynamical model which identifies relationship existing between its variables. The proposed dynamic model of uncertainty with incomplete information is modeled using Fuzzy Relational Maps (FRMs). An n-dimensional approach is applied to study and predict the long run behavior of the stock market trend using FRMs.

Keywords: n-dimension; Fuzzy Relational Maps; Long run behavior; Stock market trend; Principles of uncertainty

1. Introduction

Stock market is the aggregation of buyers and sellers. It is a place for people, wishing to attain their financial goals. To reach their financial goals, one should first set the goals and aim for it. Understanding a comprehensive knowledge makes the task easy for deciding and reaching the goals. The behavior of the stock market involves lot of uncertainty. Fuzzy logic is a tool built and designed exclusively for modeling such uncertainty to achieve the desired goals. In a free market economy, the increased complexity and uncertainty of factors affects the financial standing of entities. Fuzzy logic tool is introduced in the paper to overcome such needs of financial management.

Fuzzy sets are the base for possibility theory and are a versatile tool for both linguistic and numerical modeling of fuzzy rule based systems. Though applications of fuzzy logic has resulted in stock market forecasting, still the limitation reported with tool remains. And it is the requirement of comprehensive knowledge prior to design fuzzy methods for forecasting, which will require further research.

In this paper a new approach is proposed by employing a qualitative trend analysis for predicting the trend of the stock market using FRM model. It identifies the relationship existing between the defined domain and the corresponding co-domain.

The paper is organized as follows: Section 2 describes the Background, Section 3 deals with Preliminaries. Details of the Proposed Model are given in Section 4, Sections 5 and 6 present the Experimental Calculations and Results. Section 7 reports Discussion and is followed by a brief Conclusion in Section 8.

2. Background

Lofti Zadeh [19] introduced the concept of fuzzy logic. He succeeded with fruitful achievements both in theory and applications. Apparently, Zadeh [20] used the concept of a linguistic variable and its application to approximate reasoning. Kosko [9] worked on fuzzy cognitive maps. FRMs new notion was introduced by Vasantha Kandasamy [16] to analyse the employee-employer relationship model. Subsequently, the FRMs have been widely studied by many researchers and applied for solving the prediction problem of stocks. In recent years, fuzzy logic along with various other modern soft computing techniques are involved in predicting the stock market with better accuracy.

Fuzzy logic based on expert knowledge is used as a tool in the domain of the stock market forecasting. One can extract rules from the dataset, whenever an expert knowledge is missing. Fuzzy logic can also be utilized independently for developing effective models in real life scenarios. For instance, Romahi and Shen [11] have developed an evolving rule based expert system, where fuzzy logic and the rule induction were merged together to obtain a promising method to forecast financial market activity. Another interesting model is EFuNN, a hybridization of Fuzzy logic, ANN and the Evolutionary computation which has been effectively used by Abraham et al. [1] to analyse and forecast the trend of stock market. Further Chin-Shienlin et al. [4] developed a model of a trading system by using neuro fuzzy framework in order to predict the stock index in a better way. Wee Mien Cheung et al. [17] also examined a trading model that combines fuzzy logic and technical analysis to find patterns and trends in financial indices.

George S. Atsalakis et al. [6] surveyed more than hundred published articles that focused on neural and neuro-fuzzy techniques derived and applied to stock markets. When there were no matching historical patterns, vector quantization technique was utilized by Sheng-Tun Li et al. [14] to support long –term forecasting. Jui-Chung Hung [8] studied volatility forecasting in the financial stock market to improve the performance. Saima Hassan et al. [12] used ARIMA model to select appropriate coefficients from the observed data set and utilized interval type-2 fuzzy logic system to forecast the result. Mehrdad Madhoushi et al. [10] developed a Hybrid Approach

based on Neuro Fuzzy Model and Emotional Learning for prediction of stock exchange market. The efficiency of interval type-2 fuzzy logic system model, the flow and trend of underground economy in Malaysia is discussed in Abd. Fatahwahab et al. [2]. Govindasamy and Thambidurai [7] developed a probabilistic fuzzy logic based stock prediction to buy and to sell share in real time.

FRMs analyse the cognitive relationship that exists between specific domains and co-domains by building new models. FRMs have been used by many researchers to analyse various social problems in real world situations. Comprehensive knowledge based expert system is developed and presented using an effective fuzzy technique for studying long run behavior of the stock market trend. It is based on the behavior of predictions using n-dimensional analysis model.

