Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 1 (2016), pp. 643-655

http://www.ripublication.com

Problems of measurement of high-frequency fields in linear electron accelerators

Aleksandr Evgen'evich Novozhilov
National Research Nuclear University "MEPhI",
Kashirskoye Avenue 31, Moscow, 115409, Russian Federation.

Aleksandr Nikolaevich Filatov
National Research Nuclear University "MEPhI".
Kashirskoye Avenue 31, Moscow, 115409, Russian Federation/

Vladimir Kuz'mich Shilov
National Research Nuclear University "MEPhI",
Kashirskoye Avenue 31, Moscow, 115409, Russian Federation.

Abstract

The study presents a numerical simulation of processes of high-frequency waves' changes in acceleration sections of linear electron accelerators of various types. Both running wave type linear electron accelerators and standing wave type linear electron accelerators based on a biperiodic slow-wave structure are discussed. Errors of various methods of measurement are evaluated and comparison analysis of those methods is carried out. It is demonstrated that an application of small resonance perturbances method is unreasonable in a case quality factors of an accelerating section and excited resonator are quite small.

Keywords: reference plane, accelerating section, accelerating system, small resonance perturbances method, running wave, standing wave, accelerating resonator, shunt resistance, coupling coefficient, amplitude of accelerating field's intensity, phase of accelerating field's intensity, complex amplitude of accelerating field's intensity, small perturbing body, form factor of perturbing body.

Introduction

The most important system of any resonance electron accelerator is its accelerating system [1], which includes a number of accelerating sections. Depending on a purpose of an accelerator [2] accelerating sections with running wave are implemented, as well as standing wave type accelerating sections [3, 4, 5], i.e. accelerating resonators. An integral part of a creation of accelerating sections is a process of their tuning, which necessarily includes a process of a measuring of intensity of an accelerating field in a drift tube of accelerating section [6, 7]. In some cases, in addition to a measurement of an accelerating field it is required to measure fields of higher modes, which can be excited in accelerating sections.

A measurement of fields in high-frequency systems in a whole and, in particular, in accelerating section of particle accelerators is based on so-called small perturbances methods. Those methods essentially comprise an obtainment of information on a certain component of electromagnetic field in a certain point of radio-frequency system via changes of a corresponding integral characteristic of a studied radio-frequency system in a case of an introduction of a small perturbing body in a certain point of that system. In a case of a measurement of an accelerating field in a drift tube of an accelerating section a small perturbing body usually has cylinder shape with a very small diameter (needle shape) and is positioned along axis of a drift tube of a section. At the same time, complex reflection coefficient, complex transmission coefficient and relative resonance frequency shift are measured; they are used for a determination of an accelerating field's value in a given point of a drift tube of a studied accelerating section.

There are two small perturbances methods: methods of non-resonance and resonance small perturbances, which were theoretically justified in the studies of Steele and Nakamura [8, 9]. In particular, in the study of Nakamura [9] the following fundamental relationships for methods of non-resonance and resonance small perturbances are obtained:

$$\Delta S_{2,1} = \frac{1}{4\sqrt{P_1 P_2}} \sum_{i=x,y,z} \left\{ j\omega \left[K_i^{(B)} B_i^{(1)} B_i^{(2)} - K_i^{(E)} E_i^{(1)} E_i^{(2)} \right] - K_i^{(\sigma)} E_i^{(1)} E_i^{(2)} \right\}, \quad (1)$$

$$\frac{f_0 - f_0^{(p.b.)}}{f_0} = \frac{\sum_i \left[K_i^{(E)} |E_i|^2 + K_i^{(B)} |B_i|^2 \right]}{4W}; \Delta\left(\frac{1}{Q}\right) = \frac{1}{Q^{(p.b.)}} - \frac{1}{Q} = \frac{1}{\pi f_0} \frac{\sum_i K_i^{(\sigma)} |E_i|^2}{4W}. \quad (2)$$

Where $j -$ is imaginary unit, coefficients $K_i^{(E)}, K_i^{(B)}$ and $K_i^{(\sigma)}$ in the equations (1) and (2) are called small perturbing body form-factors by corresponding components $(i = x, y, z)$ of intensity of electric field (E_i) and magnetic induction (B_i) in a position of a small perturbing body.

A perturbing body's form factors $(p.b.)$ depend on a shape and sizes of a body; they determine magnetization, polarization and electrical current conduction capability of a perturbing body's material (σ) in a case it is subjected to an external electromagnetic field. It worth mentioning, that form factors of a perturbing body in the equations (1) and (2) have the same values

The equation (1) is the equation of non resonance perturbances; it is generally used for a measurement of fields in acceleration sections with running wave; however they can be applied for a measurement of fields in accelerating sections with standing wave. In that equation the values $\Delta S_{2,1} = \Delta S_{1,2} = S_{2,1}^{(p.b.)} - S_{2,1} = S_{1,2}^{(p.b.)} - S_{1,2}$ are changes of a complex transmission coefficient from a certain reference plane of $1 -$

arm to a certain reference plane of $2-$ arm of a studied multiple arm electrodynamic system in a case of an introduction of a perturbing body in it, at the same time, the studied electrodynamic system is considered symmetric and $\omega = 2\pi f -$ is angular frequency, at which measurements are carried out. Where $S_{2,1} = S_{1,2} -$ a complex transmission coefficient in a case a small perturbing body is positioned outside a studied electrodynamic system and $S_{2,1}^{(p.b.)} = S_{1,2}^{(p.b.)} -$ the same complex transmission coefficient, in a case perturbing body is introduced into a studied system.

