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Solution of Problem of Heating Elements' Location of Distributed Control Objects

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Abstract

At the present stage of development of automatic control systems the issue of maintaining of a given temperature regime appears. The authors have developed a method of synthesis of nonlinear regulators to stabilize a control object temperature field. The resulting controller allows creating an adaptive regulated system to maintain a temperature field – to do that, it is necessary to analyze a control system in order to find an optimal number of heating elements to stabilize a temperature field. In this there have been article obtained functions for calculating an optimal number of heating elements of silicon-carbide heaters, used for a temperature treatment of various kinds of products (bread baking, metal hardening, etc.). The article describes a method to calculate the number of heating elements of n-dimensional control objects. A check of an adequacy of a mathematical model, obtained within a synthesis process is provided as well. Obtained hodographs of the system demonstrate a stability of running processes and therefore an adequacy of results.

Keywords: Green's function, heat field, discretization interval, control object, analysis, synthesis.

Introduction

At the present stage of development of automatic control systems there is a problem of considering of control objects with spatial coordinates, methods of research and analysis. For a more detailed understanding of the world around us, many objects need to be considered from the point of view of distributed objects. Fields of different nature, thermal conductivity and diffusion, synergetic technology, magnetism, solid body physics and much more − all these are facilities and systems with distributed parameters.

If we consider the classical automatic control system, it is possible to pay attention to a great mathematical apparatus reinforcing this system.

A theory of distributed parameter systems began to develop with the first works published by A.G. Butkovskiy and was offered in the works of T.K. Sirazetdinova, E.Y. Rapoport, I.M. Pershin, V.A. Koval and others in the territory of our country.

Also, works performed by foreign scholars such as J.-L. Lyons, S.G. Tzafistas, V. Wertz, P. Demise, I.S. Meditch, J.S Gibson, I.G. Rosen and several others. Formation of a theory of systems with distributed parameters is conditioned by a complexity and originality of a mathematical apparatus. The aim of automatic control of distributed systems is much more difficult than focused ones. This is mainly due to the necessity of spatially-distributed control of object state. Control of simulation results, including feedback closed systems. In a spatially-distributed control system the class of possible impacts on a control object becomes wider (e.g., spatial-temporal control). To consider these effects a theory of lumped systems becomes unacceptable. A feature of a study of systems with distributed parameter is a development of a mathematical apparatus and methods of their study.

Most of the results, obtained in a theory of systems with distributed parameters used in linear systems. Real automatic control systems are not purely linear. There are some cases where behavior of such systems can not be even approximately described by linear differential equations. Nonlinear automatic system is a system in which there is at least one unit, represented by a nonlinear equation. Non-linear equation is an equation that includes some coordinates or their derivatives in a form of products or power, different from the first, or their coefficients are functions of some coordinates or their derivatives. Techniques, developed for linear control systems, can not be applied to non-linear systems of general form. However, it is worth pointing out that among non-linear systems, there is a class of systems with one nonlinear element, for which this apparatus with a few changes is applied. The main objective of analysis, synthesis and simulation of nonlinear systems of automatic control is to find possible states of a system to ensure stable operation of a system. There is an

enormous amount of possible solutions, depending on various initial and boundary conditions, type of response to deviations from a set mode, etc.
Characteristic of nonlinear units is described by logical conditions indication. In connection with a non-linear characteristic, an output variable is not proportional to an input variable. Therefore, a system reaction, for instance, to a relay signal will depend on power of this signal. In some cases, it is needed to change an input signal to disbalance a system. If we consider a dynamics of vibrational dynamic control systems, it can be seen that within an attenuation of a transition process minor changes of an oscillation period appear.
Due to the absence of a uniform method for solving of nonlinear systems we have to conduct an analysis and synthesize a private method to solve a stated problem, overcoming considerable mathematical difficulties.
A peculiarity of relay control systems is a shape of an output variable, which is independent of an input variable. In these systems, a control action, changes abruptly whenever a control signal at an input of a relay element passes a predetermined range of permissible values.
At a current stage a problem of synthesis of closed, non-linear control systems with distributed parameters are poorly studied. A mathematical apparatus of synthesis of nonlinear distributed systems are not sufficiently developed. Systems created at the moment have eminently linear structure, which does not take into account geometry of an object. Scientific literature knows a number of basic methods for a synthesis of distributed objects such as:

- analytical design of optimal controllers (ADOC);
- parametric synthesis of regulators;
- finite-dimensional approximation of systems with distributed parameters and solution of a problem of synthesis of regulators by methods, used in concentrated systems;
- synthesis of control systems with movable effects;
- frequency method of synthesis.