3. Preliminaries

In this section the notion of Fuzzy Relational Maps (FRMs) [15] is introduced. In FRMs we divide the very causal associations into two disjoint units. To define FRMs, one needs a domain space and a range space which are disjoint in the sense of concepts. It is further assumed that no intermediate relation exists within the domain elements and the range space elements. The number of elements in the domain space need not in general be equal to the number of elements in the range space.

We assume the elements of the domain space are taken from the real vector space of dimension m and that of the range space are real vectors from the vector space of dimension n (m need not be equal to n). D denotes a set of nodes D_1, \dots, D_m of the domain space and R , a set of nodes R_1, \dots, R_n of the range space.

$$D = \{(x_1, \dots, x_m) / x_i = 0 \text{ or } 1; 1 \leq i \leq m\}$$

$$R = \{(y_1, \dots, y_n) / y_j = 0 \text{ or } 1; 1 \leq j \leq n\}$$

Definition 3. 1. A FRM is a directed graph or a map from D to R with concepts like policies or events etc, as nodes and causalities as edges. It represents the causal relations between spaces D and R . Let D_i to R_j denotes the causality of D_i on R_j called relations.

Every edge in the FRM is weighted with a number from the set $\{0, \pm 1\}$. Let e_{ij} be the weight of the edge $D_i R_j$; $e_{ij} \in \{0, \pm 1\}$. The weight of the edge $D_i R_j$ is positive if increase in D_i implies increase in R_j or decrease in D_i implies decrease in R_j , i. e., causality of D_i on R_j is 1. If $e_{ij} = 0$ then D_i does not have any effect on R_j . When increase in D_i implies decrease in R_j or decrease in D_i implies increase in R_j it is denoted by -1, i.e., $e_{ij} = -1$.

Definition 3. 2. When the nodes of the FRM are fuzzy sets then they are called fuzzy nodes. FRMs with edge weights $\{0, \pm 1\}$ are called simple FRMs.

Definition 3. 3. Let D_1, \dots, D_m be the nodes of the domain space D of an FRM and R_1, \dots, R_n be the nodes of the range space R of an FRM. Let the matrix E be defined as $E = (e_{ij})$ where e_{ij} is the weight of the directed edge $D_i R_j$ (or $R_j D_i$). E is called the relational matrix of the FRM.

E. This is done by multiplying A_1 with the relational matrix E. Let $A_1E = (r_1, r_2, \dots, r_n)$. After thresholding and updating the resultant vector we get $A_1E \in R$. Now let $A_1E = B$, then we pass B into E^T and obtain BE^T . We update and threshold the vector BE^T so that $BE^T \in D$. This procedure is repeated till we get a limit cycle or a fixed point. After a finite number of steps we would arrive at a fixed point or a limit cycle.

4. Modelling stock market using FRMs

4. 1. Facts about Financial Forecasting Stock Market Models

In literature, forecasting models are categorized into three main groups: Statistical models, Theoretical models and Soft Computing technique models. Soft Computing technique models are part of a separate field of science named computational or artificial intelligence for solving various decision making problems. According to literature, 64% of case studies used statistical models, 25% soft computing techniques, and 11% other models [3].

4. 2. Principles of Uncertainty

The three fundamental principles [5] for managing uncertainty: the principle of minimum uncertainty, the principle of maximum uncertainty, and the principle of uncertainty invariance. Since measures of uncertainty and making decisions differ substantially in different uncertainty theories, the principles result in different mathematical problems considerably when we move from one theory to another. Once justification is done for uncertainty measure, it can be very efficiently used for managing the incomplete information. Optimization problems emerge from the maximum uncertainty principle and it has to be properly investigated and tested.

4. 3. Financial Model Set up

The new notion called FRMs, introduced by Vasantha Kandasamy [16] is used in our model to analyze the interrelations between different dimensions. By varying the dimensions of the problem, one can study directly or indirectly the impact or interrelation existing between the dimensions considered. The uniqueness of this model is that it identifies a pair of hidden pattern for the dimension taken. Using this model, the underlying effect over a dynamical system of a pair of different dimensions can be analyzed and also it helps in comparing a step by step effect of the states on the system.

The term FRMs is usually used for modeling uncertainty of a complex system, taking into account the dynamic or static relations existing between them. Most fuzzy relations used in real-world applications do not represent a concept, instead they denote a functional mapping from a set of input variables to one or more output variables. A fuzzy relation represents a multi-dimensional fuzzy set. An important fuzzy technique is the composition of two fuzzy relations. The notion of fuzzy relational equations based on max-min composition was first investigated by Sanchez [13]. The max-min composition is used when conservative solutions are required by the system in the sense that the goodness of one value cannot compensate the flaw of another value [18].