In the equation (1) $E_i^{(1)}, B_i^{(1)} -$ complex amplitudes of components of electric field's intensity and magnetic induction of not perturbed electromagnetic field, which is created in a studied electrodynamic system in a case it is excited through $1-$ arm and matched $2-$ arm. In turn, $E_i^{(2)}, B_i^{(2)} -$ are complex amplitudes of components of electric field's intensity and magnetic induction of not perturbed electromagnetic field, which is created in a studied electrodynamic system in a case it is excited through $2-$ arm and matched $1-$ arm. At the same time $P_1 -$ is incident power in a reference plane of $1-$ arm and $P_2 -$ is incident power in a reference plane of $2-$ arm. In general, remaining arms, if they exist, can be terminated by matched and unmatched loads.

In a case of a measurement of fields in an accelerating section, it can be connected into a measuring circuit using quadrapole or dipole scheme. In the first case complex transmission coefficient $S_{2,1} = S_{1,2}$ is measured in a case there is no perturbing body in a drift tube of an accelerating section, complex transmission coefficient $S_{2,1}^{(p.b.)} = S_{1,2}^{(p.b.)}$ is also measured in a case a perturbing body is introduced into a drift tube. In general, complex amplitudes of components of electromagnetic field in a drift tube of an accelerating section in a case of an excitement through $1-$arm and $2-$ arms are different, i.e. $E_i^{(1)} \neq E_i^{(2)}, B_i^{(1)} \neq B_i^{(2)}$ even if $P_1 = P_2$.

In the second case $P_1 = P_2 = P$, $E_i^{(1)} E_i^{(1)} = \left(E_i^{(1)}\right)^2$ and $B_i^{(1)} B_i^{(1)} = \left(B_i^{(1)}\right)^2$ are squares of complex amplitudes of components of not perturbed electromagnetic field created in a studied electrodynamic system in a case it is excited through $1-$ arm and incident power in that arm equal to P. At that, remaining arms, if they exist, can be terminated by matched and unmatched loads. In that case $\Delta S_{1,1} = \Delta G_1 = G_1^{(p.b.)} - G_1$ is a change of complex reflection coefficient in a certain reference plane of $1-$ arm, through which a section's electromagnetic field is excited. The method of non-resonance small perturbances exactly in that form is the most frequently used for a measurement of fields in accelerating sections both with running and standing waves. Scattering parameters can be determined experimentally using $HP\ Network\ Analyzer$.

Because at axis of a drift tube in accelerating sections of linear electron accelerators, in the majority of cases, there is only one component of electromagnetic field, which is longitudinal component of electric field's intensity, let's define its complex amplitude as E_z, and then the equation (1) will have the following form

$$\Delta G_1 = -\frac{j\omega}{4P} K_z^{(E)} E_z^2. \qquad (1.1)$$

The equations (1) and (1.1) show that the method of non resonance perturbances allows to obtain information both about absolute value and phase of components of unperturbed electromagnetic field of forced oscillations created in a studied electrodynamic system in a case it is excited on a given frequency $\omega = 2\pi f$ and under given conditions.

The equation (2) is the equation of small resonance perturbances and it is used for measuring fields in accelerating sections with standing wave, i.e. in acceleration resonators. In that equation $\left(f_0 - f_0^{(p.b.)}\right)/f_0 -$ relative shift of resonance frequency of a studied mode of resonator in a case of an introduction of a small perturbing body into a drift tube and $f_0^{(p.b.)}, f_0 -$ resonance oscillation frequencies of a studied mode of a resonator in a case of an introduction of a perturbing body into a resonator or without it.

$E_i, B_i -$ complex amplitudes of components of intensity of electric filed and magnetic induction of unperturbed oscillations of a resonator at a position of a perturbing body and $W -$ total energy of electromagnetic field, which is stored in a resonator during its excitation at resonance frequency f_0 of a studied mode without a perturbing body.

