

Influence of magnetic fluid through a series of flow factors on the performance of a longitudinally rough finite slider bearing

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Abstract

In this article it has been sought to study the effect of longitudinal roughness on the behaviour of plane slider bearings with a film formed by a magnetic fluid as the lubricant, by using pressure flow factor-which is strongly dependent on the surface pattern parameter γ for longitudinally rough slider bearing. A stochastic random variable with non zero mean, variance and skewness is used to model the roughness of the underlying bearing surfaces. The associated Reynolds' equation is stochastically averaged with respect to the random roughness parameter. Solving this equation with suitable boundary conditions one obtains the pressure distribution which results in the calculation of load carrying capacity. The results are shown graphically. The graphical representations indicate that the use of a magnetic fluid

as lubricant increases the load carrying capacity in comparison with the conventional lubricant based bearing system. Moreover, the adverse effect of roughness can be minimized to certain extent by increasing the strength of the magnetic field.

AMS subject classification:

Keywords: Reynolds' equation, Finite plane slider bearing, Flow factor, Longitudinal roughness, Magnetic fluid.

1. Introduction

The study of an inclined plane slider bearing is a classical one. Slider bearing is used in various fields like clutch plates, automobile transmissions and domestic appliances. It is well known that the bearing surfaces after having some run-in and wear develop roughness. Various film shapes [1, 2] have been investigated. It has gained growing attentions after the introduction of stochastic concept [3]. For the evaluation of the effect of transverse and longitudinal roughness Christensen and Tonder [4,5,6,7] used a stochastically averaged Reynolds' type equation to analyse the hydrodynamic lubrication for slider bearings. Christensen and Tonder's approach formed the base of the analysis to study the effect of surface roughness in a number of investigations (Prakash and Tiwari [14], Guha [15], Gupta and Deheri [16], Andharia et al. [17]). The effect of surface roughness on the performance of hydrodynamic slider bearings was studied by Andharia et al. [8]. The effect of longitudinal surface roughness on the behaviour of slider bearing with squeeze film formed by a magnetic fluid was analysed by Deheri et al. [9]. Both these studies established that the roughness had a significant effect on the performance of the bearing system. The effect of surface roughness was discussed by many investigators (Davies [10], Burton [11], Michell [12], Tonder [13]).

To improve the Tribological performance of a sliding interface, magnetic fluid as a lubricant has been employed in bearings. Agrawal [18], Bhat and Deheri [19, 20] studied an inclined plane slider bearing by considering a ferrofluid as the lubricant. It was found that such a lubricant caused increase in the load carrying capacity of the bearings without affecting the friction force on the respective sliders. Patel et al. [21] examined the squeeze film performance of ferrofluid with Non-Newtonian couple stress effect on parallel rough circular disks. The effect of surface roughness on the performance of a magnetic fluid based parallel plate porous slider bearing was observed by Patel and Deheri [22]. Slip velocity and roughness effect on magnetic fluid based infinitely long bearings was analysed by Patel et al. [23]. It was concluded that the slip effect remained crucial for bearing design. It required to be kept at minimum.

Andharia et al. [17] observed that in case of longitudinal roughness, by suitably choosing the strength of the ferrofluid, the performance of the bearing could be improved to a large extent.

Further efforts have been made to analyse the effect of surface roughness and flow factor-which is strongly dependent on the surface pattern parameter ($\gamma > 1$) on the

behaviour of a longitudinally rough plane slider bearing with a film formed by a magnetic fluid.

2. Analysis

For a rough planer slider bearing as shown in Figure 1, we assume that the bearing is infinite in Y -direction and the slider is moving in X -direction with the uniform velocity U , while h_m and h_M are minimum and maximum film thickness respectively. The length of the bearing is l .