4. 4. Interval varying parameter dependent approximate reasoning

The model proposed characterizes approximate reasoning with interval-valued fuzzy sets. It develops inference engines of expert systems in any application area which is not sensible enough to represent exact membership grades.

For instance, Let A denote an interval varying parameter dependent fuzzy set. Then $A(x) = [L_A(x), U_A(x)] \subseteq [0, 1]$ for each $x \in X$, where L_A, U_A are fuzzy sets that are called the lower bound of A and the upper bound of A respectively.

The reasoning process is facilitated once a relation is determined. A relation can be written as a set of ordered tuples. A relation between two sets is called binary; if three, four, or five sets are involved, the relation is called ternary, quaternary, or quinary, respectively. In general, a relation defined on n sets is called n -ary or n -dimensional.

A fuzzy relation is a fuzzy set defined and represented by an n -dimensional membership array whose entries correspond to n -tuples in the universal set. These entries take values representing the membership grades of the corresponding n -tuples.

4. 5. Financial Model Data Design

The designed financial stock market model has fuzzy sets with input variables of n -dimension. The relation existing between fuzzy sets is identified by means of a connection matrix. The results show the prediction of the behavior of the system in the long run.

This fuzzy model deals with data matrices which are got from concrete numbers. The matrix table contains real entries from the set of reals, forming a rectangular array of numbers.

Let us consider the different dimensions for our study for the domain space and the range space. Let the dimension for the domain space be m and the dimension for the range space be n . The analysis is done for $m \times n$ rectangular matrix and conclusions are drawn depending on the pair of resultant state vectors obtained.

For instance,

The states related to the range space of varying dimensions, when $n=2, n=3, n=4, n=5, n=6$ respectively are defined as follows:

$S_1 = \text{Low}; S_2 = \text{High}$

$S_1 = \text{Low}; S_2 = \text{Medium}; S_3 = \text{High}$

$S_1 = \text{Low}; S_2 = \text{Medium}; S_3 = \text{Moderate}; S_4 = \text{High}$

$S_1 = \text{Very Low}; S_2 = \text{Low}; S_3 = \text{Moderate}; S_4 = \text{High}; S_5 = \text{Very High}$

$S_1 = \text{Very Low}; S_2 = \text{Low}; S_3 = \text{Moderate Low}; S_4 = \text{Moderate High}; S_5 = \text{High}; S_6 = \text{Very High}$

The same way, states can be defined for any number of dimensions.

Let N denotes the number of days, for which the close values of stock market for each day to be analyzed is collected as a numerical data. Let n denotes the dimension of the number of subintervals in the defined range. Consider the states S_1, S_2, \dots, S_n such that S_i takes a value in the interval (a_i, a_{i+1}) for $1 \leq i \leq n$. Take 2 consecutive days; find the one day difference in close values. Find the Upper bound and the Lower bound of the difference values. The Upper bound is the Maximum range and the

Lower bound is the Minimum range. Divide the range into n sub intervals. Each co-ordinates of the subinterval is used to define our states considered.

Now find the mini-max of each sub interval. This Range of values of the dynamical system, determines the domain and co-domain depending on different dimensions considered.

Table 1: States, Intervals and Mini-max composition

States	S ₁	S ₂	...	S _n
Intervals indicating lower and upper bounds	(a ₁ , a ₂)	(a ₂ , a ₃)	...	(a _n , a _{n+1})
Mini-max composition for the corresponding interval	a ₂	a ₃	...	a _{n+1}

A Relational matrix E can be formed for the above defined states between the domain and its corresponding range with the conditions of FRMs defined as follows. If the value in the domain as well as the value in the co-domain are both positive and both negative then we give a value 1. If the value in the domain is either positive or negative and the value in the co-domain is either negative or positive respectively then we assign a value-1.

5. Calculation

The following procedure is constructed for determining the hidden pattern of the FRM using the relational matrix formed.

- Step 1: Consider an initial input vector $A_1 = (1\ 0\ 0\ \dots\ 0)$ in D.
- Step 2: Make the data A_1 to pass through relational matrix E, i. e., multiply A_1 and E
- Step 3: The resultant vector $A_1E \in R$ after thresholding and updating.
- Step 4: Let $A_1E = B$
- Step 5: Multiply B and E^T
- Step 6: The resultant vector $BE^T \in D$ after thresholding and updating.
- Step 7: Repeat the procedure till a fixed point or limit cycle is obtained.