ADOC method for systems with distributed parameters is based on a principle of Bellman's optimality and Pontryagin's maximum principle.
Synthesis of parametric controller is based on the use of structural theory, which a concept of distributed units is introduced. A description of distributed units is given by an impulse transition function (Green's function).
Finite-dimensional approximation of distributed systems based on the use of finite-dimensional representations of partial derivatives on the basis of "networks" and "lines", as well as using the Taylor series.
For systems with a movable exposure, special methods of analysis and synthesis have been developed. The main technical difficulty is to create high-impact sources of any physical nature.
Frequency method of regulators' synthesis is the main working tool in a design of concentrated systems with one input and one output.
Synergetic approach based on self-organization of nonlinear dynamical concentrated systems. It highlighted by a clearly defined physical content of control processes.

Synergetic control concept is developed in A.A. Kolesnikov’s works and his scientific school.

Problem statement

In current time tunnel ovens with conveyor-type electric heater consume large amount of energy, which in its turn affects a value of a product of a temperature treatment. Reduction of energy costs to maintain a temperature will reduce a cost of a final product.

Let us state a problem of calculating an optimal location of heating elements to heat an isotropic rod. We will calculation of a temperature field of hexagonal silicon carbide rod under an exposure a rod by pulsed heating elements with relay control principle.

To solve this problem it is necessary to determine a place of inclusion of a heating element, in which a silicon carbide rod will maintain a necessary temperature regime [1]. Thus it is needed to find a method of finding an optimal location of heating elements in stabilization of a temperature field of a conveyor type tunnel kiln. This technique should be checked on a hexagonal silicon carbide rod, which is used in a firing chamber of building ceramics in electric tunnel kilns of conveyor type. We consider a general case of distribution of temperature field and its stabilization in a given interval.

Methods

For a general case consideration it is common to use a mathematical model of an unknown dimension, that is, the so-called N − dimensional model. Where N is a number of dimensions of an object. A mathematical model of N− dimensional control object will look as follows

$$f(x,y,..,N,t)=\frac{\partial Q(x,y,..,P,t)}{\partial t}-a^2\left[\frac{\partial^2 Q(x,y,..,P,t)}{\partial x^2}+\frac{\partial^2 Q(x,y,..,P,t)}{\partial y^2}+...+\frac{\partial^2 Q(x,y,..,N,t)}{\partial N^2}\right]\ 0\le x\le l_1\,;$$

$0\le y\le l_2$; $0\le ....\le l$; $0\le N\le l_p$; $t\ge 0$; $a>0$;

Calculation of temperature indexes can be carried out by Green's function represented as an infinite Fourier series

$$G(x,...,p,\rho,...,p,t)=\frac{2^p}{l_1\cdot ...\cdot l_p}\cdot\sum_{k,...,n=1}^{\infty}\sin\left(\frac{k\cdot\pi\cdot x}{l_1}\right)\cdot ...\cdot\sin\left(\frac{k\cdot\pi\cdot p}{l_p}\right)\times$$

$$\times\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\cdot ...\cdot\sin\left(\frac{k\cdot\pi\cdot p}{l_p}\right)\cdot\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+...+\frac{p^2}{l_p^2}\right)\right]$$

This function allows determining a value of a temperature field in an object of any dimension. It is also possible to obtain values of time and place of heating elements’ switching on [1, 2-10].

Let’s try to establish, whether time of the second heating element switching on differs from a rod. To do this, let us calculate time and place of heating element switching on. To do this, as in the case of one-dimensional object, let us use the Green's function for two-dimensional control object.

$$G(x,y,\rho,\nu,t) = \frac{4}{l_1 \cdot l_2} \cdot \sum_{k,m=1}^{\infty} \sin\left(\frac{k \cdot \pi \cdot x}{l_1}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y}{l_2}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \rho}{l_1}\right) \times$$
$$\times \sin\left(\frac{m \cdot \pi \cdot \nu}{l_2}\right) \cdot \exp\left[-a^2\pi^2 \cdot t \cdot \left(\frac{k^2}{l_1^2} + \frac{m^2}{l_2^2}\right)\right]$$

And express variables x and y from it, which are responsible for finding coordinates of a heating element:

$$x = \arcsin \frac{\frac{4}{l_1 \cdot l_2} \cdot \sum_{k,m=1}^{\infty} \sin\left(\frac{m \cdot \pi \cdot y}{l_2}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \rho}{l_1}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot \nu}{l_2}\right) \times \exp\left[-a^2\pi^2 \cdot t \cdot \left(\frac{k^2}{l_1^2} + \frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,\nu,t)} \cdot \left(\frac{l_1}{k \cdot \pi}\right)$$

$$y = \arcsin \frac{\frac{4}{l_1 \cdot l_2} \cdot \sum_{k,m=1}^{\infty} \sin\left(\frac{m \cdot \pi \cdot x}{l_1}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \rho}{l_1}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot \nu}{l_2}\right) \times \exp\left[-a^2\pi^2 \cdot t \cdot \left(\frac{k^2}{l_1^2} + \frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,\nu,t)} \cdot \left(\frac{l_2}{m \cdot \pi}\right)$$