A derivation of the equations (2) is based on a discussion of free oscillations of a studied mode of an accelerating resonator. At the same time it is accepted that that mode of a resonator has negligibly small losses and its quality factor Q is high in a case there is no perturbing body. A perturbing body is of very small sizes and introduces very small additional losses into a resonator, i.e. quality factor of a studied mode of a resonator $Q^{(p.b.)}$, in a case a perturbing body is introduced, is also small. In a single-wave approximation unperturbed and perturbed free oscillations of a studied mode of a resonator attenuate with time according to the law $exp\{-[\pi f_0/Q]t\}$ and $exp\left\{-\left[\pi f_0^{(p.b.)}/Q^{(p.b.)}\right]t\right\}$ respectively. In a case of high quality factors of a resonator's mode Q and $Q^{(p.b.)}$, unperturbed and perturbed frequency of free oscillations of the studied mode of a resonator $\left(f_{free}, f_{free}^{(p.b.)}\right)$ doesn't considerably differ from unperturbed and perturbed resonance frequency of that mode. $\left(f_0, f_0^{(p.b.)}\right)$

$$f_{free} = f_0\sqrt{1-[1/(2Q)]^2} \approx f_0, f_{free}^{(p.b.)} = f_0^{(p.b.)}\sqrt{1-[1/(2Q^{(p.b.)})]^2} \approx f_0^{(p.b.)}.$$

Thus, we are transferring from free oscillations of a studied mode of a resonator to forced harmonic oscillations of that mode, which is excited at its resonance frequencies f_0 and $f_0^{(p.b.)}$. In addition, we presume that a relatively small shift of resonance frequency of that mode of a resonator, caused by a small perturbing body, is negligibly small, i.e $\left(f_0 - f_0^{(p.b.)}\right)/f_0 \ll 1$.

For a measurement of a longitudinal component of intensity of electric field in a drift tube of an accelerating resonator we can use the equation of resonance perturbances in the following form

$$\frac{f_0 - f_0^{(p.b.)}}{f_0} = \frac{\Delta f_0}{f_0} = \frac{K_z^{(E)}|E_z|^2}{4W}\ . \tag{2.1}$$

In particular, the equation (2.1) shows that $|E_z| \sim \sqrt{\Delta f_0 / f_0}$ in a case of movement of a small cylinder-shape perturbing body (needle) along axis of a drift tube of an accelerating resonator. The method of small resonance perturbances allows to evaluate only modulus of complex amplitude of a longitudinal component of electric field and doesn't allow to evaluate its phase.

It is worth mentioning that an experimental determination of resonance frequencies of a studied mode of resonator f_0 and $f_0^{(p.b.)}$ also can be carried out using results of a measurement of scattering parameters $S_{2,1}(f) = S_{1,2}(f)$ and $S_{2,1}^{(p.b.)}(f) = S_{1,2}^{(p.b.)}(f)$ or $S_{1,1}(f)$ and $S_{1,1}^{(p.b.)}(f)$ as a function of frequency.

In the following part of the study numerical simulation of a measurement of an accelerating field in a drift tubes of accelerating sections of various type is presented. Determination of scattering parameters of accelerating sections with and without a perturbing body is also carried out using numerical simulation. On the basis of that modeling method measurement error will be evaluated.

Methodology

Let's consider that accelerating sections are chains of connected cells [10, 11]. It is reasonable to use equivalent circuit, which is presented in figure 1 for a calculation of complex amplitudes of accelerating stresses in cells of those sections [12, 13, 14, 15].

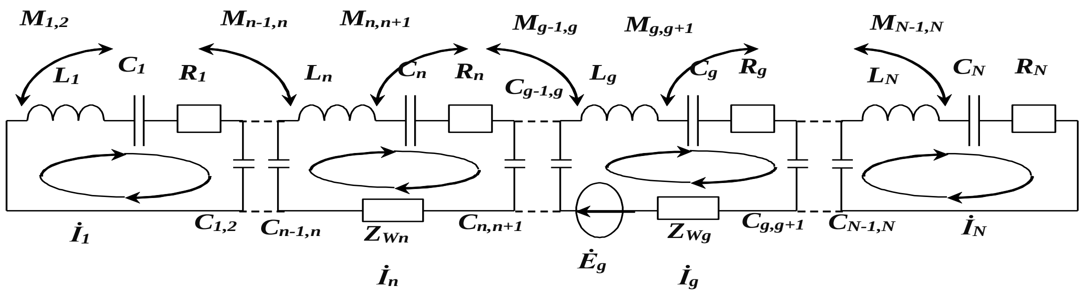

Figure 1: Equivalent circuit of accelerating section of linear electron accelerator.

A longitudinal accelerating component of electric field in the circuit is presented by a longitudinal capacitive element C_k and an azimuthal component of magnetic field is presented by means of inductive elements L_k. Inherent losses in each cell are presented by means of resistance element R_k. Where $k = 1,2,3, \ldots, N -$ number of a cell and $N -$ number of cells in an accelerating section. Cells are connected both via electric field at a section's axis (capacitive coupling) and via magnetic field, e.g. by means of peripheral coupling slots (inductive coupling). Both of those types of coupling between cells are presented in a figure by means of transversal capacitive elements with capacitance $C_{k,k+1}$ and inductive coupling between inductive elements L_k and L_{k+1}, which is characterized by mutual inductance $M_{k,k+1}$.

An excitation of electromagnetic field in accelerating sections is carried out via a cell with the number g, which is presented in the circuit by means of an introduced into that cell voltage source with complex amplitude $\dot{E}_g$ and an introduced wave resistance of supply waveguide line Z_{Wg}, through which electromagnetic field in an accelerating section is excited. In order to model connection of an accelerating section into circuit using quadrapole scheme there is a waveguide line connected with $n-$ cell, which is presented in the circuit by means of $n-$ cell of an introduced wave resistance of that waveguide line Z_{Wn}. Loop currents are also presented in the circuit, their are complex amplitudes are equal to $\dot{I}_k$.