The mean pressure in a rough slider bearing is governed by the averaged Reynolds' type equation (Patir [24] and Patir and Cheng [25, 29])

$$\frac{\partial}{\partial x} \left[\phi_x \frac{h^3}{12\mu} \frac{\partial \bar{p}}{\partial x} \right] + \frac{\partial}{\partial y} \left[\phi_y \frac{h^3}{12\mu} \frac{\partial \bar{p}}{\partial y} \right] = \frac{U}{2} \frac{\partial \bar{h}_T}{\partial x} + \frac{U\sigma}{2} \frac{\partial \phi_s}{\partial x} \quad (2.1)$$

where

$$\bar{h}_T = \int_{-h}^{\infty} (h + \delta) f(\delta) d\delta = E(h_T) \quad (2.2)$$

while

$$h_T = h + \delta \quad (2.3)$$

This stochastically averaged Reynolds' type equation took surface topography into account. It has been assumed that the flow of lubricant is steady and in X -direction only while $U_1 = U$, $U_2 = 0$. Moreover for longitudinally rough surface ($\gamma > 1$), the variation in roughness heights in X -direction is negligible. Thus the effect of shear flow factor (ϕ_s) is also negligible. Equation (2.1) then can be modified as

$$\frac{d}{dx} \left[\phi_x \frac{h^3}{12\mu} \frac{d\bar{p}}{dx} \right] = \frac{U}{2} \frac{d\bar{h}_T}{dx} \quad (2.4)$$

The use of magnetic fluid as a lubricant modifying the performance of the bearing system is well known. The magnetic field M is oblique to the stator as in Agrawal [18] having magnitude

$$M^2 = x(l - x) \quad (2.5)$$

In case of magnetic fluid as a lubricant, the pressure generated in a fluid film can be assumed to be increased with respect to the applied magnetic field. Also, the lubricant is considered to be incompressible and its flow is laminar. Under the usual assumptions of the hydrodynamic magnetic lubrication, the modified Reynolds' equation (Agrawal [18], Bhat [27], Bhat and Patel [28]) in present case, turns out to be

$$\frac{d}{dx} \left[\phi_x \frac{h^3}{12\mu} \frac{d}{dx} (\bar{p} - 0.5\mu_0\mu_f M^2) \right] = \frac{U}{2} \frac{d\bar{h}_T}{dx} \quad (2.6)$$

where μ_0 is magnetic susceptibility, which is a dimensionless proportionality constant that includes the magnetization of the lubricant in response to an applied magnetic field. μ_f is free space permeability. The free space permeability (permeability in vacuum) of a material characterizes the response of that material to electric or magnetic field. In simplified models, it is often regarded as constants $\left(4\pi \times 10^{-7} \frac{N}{A^2}\right)$ for a given material.

δ is assumed to be stochastic in nature and is governed by the probability density function $f(\delta)$, $-c < \delta < c$, where c is maximum deviation from the mean film thickness. Then, mean- α , the standard deviation- σ and the measure of symmetry- ε are described in Christensen and Tonder [4, 5, 6, 7] in terms of the expected values as:

$$E(R) = \int_{-c}^c R f(\delta) d\delta \quad (2.7)$$

$$\alpha = E(\delta) \quad (2.8)$$

$$\sigma^2 = E[(\delta - \alpha)^2] \quad (2.9)$$

$$\varepsilon = E[(\delta - \alpha)^3] \quad (2.10)$$

It is to be noted that while α and ε can assume both positive and negative values, σ is always positive.

Following the discussion of Chiang, Hsiu-Lu, et al.[30] an approximation to $f(\delta)$ is

$$f(\delta) = \begin{cases} \frac{32}{35c} \left(1 - \frac{\delta^2}{c^2}\right)^3 & \text{if } -c \leq \delta \leq c, \\ 0 & \text{otherwise} \end{cases} \quad (2.11)$$