Similarly, let us express the variable t, which is responsible for switching on of a heating element with coordinates x and y.

$$t = \frac{\ln\left(\frac{\frac{4}{l_1 \cdot l_2} \cdot \sum_{k,m=1}^{\infty} \sin\left(\frac{k \cdot \pi \cdot x}{l_1}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot y}{l_2}\right) \cdot \sin\left(\frac{k \cdot \pi \cdot \rho}{l_1}\right) \cdot \sin\left(\frac{m \cdot \pi \cdot \nu}{l_2}\right) \cdot \exp\left[-a^2\pi^2 \cdot t \cdot \left(\frac{k^2}{l_1^2} + \frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,\nu,t)}\right) \cdot \left(\frac{l_1^2}{k^2} + \frac{l_2^2}{m^2}\right)}{a^2\pi^2}$$

Due to a complexity of calculation, let us introduce a number of restrictions for a calculation of current values. Namely, suppose that at some point of time a function takes a form in which it can be described by the first term of the Fourier's series.

$$T(x,y,t) =$$
$$= \frac{4}{l_1 l_2} \exp\left[-a^2\pi^2 t\left(\frac{1}{l_1^2} + \frac{1}{l_2^2}\right)\right] \cdot \sin\frac{\pi}{l_1} x \cdot \sin\frac{\pi}{l_2} y \cdot \sum_{i=1}^{d} \sin\frac{\pi}{l_1}\rho_i \cdot \sin\frac{\pi}{l_2}\nu_i$$

Having suppose that $x = \rho_1$, $y = \nu_1$, and specify time $t = \tau_1$, we get:

$$x = \arcsin \frac{\frac{4}{l_1 l_2} \exp\left[-a^2\pi^2 t\left(\frac{1}{l_1^2} + \frac{1}{l_2^2}\right)\right] \cdot \sin\frac{\pi}{l_2} y \cdot \sum_{i=1}^{d} \sin\frac{\pi}{l_1}\rho_i \cdot \sin\frac{\pi}{l_2}\nu_i}{T(x,y,t)} \cdot$$
$$\cdot \frac{l_1}{\pi}$$

$$y = \arcsin \frac{\frac{4}{l_1 l_2} \exp\left[-a^2\pi^2 t\left(\frac{1}{l_1^2} + \frac{1}{l_2^2}\right)\right] \cdot \sin\frac{\pi}{l_2} x \cdot \sum_{i=1}^{d} \sin\frac{\pi}{l_1}\rho_i \cdot \sin\frac{\pi}{l_2}\nu_i}{T(x,y,t)} \cdot$$
$$\cdot \frac{l_1}{\pi}$$

$$\tau_1 = \frac{1}{a^2\pi^2\left(\frac{1}{l_1} + \frac{1}{l_2}\right)} \ln\left(\frac{8\sin\frac{\pi}{l_1}\rho_1 \sin\frac{\pi}{l_2}\nu_1 \sum_{i=1}^{N} \sin\frac{\pi}{l_1}\rho_i \sin\frac{\pi}{l_2}\nu_i}{l_1 l_2 T_{set}}\right)$$

Results

Let us carry out a research on a location of heating elements of a plate and try to compare these data with research date of one-dimensional control object. To ensure a coordination of output data let us perform calculations on coordinate slices. A slice of a plate will be placed in a heating element location coordinate, since there are no heating elements outside a slice line, therefore, in finding of place and time of inclusion of a heating element is absent in a given coordinate. To conduct a study let us take the same constants as for one-dimensional control object, which have the following values l_1=10, a^2=0.01, k=10, d=9, τ=3, t=1..500, $x_1 = \xi_1 = 1$, $T_{set} = 0,2$, $\xi_i \in \{1,2,3,4,5,6,7,8,9\}$. Calculation result is represented in the table 1.