For the purposes of the further analysis it is convenient to pass from electrical engineering parameters of cells $L_k, C_k, R_k, C_{k.k+1}, M_{k,k+1}$ radio engineering parameters: $f_{ck} = 1/\left(2\pi\sqrt{L_k C_{\Sigma k}}\right)-$ frequency of $k-$ cell; $C_{\Sigma k} = \left(C_k^{-1} + C_{k-1,k}^{-1} + C_{k,k+1}^{-1}\right)^{-1}$; $k \neq 1, N$; $C_{\Sigma 1} = \left(C_1^{-1} + C_{1,2}^{-1}\right)^{-1}$; $C_{\Sigma N} = \left(C_N^{-1} + C_{N-1,N}^{-1}\right)^{-1}$; $K_{ck,k+1}/2 = \sqrt{C_{\Sigma k} C_{\Sigma k+1}}/C_{k,k+1}-$ coefficient of capacitive coupling between cells with the numbers k and $k+1$ $\left(K_{cN,N+1}/2 = 0\right)$;

$K_{Lk,k+1}/2 = M_{k,k+1}/\sqrt{L_k L_{k+1}}-$ coefficient of inductive coupling between the same cells $\left(K_{LN,N+1}/2 = 0\right)$; $r_{shk}/Q_{0k}-$ ratio of shunt resistance of $k-$ cell to its quality factor $Q_{0k} = \sqrt{L_k/C_{\Sigma k}}/R_k$; $\chi_g = Z_{Wg}/R_g$ and $\chi_n = Z_{Wn}/R_n-$ coupling coefficients of cells with the numbers g and n with corresponding waveguide lines.

Considering that an accelerating section consists of similar cells with the same inherent quality factors Q_0, coefficients of capacitive and inductive coupling between adjacent cells $(K_c/2, K_L/2)$ and r_{sh}/Q_0, it is possible to obtain a complete system of equations relatively to complex amplitudes of loop currents

$$\left.\begin{array}{c}
\left(1-\frac{f^2}{f_{c1}^2}+j\frac{f}{f_{c1}}\frac{1}{Q_0}\right)\dot{I}_1-\left(\frac{K_c}{2}+\frac{f^2}{f_{c1}^2}\frac{K_L}{2}\right)\dot{I}_2=0;\\
\text{==}\\
-\left(\frac{K_c}{2}+\frac{f^2}{f_{cg}^2}\frac{K_L}{2}\right)\dot{I}_{g-1}+\left(1-\frac{f^2}{f_{cg}^2}+j\frac{f}{f_{cg}}\frac{1+\chi_g}{Q_0}\right)\dot{I}_g-\left(\frac{K_c}{2}+\frac{f^2}{f_{cg}^2}\frac{K_L}{2}\right)\dot{I}_{g+1}=\\
=j2\frac{f}{f_{cg}}\frac{\chi_g}{Q_0}\left(\frac{\dot{E}_g}{2Z_{Wg}}\right);\\
\text{==}\\
-\left(\frac{K_c}{2}+\frac{f^2}{f_{cn}^2}\frac{K_L}{2}\right)\dot{I}_{n-1}+\left(1-\frac{f^2}{f_{cn}^2}+j\frac{f}{f_{cn}}\frac{1+\chi_n}{Q_0}\right)\dot{I}_n-\left(\frac{K_c}{2}+\frac{f^2}{f_{cn}^2}\frac{K_L}{2}\right)\dot{I}_{n+1}=0;\\
\text{==}\\
-\left(\frac{K_c}{2}+\frac{f^2}{f_{ck}^2}\frac{K_L}{2}\right)\dot{I}_{k-1}+\left(1-\frac{f^2}{f_{ck}^2}+j\frac{f}{f_{ck}}\frac{1}{Q_0}\right)\dot{I}_k-\left(\frac{K_c}{2}+\frac{f^2}{f_{ck}^2}\frac{K_L}{2}\right)\dot{I}_{k+1}=0;\\
k\neq 1,g,n,N\\
\text{==}\\
-\left(\frac{K_c}{2}+\frac{f^2}{f_{cN}^2}\frac{K_L}{2}\right)\dot{I}_{N-1}+\left(1-\frac{f^2}{f_{cN}^2}+j\frac{f}{f_{cN}}\frac{1}{Q_0}\right)\dot{I}_N=0.
\end{array}\right\} \quad (3)$$