Hence, \bar{h}_T can be approximated as

$$\bar{h}_T = \frac{13}{8}h \approx h \quad (2.12)$$

Now, following the averaging process of Andharia et al. [8], equation (2.6) can be modified as

$$\frac{d}{dx} \left[\phi_x \frac{m(h)^{-1}}{12\mu} \frac{d}{dx} ((\bar{p}) - 0.5\mu_0\mu_f M^2) \right] = \frac{U}{2} \frac{d}{dx} [n(h)^{-1}] \quad (2.13)$$

where (\bar{p}) is expected value of the mean pressure level \bar{p} ,

$$m(h) = h^{-3} [1 - 3\alpha h^{-1} + 6h^{-2}(\sigma^2 + \alpha^2) - 20h^{-3}(\varepsilon + 3\sigma^2\alpha + \alpha^3)] \quad (2.14)$$

and

$$n(h) = h^{-1}[1 - \alpha h^{-1} + h^{-2}(\sigma^2 + \alpha^2) - h^{-3}(\varepsilon + 3\sigma^2\alpha + \alpha^3)] \quad (2.15)$$

Patir [24] provided that experimental relation for ϕ_x

$$\phi_x = 1 + CH^{-r} \quad (\text{for } \gamma > 1) \quad (2.16)$$

The dimensionless form of this is

$$\phi_X = 1 + C(h^* H_m)^{-r} \quad (\text{for } \gamma > 1) \quad (2.17)$$

where

$$H = \frac{h}{\sigma}, \quad H_m = \frac{h_m}{\sigma} \quad (2.18)$$

while the constants C and r are given as functions of γ in the table-1 (Patir [24]).

Making use of the following dimensionless quantities

$$h^* = \frac{h}{h_m}, \quad X = \frac{x}{l}, \quad m^* = \frac{ml}{h_m}, \quad \bar{P} = \frac{h_m^2(\bar{p})}{\mu Ul}, \quad \mu^* = \frac{\mu_0 \mu_f h_m^2 l}{2\mu U}, \quad M^2 = X(1 - X) \quad (2.19)$$

$$M(h^*) = h^{*-3}[1 - 3\alpha^* h^{*-1} + 6h^{*-2}(\sigma^{*2} + \alpha^{*2}) - 20h^{*-3}(\varepsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3})] \quad (2.20)$$

and

$$N(h^*) = h^{*-1}[1 - \alpha^* h^{*-1} + h^{*-2}(\sigma^{*2} + \alpha^{*2}) - h^{*-3}(\varepsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3})] \quad (2.21)$$

equation (2.13) leads to the dimensionless form

$$\frac{d}{dX} \left[\phi_X M(h^*)^{-1} \frac{d}{dX} (\bar{P} - \mu^* X(1 - X)) \right] = 6 \frac{d}{dX} [N(h^*)^{-1}] \quad (2.22)$$

The associated boundary conditions are

$$\bar{P} = 0, \quad \text{at } X = 0 \quad (2.23)$$

$$\bar{P} = 0, \quad \text{at } X = 1 \quad (2.24)$$

Solving equation (2.22) on the boundary conditions one obtains the expression for non-dimensional pressure distribution

$$\bar{P}(X) = \mu^* X(1 - X) + \int_0^X \frac{1}{\phi_X} \frac{1}{M(h^*)^{-1}} [6N(h^*)^{-1} - Q^*] dX \quad (2.25)$$

where,

$$Q^* = \frac{\int_0^1 \frac{6M(h^*)}{\phi_X N(h^*)} dX}{\int_0^1 \frac{M(h^*)}{\phi_X} dX} \quad (2.26)$$

while

$$\frac{d\bar{P}}{dX} = 0, \text{ at which the mean gap is maximum, say } Q^* \text{ (Constant)} \quad (2.27)$$

The dimensionless load carrying capacity per unit width is then given by

$$W^* = \frac{w \cdot h_m^2}{\mu U l} = \int_0^1 \bar{P} dX \quad (2.28)$$

3. Results and discussion

Figures 2-3 dealing with the effect of variance on the load carrying capacity establishes that the variance (+ve) decreases the load carrying capacity while the variance (-ve) causes increased load carrying capacity. Also, in the initial stages the effect of standard deviation remains quite significant.