Table 1: Place and time of a source actuation

l=5	l=6	l=7	l=8
source=4.83. switched on = 9.43 sec	source =1.16. switched on =10 sec	source =1.04. switched on=20 sec	source =1.40. switched on=25.3 sec
source=3.48. switched on =9.33 sec	source=8.16. switched on =10 sec	source=6.93. switched on=20 sec	source =9.00. switched on=25.3 sec
source=2.93. switched on =9.27 sec	source=6.65. switched on =10 sec	source=5.31. switched on=20 sec	source=6.42. switched on=25.3 sec
source=2.76. switched on =9.26 sec	source=6.03. switched on =10 sec	source=4.50. switched on=20 sec	source=4.98. switched on=25.4 sec
source=2.87. switched on =9.23 sec	source=6.06. switched on =10 sec	source=4.24 switched on=20 sec	source=4.28. switched on=25.3 sec
source=3.34. switched on =9.23 sec	source=6.84. switched on =10 sec	source=4.54 switched on=20 sec	source=4.21. switched on=25.2 sec
source=4.57. switched on =9.23 sec	source=9.13. switched on =10 sec	source=5.80 switched on=20 sec	source=5.06. switched on=25.4 sec
		source=1.89. switched on=30 sec	source=2.31. switched on=25.1 sec
		source=1.78. switched on=30 sec	source=1.91. switched on=25.1 sec
		source=1.85. switched on=30 sec	source=1.94. switched on=24.7 sec

The results show that for a fixed length of a rod not all the heating elements are actuated. For example, within a fixed length l = 10 only heating elements 4, 3, 2 are actuated, thus there is no need to place 10 heating elements there. Also noteworthy is the fact that according to studies, intervals between actuation of subsequent sources are approximately equal, which in turn indicates that thermal processes, taking place in homogeneous bodies are stable. Let us try to establish whether an actuation time of the second heating element in a plate differs from a rod. To do this, let us calculate time and place of an actuation of a heating element. To do this, as in the case of a one-dimensional object let us use the Green's function for the two-dimensional control object.

$$G(x,y,\rho,v,t)=\frac{4}{l_1\cdot l_2}\cdot\sum_{k,m=1}^{\infty}\sin\left(\frac{k\cdot\pi\cdot x}{l_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot y}{l_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\times$$
$$\times\cdot\sin\left(\frac{m\cdot\pi\cdot v}{l_2}\right)\cdot\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}\right)\right]$$

And expressed variables x and y from it, which are responsible for finding coordinates of a heating element:

$$x=\arcsin\frac{\frac{4}{l_1\cdot l_2}\cdot\sum_{k,m=1}^{\infty}\sin\left(\frac{m\cdot\pi\cdot y}{l_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot v}{l_2}\right)\times\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,v,t)}\cdot\left(\frac{l_1}{k\cdot\pi}\right)$$

$$y=\arcsin\frac{\frac{4}{l_1\cdot l_2}\cdot\sum_{k,m=1}^{\infty}\sin\left(\frac{m\cdot\pi\cdot x}{l_1}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot v}{l_2}\right)\times\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,v,t)}\cdot\left(\frac{l_2}{m\cdot\pi}\right)$$

Similarly, let us express the variable t, which is responsible for switching on of a heating element with coordinates x and y.

$$t=\frac{\ln\left(\frac{\frac{4}{l_1\cdot l_2}\cdot\sum_{k,m=1}^{\infty}\sin\left(\frac{k\cdot\pi\cdot x}{l_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot y}{l_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot v}{l_2}\right)\cdot\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,v,t)}\right)\cdot\left(\frac{l_1^2}{k^2}+\frac{l_2^2}{m^2}\right)}{a^2\pi^2}$$

Due to a complexity of calculation, let us introduce a number of restrictions for a calculation of current values. Namely, suppose that at some point of time a function takes a form in which it can be described by the first term of the Fourier's series.

$$T(x,y,t)=\frac{4}{l_1l_2}\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot\sin\frac{\pi}{l_2}y\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}v_i$$

Having supposed that $x=\rho_1$, $y=v_1$, and specify $t=\tau_1$, we will get:

$$x=\arcsin\frac{\frac{4}{l_1l_2}\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}\right)\right]\cdot\sin\frac{\pi}{l_2}y\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}v_i}{T(x,y,t)}\cdot$$
$$\cdot\frac{l_1}{\pi}$$

$$y=\arcsin\frac{\frac{4}{l_1l_2}\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}\right)\right]\cdot\sin\frac{\pi}{l_2}x\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}v_i}{T(x,y,t)}\cdot$$
$$\cdot\frac{l_1}{\pi}$$

$$\tau_1=\frac{1}{a^2\pi^2\left(\frac{1}{l_1}+\frac{1}{l_2}\right)}\ln\left(\frac{8\sin\frac{\pi}{l_1}\rho_1\sin\frac{\pi}{l_2}v_1\sum_{i=1}^{N}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}v_i}{l_1l_2T_{set}}\right)$$

Let us carry out a research on a location of heating elements of a plate and try to compare these data with research date of one-dimensional control object. To ensure a coordination of output data let us perform calculations on coordinate slices. A slice of a plate will be placed in a heating element location coordinate, since there are no heating elements outside a slice line, therefore, in finding of place and time of inclusion of a heating element is absent in a given coordinate. To conduct a study let us take the same constants as for one-dimensional control object, which have the following values $l = 10$, $a^2 = 0.01$, $x_1 = \xi_1 = 1$, $T_{set} = 0.2$, $k = 10$, $d = 9$. $\xi_i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Calculation result is represented in table 2.