It is worth mentioning that coefficients of capacitive coupling between cells are always positive, but coefficient of inductive coupling can be both positive and negative.
In a case of a connection of a studied accelerating section into a measuring circuit using dipole scheme, complex reflection coefficient is measured in a certain cross-section of a supply waveguide line, which is connected to $g-$ cell of an accelerating section. By means of equivalent scheme of an accelerating section and the system of equation (3), it is possible to obtain the following expression for a complex reflection coefficient in a certain cross-section of a supply waveguide line

$$G_{inp}(f) = -[(\dot{E}_g/\dot{I}_g - Z_{Wg}) - Z_{Wg}]/[(\dot{E}_g/\dot{I}_g - Z_{Wg}) + Z_{Wg}] = \frac{2\dot{I}_g Z_{Wg}}{\dot{E}_g} - 1. \quad (4)$$

That complex reflection coefficient can be calculated as follows. Setting generator parameters $\dot{E}_g/(2Z_{Wg})$, e.g. equal to 1 (A) and calculating, using the system of equations (3), complex amplitudes of all loop currents. Then, according to the expression (4) calculating a value of reflection coefficient. At the same time generator frequency f, frequencies of all cells f_{ck}, coefficients of coupling between adjacent cells $K_c/2$ and $K_L/2$, inherent quality factors of cells Q_0, as well as coupling coefficient χ_g $g-$ of a cell with a supply waveguide are considered known $(\chi_n = 0)$.
After connection of an accelerating section using quadrapole scheme a measurement of complex coupling coefficient $(S_{n,g} = S_{g,n})$ is carried out from a certain cross-section of a supply waveguide line connected with $g-$ cell, to a certain cross-section of waveguide line connected with $n-$ cell and terminated by matched load. Using the equivalent circuit of an accelerating section presented in figure 1 and the system of the equations (3), it is possible to obtain the following equation for complex coupling coefficient $\chi_n \neq 0)$

$$S_{n,g}(f) = \frac{\dot{I}_n}{\dot{I}_g} \times \frac{\chi_n}{\chi_g} \times \frac{1+G_{inp}(f)}{1-G_{inp}(f)}. \quad (5)$$

Thus, we can calculate complex amplitudes for all loop currents $\dot{I}_k(f)$, complex reflection coefficient $G_{inp}(f)$ and complex transmission coefficient $S_{n,g}(f)$ as functions of frequency f.
Absence of presence of a perturbing body in a certain cell of an accelerating section is simulated by the frequency of each cell can become equal f_{ck} (frequency of unperturbed cell) and $f_{ck}^{(p.b.)}$ (frequency of perturbed cell). At the same time, a relative detuning of a cell caused by a perturbing body is the same for all cells and quite small, i.e. $\left(f_{ck} - f_{ck}^{(p.b.)}\right)/f_{ck} = (\Delta f_c/f_c)_{p.b.} \ll 1$.

Results

Simulation of a measurement of high-frequency field in an accelerating section with running wave:

An excitation of electromagnetic field in an accelerating section with running wave is carried out using supply waveguide line connected with the first cell (Kalyuzhny and Kalyuzhny, 2008). In order to measure an accelerating field at axis of a drift tube a section if connected using dipole scheme, reflection coefficient is measured in a reference plane of that supply waveguide line $(g = 1)$.

In order to provide pure running wave propagation in an uniform accelerating section and absence of reflections in a waveguide line it is necessary that frequencies of cells, loaded quality factor of the last cell relatively to a matched waveguide line, connected with that cell, and coupling coefficient of the first cell with a supply waveguide line are equal to:

$$\begin{cases} f_{c1} = f_{op}\sqrt{\dfrac{1+\frac{K_L}{2}exp(-\alpha_{op}D)cos(\varphi_{op})}{1-\frac{K_c}{2}exp(-\alpha_{op}D)cos(\varphi_{op})}}, \\ f_{cn} = f_{op}\sqrt{\dfrac{1+K_L ch(\alpha_{op}D)cos(\varphi_{op})}{1-K_c ch(\alpha_{op}D)cos(\varphi_{op})}}, n = 2,3,4,,N-1, \\ f_{cN} = f_{op}\sqrt{\dfrac{1+\frac{K_L}{2}exp(\alpha_{op}D)cos(\varphi_{op})}{1-\frac{K_c}{2}exp(\alpha_{op}D)cos(\varphi_{op})}}, \\ \chi_1 = 1 + Q_0\left|\dfrac{f_{c1}}{f_{op}}\dfrac{K_c}{2}+\dfrac{f_{op}}{f_{c1}}\dfrac{K_L}{2}\right| exp(-\alpha_{op}D)sin(\varphi_{op}), \\ \dfrac{Q_0}{1+\chi_N} = \dfrac{1}{\left|\frac{f_{cN}}{f_{op}}\frac{K_c}{2}+\frac{f_{op}}{f_{cN}}\frac{K_L}{2}\right| exp(\alpha_{op}D)sin(\varphi_{op})}. \end{cases} \quad (6)$$

In those expression $N-$ is a number of cells in a uniform sections $\chi_N -$ is coupling coefficient of the last cell with a withdraw waveguide line, $f_{op} -$ is a working frequency, with which running wave is propagating with working type of oscillations φ_{op} and working attenuation $exp(-\alpha_{op}D)$ on one cell ($D -$ geometry period of an accelerating section).