The increase in the load carrying capacity due to the increase in standard deviation gets further increased owing to the combined effect of negatively skewed roughness and variance (-ve) (Figure 4–5) which does not happen in the case of transverse roughness pattern.

However, the effect of standard deviation on the variation of the load carrying capacity with respect to skewness is nominal (Figure 6).

In this type of bearing system the roll of variance (-ve) remains more prominent as compared to the negatively skewed roughness, unlike the case of transverse roughness pattern (Deheri et al. [31]).

The effect of standard deviation on the variation of the load carrying capacity with respect to magnetization is approximately marginal (Figure 8).

4. Conclusion

The current investigation shows that this type of bearing system may turn out to be very effective as there are many parameters contributing towards the increase in the load carrying capacity of the bearing. The adverse effect of γ can be compensated by the combined positive effect of standard deviation, negatively skewed roughness and variance (-ve) by choosing the magnetic strength. However, from bearing's life period point of view the roughness aspect deserves to be evaluated at the time of designing the bearing system.

5. Tables and Figures

Table-1. Relation among C , r and H

γ	C	r	H
3	0.225	1.5	$H > 0.5$
6	0.520	1.5	$H > 0.5$
9	0.870	1.5	$H > 0.5$

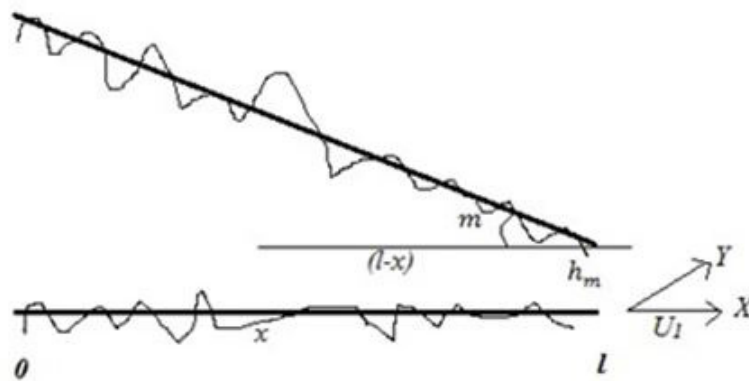


Fig.1 . Bearing geometry

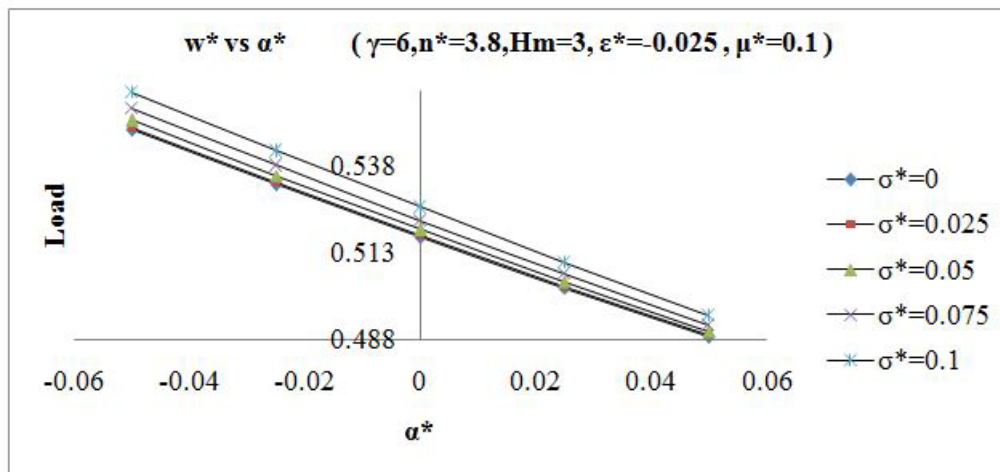
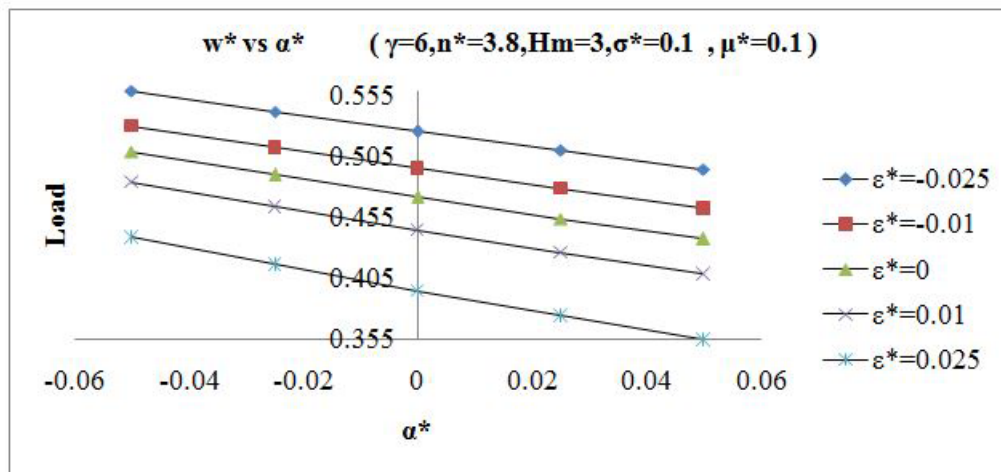
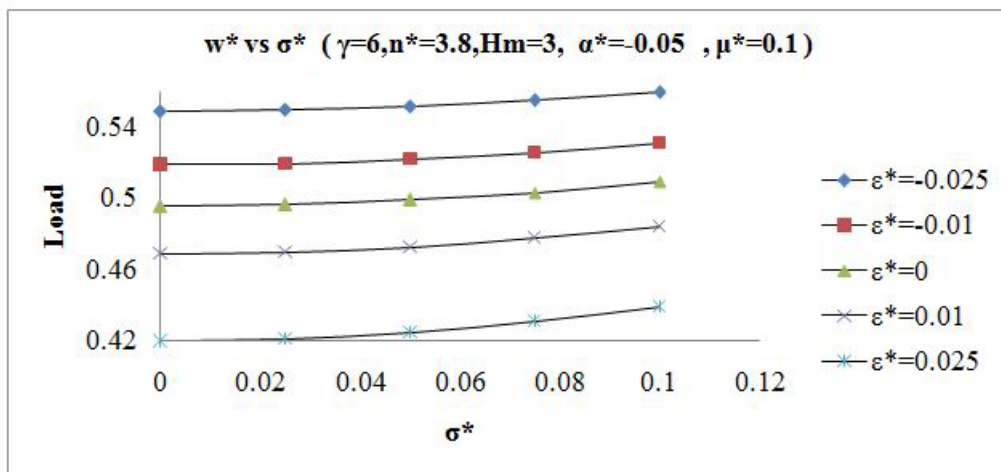
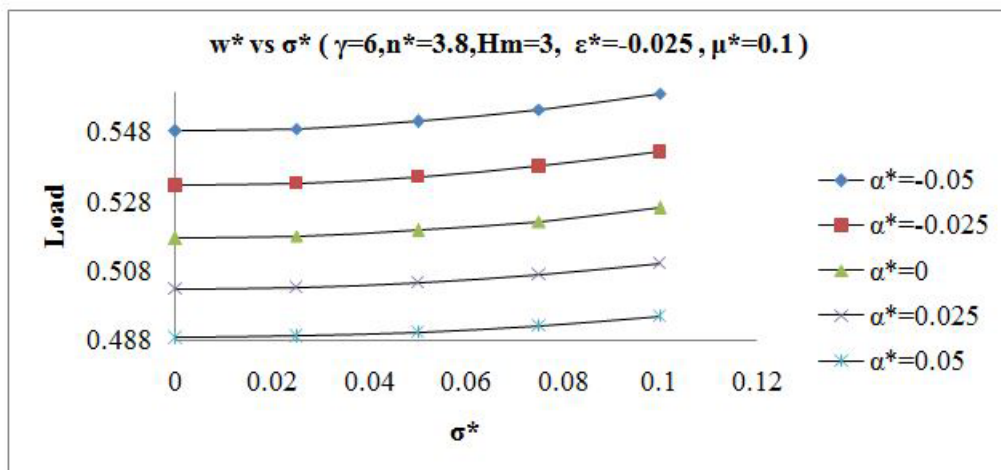
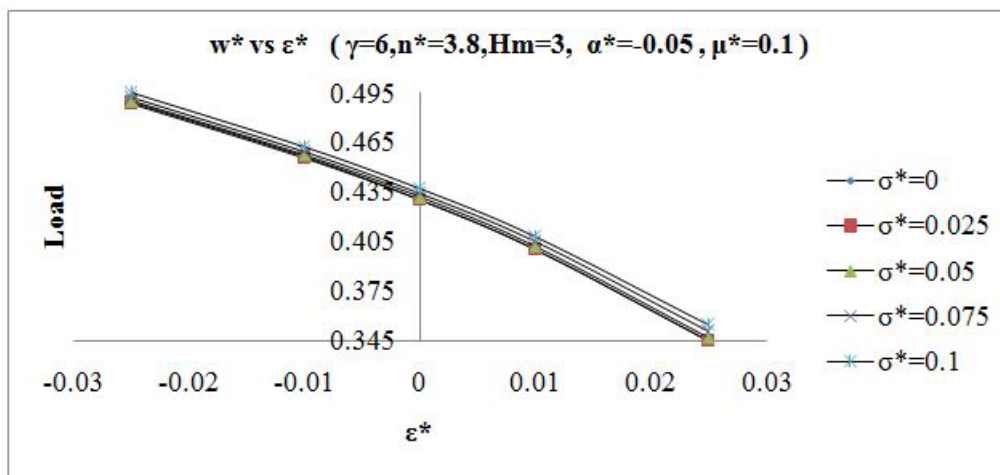
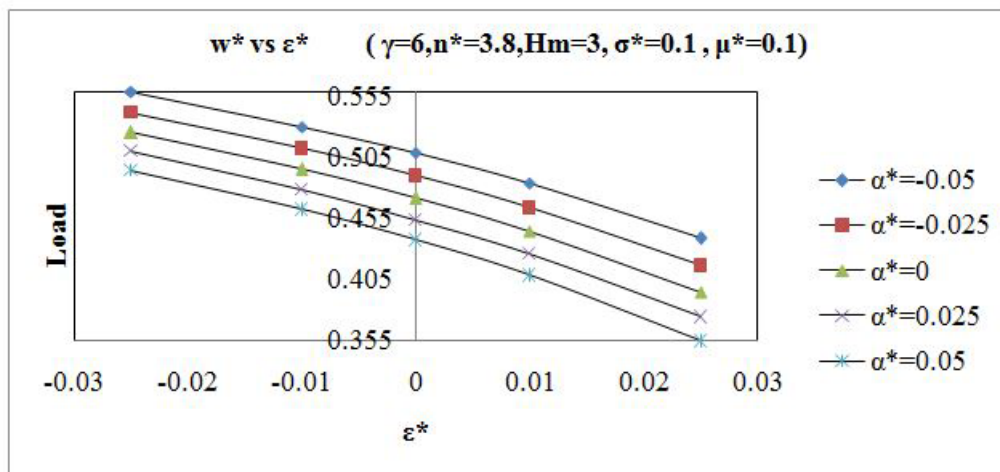
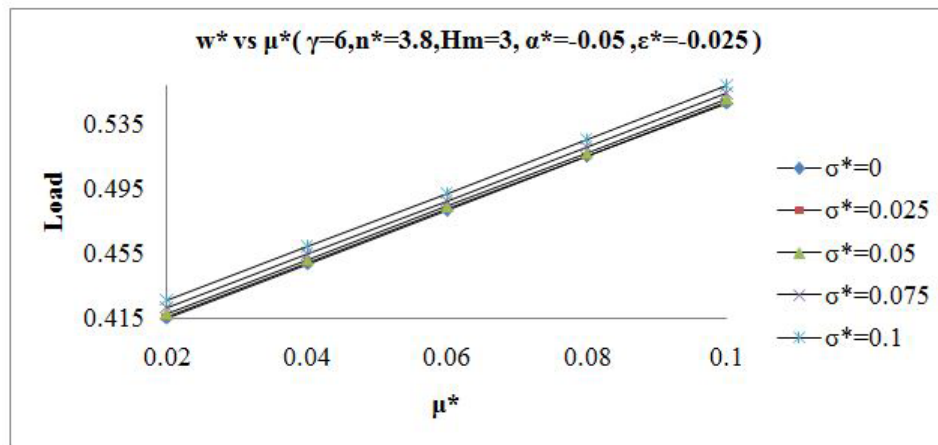
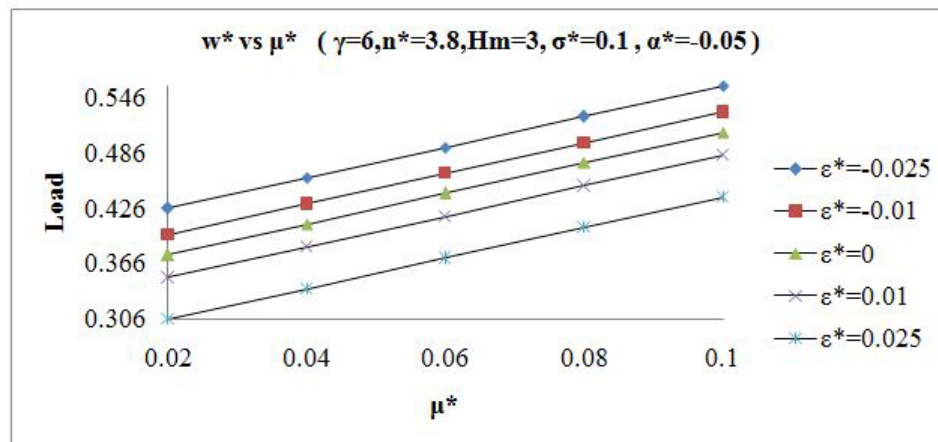
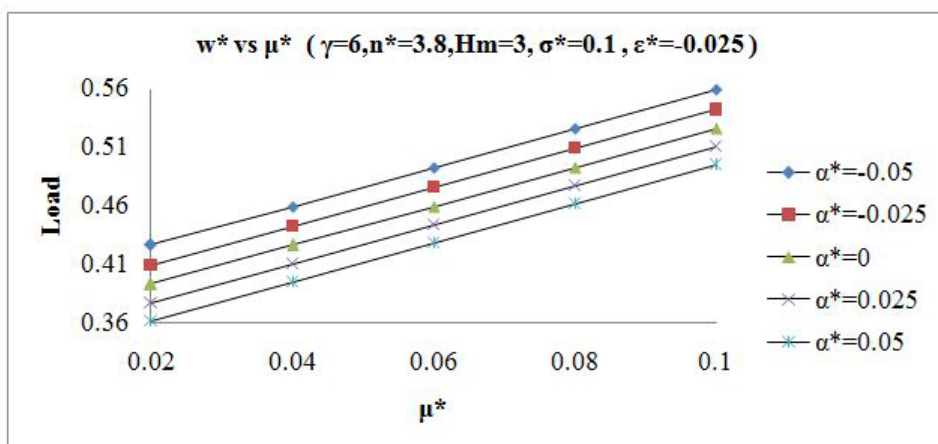
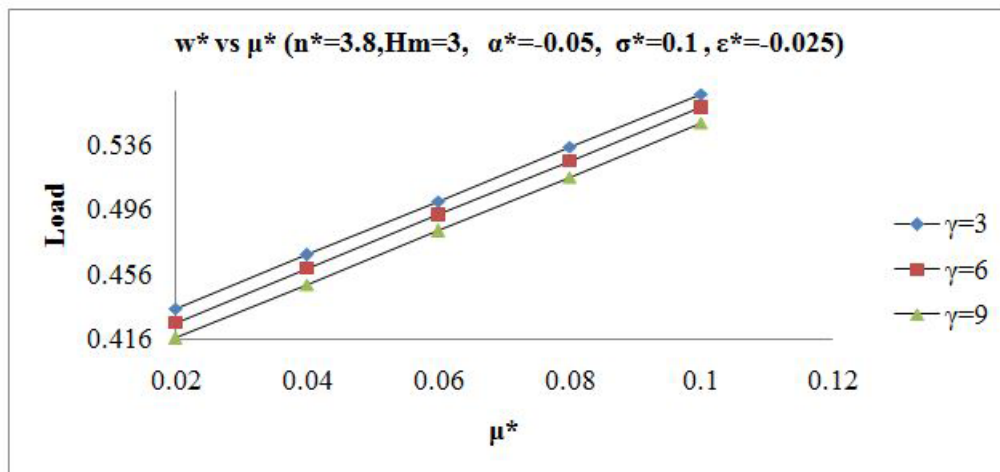