6. Results

Table 2: Input set of variables and its resultant binary pair of vectors for X_1 paired with X_2 , X_1 paired with X_3 , X_1 paired with X_4 , ...

S. No	Input set of variables	Initial vector	Resultant binary pair of vectors
1.	$\langle X_1, X_2 \rangle$	(1 0 0)	{(1 0 0), (1 1 0 0)}
		(0 1 0)	{(0 1 1), (0 0 1 1)}
		(0 0 1)	{(0 1 1), (0 0 1 1)}
2.	$\langle X_1, X_3 \rangle$	(1 0 0)	{(1 0 0), (1 1 0 0 0)}
		(0 1 0)	{(0 1 1), (0 0 1 1 1)}
		(0 0 1)	{(0 1 1), (0 0 1 1 1)}

3.	$\langle X_1, X_4 \rangle$	(1 0 0)	{(1 0 0), (1 1 1 0 0 0)}
		(0 1 0)	{(0 1 1), (0 0 0 1 1 1)}
		(0 0 1)	{(0 1 1), (0 0 0 1 1 1)}
.	.	.	.
.	.	.	.
.	.	.	.

Similarly, X_1 can be paired with any X_n and the corresponding resultant binary pair of vectors can be obtained.

Table 3: Input set of variables and its resultant binary pair of vectors for X_2 paired with X_1 , X_2 paired with X_3 , X_2 paired with X_4 , ...

S. No	Input set of variables	Initial vector	Resultant binary pair of vectors
1.	$\langle X_2, X_1 \rangle$	(1 0 0 0)	{(1 1 0 0), (1 0 0)}
		(0 1 0 0)	{(1 1 0 0), (1 0 0)}
		(0 0 1 0)	{(0 0 1 1), (0 1 1)}
		(0 0 0 1)	{(0 0 1 1), (0 1 1)}
2.	$\langle X_2, X_3 \rangle$	(1 0 0 0)	{(1 1 0 0), (1 1 0 0 0)}
		(0 1 0 0)	{(1 1 0 0), (1 1 0 0 0)}
		(0 0 1 0)	{(0 0 1 1), (0 0 1 1 1)}
		(0 0 0 1)	{(0 0 1 1), (0 0 1 1 1)}
3.	$\langle X_2, X_4 \rangle$	(1 0 0 0)	{(1 1 0 0), (1 1 1 0 0 0)}
		(0 1 0 0)	{(1 1 0 0), (1 1 1 0 0 0)}
		(0 0 1 0)	{(0 0 1 1), (0 0 0 1 1 1)}
		(0 0 0 1)	{(0 0 1 1), (0 0 0 1 1 1)}
.	.	.	.
.	.	.	.
.	.	.	.

Similarly, X_2 can be paired with any X_n and the corresponding resultant binary pair of vectors can be obtained.

Table 4: Input set of variables and its resultant binary pair of vectors for X_3 paired with X_1 , X_3 paired with X_2 , X_3 paired with X_4 , ...

S. No	Input set of variables	Initial vector	Resultant binary pair of vectors
1.	$\langle X_3, X_1 \rangle$	(1 0 0 0 0)	{(1 1 0 0 0), (1 0 0)}
		(0 1 0 0 0)	{(1 1 0 0 0), (1 0 0)}
		(0 0 1 0 0)	{(0 0 1 1 1), (0 1 1)}
		(0 0 0 1 0)	{(0 0 1 1 1), (0 1 1)}
		(0 0 0 0 1)	{(0 0 1 1 1), (0 1 1)}

2.	$\langle X_3, X_2 \rangle$	(1 0 0 0 0)	{(1 1 0 0 0), (1 1 0 0)}
		(0 1 0 0 0)	{(1 1 0 0 0), (1 1 0 0)}
		(0 0 1 0 0)	{(0 0 1 1 1), (0 0 1 1)}
		(0 0 0 1 0)	{(0 0 1 1 1), (0 0 1 1)}
		(0 0 0 0 1)	{(0 0 1 1 1), (0 0 1 1)}
3.	$\langle X_3, X_4 \rangle$	(1 0 0 0 0)	{(1 1 0 0 0), (1 1 1 0 0 0)}
		(0 1 0 0 0)	{(1 1 0 0 0), (1 1 1 0 0 0)}
		(0 0 1 0 0)	{(0 0 1 1 1), (0 0 0 1 1 1)}
		(0 0 0 1 0)	{(0 0 1 1 1), (0 0 0 1 1 1)}
		(0 0 0 0 1)	{(0 0 1 1 1), (0 0 0 1 1 1)}
.	.	.	.
.	.	.	.
.	.	.	.