Working type of oscillations and working attenuation on one cell are connected with each other via the following relationship:

$$Q_0^2 sin^2(\varphi_{op})[ch^2(\alpha_{op}D)-1]\left\{\begin{matrix} K_c^2[1+K_L cos(\varphi_{op})ch(\alpha_{op}D)]^2 + \\ +2K_cK_L\begin{bmatrix} 1-K_c cos(\varphi_{op})ch(\alpha_{op}D)+ \\ +K_L cos(\varphi_{op})ch(\alpha_{op}D)- \\ -K_cK_L cos^2(\varphi_{op})ch^2(\alpha_{op}D)\end{bmatrix} + \\ +K_L^2[1-K_c cos(\varphi_{op})ch(\alpha_{op}D)]^2 \end{matrix}\right\} - \\ -\begin{bmatrix} 1-K_c cos(\varphi_{op})ch(\alpha_{op}D)+ \\ +K_L cos(\varphi_{op})ch(\alpha_{op}D)- \\ -K_cK_L cos^2(\varphi_{op})ch^2(\alpha_{op}D)\end{bmatrix} = 0. \quad (7)$$

In the expression (7) $ch(\alpha_{op}D) -$ hyperbolic cosine of argument $(\alpha_{op}D)$. An analogous equation is connecting unconditioned type of oscillations φ $(0 < \varphi < \pi)$ and corresponding attenuation on one cell $exp(-\alpha D)$.

Numerical simulations calculations were carried out for an accelerating section with opposite running wave having the following parameters $f_{op} = 3$ GHz, $\varphi_{op} = 120^0, Q_0 = 15000, K_c/2 = 0.01, K_L/2 = -0.02$.
Frequencies of cells, for which attenuating opposite running wave is propagating in a structure at working frequency, are equal to $f_c = 3.014815$ GHz and attenuation on one cell at working frequency is equal to $exp(-\alpha_{op}D) = 0.9961$. Figure 2 shows results of calculation of phase voltage distribution on a section's cells (o) and results of simulation of a measurement of those values (×). Number of cells in a section $N = 40$, ideally tuned cells must have the following parameters:
$f_{c1} = 3.007425, f_{c2} = \cdots = f_{cN-1} = 3{,}014815, f_{cN} = 3{,}007482\ Ghz$
$\chi_1 = 129.438173, \chi_N = 128.438115$

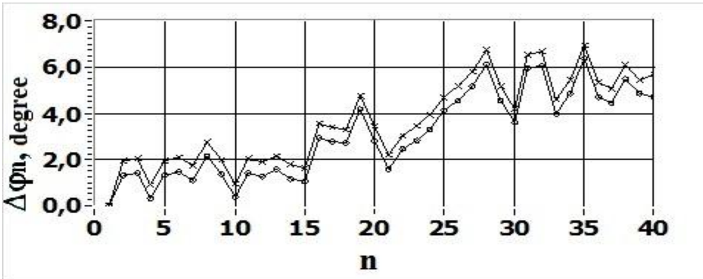

Figure 2: Distribution of accelerating voltage's phase: o – without perturbing body; × − simulation of changes.

Complete attenuation at working frequency f_{op} for ideally tuned section is equal to $exp[-(N-1)\alpha_{op}D] = 0.858679$. For the conducted calculation frequencies of cells have random relative scattering with uniform distribution in the range $(\delta f_c/f_c)_{random} = \pm 10^{-4}$ and relative detuning of cells cause by a perturbing body is $(\Delta f_c/f_c)_{p.b.} = 10^{-4}$.
The analysis demonstrated that a maximum difference of calculated relative amplitude of voltage and its phase and relative amplitude of voltage and its phase obtained as a result of simulation their measurements is $0.76 - 0.82$ % and $0.96^0 - 1.00^0$ respectively,depending on sample of random scattering of cells' frequencies.
In the figure 2 the value $\Delta\varphi_n$ is phase difference on a cell with the number n and the phase, which must correspond to that voltage in a case of ideal tuning of all cells of a section. Because dispersion of ideal structure, on which basis the discussed section was created, is negative, then voltage phase on each cell for ideally tuned section is equal to $\varphi_{ideal\ n} = +(n-1)120^0$.

Simulation of a measurement of high-frequency field in an accelerating section with standing wave:
Cells of accelerating resonators and resonators themselves generally have high inherent quality factor. For a measurement of accelerating field at axis of a drift tube the

most used method is the method of small resonance perturbances. At that, it is necessary to carry out measurements of resonance frequencies of studied resonator mode f_0 and $f_0^{(p.b.)}$. In the major part of cases resonance frequencies of a studied mode of a resonator are determined as frequencies, for which reflection in a supply waveguide line $G(f), G_{inp}^{(p.b.)}(f)$ are minimal by modulus (resonator is set up using dipole scheme) or as frequencies, for which coupling coefficients $S_{n,g}(f), S_{n,g}^{(p.b.)}(f)$ are maximum by module.