Fig.2. Variation of load carrying capacity with respect to α^*

Fig.3. Variation of load carrying capacity with respect to α^* Fig.4. Variation of load carrying capacity with respect to σ^* Fig.5. Variation of load carrying capacity with respect to σ^*

Fig.6. Variation of load carrying capacity with respect to ϵ^* Fig.7. Variation of load carrying capacity with respect to ϵ^*

Fig.8. Variation of load carrying capacity with respect to μ^* Fig.9. Variation of load carrying capacity with respect to μ^* Fig.10. Variation of load carrying capacity with respect to μ^*

Fig.11. Variation of load carrying capacity with respect to μ^*

Nomenclature

\bar{h}_T	Average film thickness-Mean gap (m)
$f(\delta)$	Frequency density function of combined roughness amplitude- δ (m^{-1})
m	Inclination of slider bearing
l	Length of slider bearing (m)
w	Load carrying capacity (N)
W^*	Load carrying capacity in dimensionless form
h_T	Local film thickness (m)
p	Local pressure (Nm^{-2})
M	Magnitude of the Magnetic field (Am^{-1})
\bar{p}	Mean pressure level (Nm^{-2})
\bar{P}	Mean pressure level in dimensionless form
h_m	Minimum film thickness at the trailing edge of slider bearing (m)
H_m	Minimum film thickness - Roughness ratio in dimensionless form
h	Nominal film thickness (m)
H	Nominal film thickness - Roughness ratio in dimensionless form
U_1, U_2	Velocities of surfaces in X-Direction (ms^{-1})
σ	Composite rms roughness given by Gaussian distribution of heights (m)
ρ	Density of the lubricant (Kgm^{-3})
ϕ_x, ϕ_y	Pressure flow factors
$\delta = \delta_1 + \delta_2$	Random roughness amplitudes of the two surfaces measured from their mean level (m)
ϕ_s	Shear flow factor
σ_1, σ_2	Standard deviations of the surfaces (m)
μ	Viscosity of lubricant ($Kgm^{-1}s^{-1}$)
μ^*	Magnetization parameter in dimensionless form
μ_0	Magnetic susceptibility
μ_f	Free space permeability ($KgmA^{-2}s^{-2}$)

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