Similarly, X_3 can be paired with any X_n and the corresponding resultant binary pair of vectors can be obtained.

Table 5: Input set of variables and its resultant binary pair of vectors for X_4 paired with X_1 , X_4 paired with X_2 , X_4 paired with X_3 , ...

S. No	Input set of variables	Initial vector	Resultant binary pair of vectors
1.	$\langle X_4, X_1 \rangle$	(1 0 0 0 0 0)	{(1 1 1 0 0 0), (1 0 0)}
		(0 1 0 0 0 0)	{(1 1 1 0 0 0), (1 0 0)}
		(0 0 1 0 0 0)	{(1 1 1 0 0 0), (1 0 0)}
		(0 0 0 1 0 0)	{(0 0 0 1 1 1), (0 1 1)}
		(0 0 0 0 1 0)	{(0 0 0 1 1 1), (0 1 1)}
		(0 0 0 0 0 1)	{(0 0 0 1 1 1), (0 1 1)}
2.	$\langle X_4, X_2 \rangle$	(1 0 0 0 0 0)	{(1 1 1 0 0 0), (1 1 0 0)}
		(0 1 0 0 0 0)	{(1 1 1 0 0 0), (1 1 0 0)}
		(0 0 1 0 0 0)	{(1 1 1 0 0 0), (1 1 0 0)}
		(0 0 0 1 0 0)	{(0 0 0 1 1 1), (0 0 1 1)}
		(0 0 0 0 1 0)	{(0 0 0 1 1 1), (0 0 1 1)}
		(0 0 0 0 0 1)	{(0 0 0 1 1 1), (0 0 1 1)}
3.	$\langle X_4, X_3 \rangle$	(1 0 0 0 0 0)	{(1 1 1 0 0 0), (1 1 0 0 0)}
		(0 1 0 0 0 0)	{(1 1 1 0 0 0), (1 1 0 0 0)}
		(0 0 1 0 0 0)	{(1 1 1 0 0 0), (1 1 0 0 0)}
		(0 0 0 1 0 0)	{(0 0 0 1 1 1), (0 0 1 1 1)}
		(0 0 0 0 1 0)	{(0 0 0 1 1 1), (0 0 1 1 1)}
		(0 0 0 0 0 1)	{(0 0 0 1 1 1), (0 0 1 1 1)}
.	.	.	.
.	.	.	.
.	.	.	.

Similarly, X_4 can be paired with any X_n and the corresponding resultant binary pair of vectors can be obtained.

In general, X_n can be paired with any X_i , $i=1, 2, \dots, i-1$ and the corresponding resultant binary pair of vectors can be obtained.

Discussion

FRMs associate 2 disjoint units. Here we try to associate the different dimensions of the problem and analyze its long run behavior. An optimal solution is obtained for the stock market problem. It is observed from the resultant binary pair of vectors of Table 1 that when Low state S_1 is 'ON', the corresponding Low state S_1 and Medium state S_2 are 'ON' in the binary pair, for the dimension $m=3$ and $n=4$. Likewise all the other resultant pairs are listed in the Tables [3, 4, 5] for different dimensions of m and n . All the resultant pair of vectors implies that the stock market cannot undergo sudden drastic changes unless it is affected by some other external factors or forces. The deviation is seen only for a minute change in value. Major difference in price of stock values could not be foreseen even for higher dimensions. As the dimension increases to n , the study becomes too optimized.

Conclusion

Making decisions under uncertainty is a difficult problem. Accepting and structuring uncertainty leads to effective decision making. A dynamical model of uncertainty with incomplete information has been effectively modeled using Fuzzy Relational Maps (FRMs). In this paper, we tried an n -dimensional approach to study and predict the long run behavior of the stock market trend. It is found that the proposed model has discovered a fuzzy dependency relationship in n -dimensional approach for a very huge noisy data. The approach is very promising in capturing the non-linearity that exists in our dynamical system. The model handles vagueness by accepting the degree of correlations as numerical certainty factors thereby finding solution to the real world situation. As an extension, it is suggested that apart from the real time data set a dynamical model can be designed and analyzed using other fuzzy techniques considering other external constraints that affect the stock market trend.

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