The presented study is limited only for discussion of resonators with working $\boldsymbol{\pi}$ −type of oscillations [17, 18]. For that kind of ideally tuned resonator without losses cells' frequencies must be equal to:

$$f_{c1} = f_{cN} = f_{op}\sqrt{\frac{1-\frac{K_L}{2}}{1+\frac{K_c}{2}}}; f_{ck} = f_{op}\sqrt{\frac{1-K_L}{1+K_c}}, k \neq 1, N, g. \quad (8)$$

However, due to losses difference of voltage phases on adjacent cells will not be exactly equal to 180^0.

The value of frequency f_{cg} and coupling coefficient χ_g will be selected in such a way in order that reflection coefficient $G_{inp}(f_{op})$ at working frequency will be minimal (equal to zero). Let's presume that coupling coefficient is$\chi_n \ll 1$, while coupling coefficient χ_g has comparatively big value. In a case of high quality factor χ_g has a value, which is close to a number of cells N. With a decrease of quality factor of cells χ_g is decreasing.

Using a system of the equations (3) and the equations (4) and (5) for reflection coefficient $G_{inp}(f), G_{inp}^{(p.b.)}(f)$ and transmission coefficient $S_{n,g}(f), S_{n,g}^{(p.b.)}(f)$ we can determine resonance frequencies of a studied mode of a resonator f_0 and $f_0^{(p.b.)}$ as frequencies, for which modulus of those values having, correspondingly, minimum and maximum. Moreover we can simulate measurements also by means of the method of small non resonance perturbances.

A calculation of such simulation was carried out for a resonator with the following parameters: $f_{op} = 3{,}0$ GHz, $= 19, g = 10, n = 1, Q_0 = 15000, K_c/2 = 0.01, \frac{K_L}{2} = -0.02, \chi_n = 10^{-3}$. For an ideally tuned resonator cells' frequencies must be equal to: $f_{c1} = f_{c19} = 3.014815, f_{c2} = \cdots = f_{c9} = 3.029269, f_{c10} = 3.028909, \quad f_{c11} = \cdots = f_{c18} = 3.029269$ GHz and for matching at a working frequency it is necessary that $\chi_g = 18.069638$. In that case $G_{inp}(f_{op}) = 0$ and $|S_{n,g}(f_{op})| = 5.365 \cdot 10^{-5}$.

Frequencies of cells have random relative scattering with uniform distribution in the range $(\delta f_c/f_c)_{random} = \pm 5 \cdot 10^{-5}$ and relative detuning of cells cause by a perturbing body is $(\Delta f_c/f_c)_{p.b.} = 10^{-5}$. Distribution ofphase is obtained using the method of non-resonance small perturbances.

Calculation results show that the best results is achieved using the method of non-resonance small perturbances, which have the lowest systematic error for changes of amplitude (in our case 0.2%), in a case of a change of phase a perturbance can be lower than 0.5^0.

Discussion

A comparative analysis of the discussed methods for a measurement of accelerating field in a drift tube of accelerating sections demonstrate that the least systematic error is obtained using the method of non-resonance small perturbances, which can be successfully used both in sections with running wave and standing wave. However, requirements for a perturbing body in the second case are stricter. Relative detuning of frequency of cells of an accelerating sections with standing wave cause by a perturbing body in that case should not exceed $(\Delta f_c/f_c)_{p.b.} = 10^{-5}$. In the first case relative detuning of frequency of cells of an accelerating section with running wave cause by a perturbing body can be greater by order of magnitude. It related with a fact that similar relative detuning of frequency of a cell leads to a different perturbance of field in a resonator and in a section with running wave with the same cell parameters (close frequencies of cells and coefficients of coupling between cells).

If a number of a cell, through which a resonator is excited, and a number of a cell, which is connected with a second arm, are not equal to $(g \neq n)$, then an implementation of small resonance perturbances method with use of a measurement of coupling coefficient $S_{n,g}$ is unreasonable, because measurement error in that case may be unacceptably big. In a case of $g = n$ both methods of a measurement of a resonance frequency of a studied mode of a resonator produce approximately the same result.

An implementation of the small resonance perturbances method is unreasonable also in the case, when quality factor of a cell and a resonator are small.

Conclusion

The results of a simulation of a measurement of an accelerating field in linear electron accelerators in a section with running wave demonstrated good conformity. The maximum value of measured and calculated relative amplitude of voltage is equal to $0.76 - 0.82$ %, and phase $0.96^0 - 1.00^0$ depending on a sample of random scattering of cells' frequencies.

For accelerating sections with standing wave the results of a simulation of a resonator with parameters $f_{op} = 3{,}0$ GHz, $N = 19, g = 10, n = 1, Q_0 = 15000, K_c/2 = 0.01, \frac{K_L}{2} = -0.02, \chi_n = 10^{-3}$ demonstrated that for an ideally tuned resonator cells' frequencies must be equal to $f_{c1} = f_{c19} = 3.014815, f_{c2} = \cdots = f_{c9} = 3.029269, f_{c10} = 3.028909, f_{c11} = \cdots = f_{c18} = 3.029269$ GHz. For matching at working frequency it is necessary that $\chi_g = 18.069638$, then $G_{inp}(f_{op}) = 0$ and $|S_{n,g}(f_{op})| = 5.365 \cdot 10^{-5}$. In the discussed example frequencies of cells have random relative scattering with uniform distribution in the range $(\delta f_c/f_c)_{random} = \pm 5 \cdot 10^{-5}$ and relative detuning of cells cause by a perturbing body is $(\Delta f_c/f_c)_{p.b.} = 10^{-5}$.

An implementation of small resonance frequencies method is reasonable, when cells and a resonator have high quality factor.

The authors propose the further application of the discussed methodology for measurement of high-frequency fields:

- irregular sections of linear electron accelerator designed for grouping of particles in bunches for increased efficiency of acceleration;
- field of higher modes, which can be excited in accelerating sections.

Acknowledgments

The authors would like to express their gratitude for all, who contributed into discussion of the presented paper and especially for Professor Eduard Yakovlevich Shkol'nikov and Professor Nikolai Pavlovich Sobenin.

References

[1] O. Val'dner, A. Vlasov, A. Shal'nov, “Linear particle accelerators”. - Moscow: Atomizdat, 1969.

[2] S. Kutsaev, N. Sobenin, A. Smirnov, et al., “Design of hybrid electron linac with standing wave buncher and travelling wave accelerating structure”, Nuclear Instruments and Methods, vol. 629, pp. 258-271, 2011. DOI: 10.1016/j.nima.2011.01.047

[3] A. Zavadtsev, “Biperiodic U-structures for Particle Accelerators”, Fourth European Accelerator Conference, London, vol. 3, pp. 2176-2178, 1994.

[4] A. Zavadtsev, Yu. Petrov, N. Sobenin, “Standing Wave Electron LINAC Accelerating Structure for Technology Purposes” Fourth European Accelerator Conference, London, vol. 3, pp. 2173-2175, 1994.

[5] V. Vikulov, B. Zverev, A. Zavadcev, et al. “Compact linear electron accelerator with standing wave and bridge power circuit. Accelerators, Issue 19”. - Moscow: Atomizdat, 1980.

[6] N. Sobenin, S. Yarigin, D. Kostin, A. Zavadtsev, “A Biperiodic Accelerating Structure for a Free Electron Laser Buncher”, Partical Accelerator Conference and International Conference on High-Energy Accelerators, Dallas, Texas, 1995.

[7] A. Zavadtsev, D. Zavadtsev, S. Kutsaev, et al., “Compact Electron Linear Accelerator Relus-5 for Radiation Technology Application”, Proceedings of EPAC'06 conference, Edinburgh, Scotland, pp. 2385-2387, 2006.

[8] Ch. Steele, “A Nonresonant Perturbation Theory”, IEEE Trans-actions on MTT, vol. 14(2), pp. 70-74, 1966.

[9] M. Nakamura, “Theory of Field Strength Determination in RF Structures by Perturbation Techniques”, Japanese Journal of Applied Physics, vol. 7(2), pp.146-155, 1968.

[10] E. Njepp, B. Njepp, D. Potter, “Accelerating structures with standing wave for linear particle accelerators for big energies”, Scientific research equipment, vol. 7(39), pp. 31-43, 1968.

[11] V. Kul'man, Je. Mirochnik, V. Pirozhenko, “Accelerating structures with ring communication resonators”, Experimental equipment and methodology, vol. 4, pp. 56-61, 1970.

[12] V. Kaluzhniy, "Analysis of accelerating structures based on round septate waveguide with equivalent circuit", Engineering Physics, vol. 3, pp. 27-37, 2006.

[13] V. Kalyuzhnyi, A. Novozhilov, A. Filatov, V. Shilov, "Relative Jitter of Accelerating Voltage Amplitude on Resonator's Cells of Linear Electron-positron collider TESLA-ILC", World Applied Sciences Journal, vol. 27(12), pp. 1620-1624, 2013. DOI: 10.5829/idosi.wasj.2013.27.12.13702

[14] N. Gavrilov, O. Novikov, N. Samargin, Yu. Strukov, Ye. Trihin, "Calculation of geometrical parameters of slowing down systems of type of chains of the connected cavity" Engineering Physics, vol. 2, pp. 14-17, 2009.

[15] D. Njegl, E. Njepp, B. Njepp, "Description of a section of an accelerator with standing wave by means of coupled oscillators' model", Equipment for scientific researches, vol. 11(38), pp. 22-26, 1967.

[16] V. Kalyuzhny, O. Kalyuzhny, "The analysis of transient and the established mode in muchcell accelerating sections with electric and magnetic communication between cells", Engineering Physics, vol. 3, pp. 27-36, 2008.

[17] B. Knapp, E. Knapp, G. Lucas, J. Potter, "Accelerating Structures Resonantly Coupled for High-current Proton Linacs", The First National Particle Accelerator Conference, Washington D.C., NS-12(3), pp. 159, 1965.

[18] V. Kaluzhniy, N. Nechaev, "Adjustment of multiperiodical structures for linear accelerators of electrons with stationary wave", Technical Physics Magazine, vol. 53(8), pp. 1517-1521, 1983.