Table 2: Place and time of a source actuation

l=5	l=6	l=7	l=8
source=4.83. switched on = 9.43 sec	source=1.16. switched on =10 sec	source=1.04. switched on=20 sec	source=1.40. switched on=25.3 sec
source=3.48. switched on =9.33 sec	source=8.16. switched on =10 sec	source=6.93. switched on=20 sec	source=9.00. switched on=25.3 sec
source=2.93. switched on =9.27 sec	source=6.65. switched on =10 sec	source=5.31. switched on=20 sec	source=6.42. switched on=25.3 sec
source=2.76. switched on =9.26 sec	source=6.03. switched on =10 sec	source=4.50. switched on=20 sec	source=4.98. switched on=25.4 sec
source=2.87. switched on =9.23 sec	source=6.06. switched on =10 sec	source=4.24 .switched on=20 sec	source=4.28. switched on=25.3 sec
source=3.34. switched on =9.23 sec	source=6.84. switched on =10 sec	source=4.54 switched on=20 sec	source=4.21. switched on=25.2 sec
source=4.57. switched on =9.23 sec	source=9.13. switched on =10 sec	source=5.80 switched on=20 sec	source=5.06. switched on=25.4 sec
		source=1.89. switched on=30 sec	source=2.31. switched on=25.1 sec
		source=1.78. switched on=30 sec	source=1.91. switched on=25.1 sec
		source=1.85. switched on=30 sec	source=1.94. switched on=24.7 sec

The results show that during a study of cross-sections the same pattern as in the case of one-dimensional control object is observed. Consequently, it is possible to make an assumption that within a uniform arrangement of heating elements time and place of these heating elements does not depend much on a size of a plate and negligibility condition of thickness of an object.

Also the conclusion has been confirmed, made within the study of one-dimensional control object, namely:

- within a fixed length l = 10 only heating elements 4, 3, 2 are actuated, thus there is no need to place 10 heating elements there.
- according to studies, intervals between actuation of subsequent sources are approximately equal, which in turn indicates that thermal processes, taking place in homogeneous bodies are stable.

Conducted studies show an identity of processes in a plate and rod of negligible thickness. Let us find a function that determines actuation time of the first control action $-\tau_1$. A function that determines values of a temperature field of three-dimensional control object, at some time t. A value of a temperature field of this function will be determined by one component of the Fourier series. Expressing a member of the Fourier series, we obtain the following equation:

$$T(x,y,z,t)=\frac{8}{l_1l_2l_3}\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot\sin\frac{\pi}{l_2}y\cdot\sin\frac{\pi}{l_3}z\times$$
$$\times\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i$$

Then, if we take into account a condition, $T(x,y,z,t)=T_{zad}$ which is necessary to provide a system stability we get

$$\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]=$$
$$=\frac{l_1l_2l_3T_{set}}{8\sin\frac{\pi}{l_1}x_{kr}\sin\frac{\pi}{l_2}y_{kr}\sin\frac{\pi}{l_3}z_{kr}\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}\nu_i\sin\frac{\pi}{l_3}\vartheta_i}$$

Having assumed that for a given equation the following equalities are peculiar $x_{kr}=\rho_1$, $y_{kr}=\nu_1$, $z_{kr}=\vartheta_1$ and having specified time $t=\tau_1$, get:

$$a^2\pi^2\left(\frac{1}{l_1}+\frac{1}{l_2}+\frac{1}{l_3}\right)\tau_1=$$
$$=\ln\left(\frac{8\sin\frac{\pi}{l_1}\rho_1\sin\frac{\pi}{l_2}\nu_1\sin\frac{\pi}{l_3}\vartheta_1\sum_{i=1}^{N}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}\nu_i\sin\frac{\pi}{l_3}\vartheta_i}{l_1l_2l_3T_{set}}\right);$$

whence:

$$\tau_1=\frac{1}{a^2\pi^2\left(\frac{1}{l_1}+\frac{1}{l_2}+\frac{1}{l_3}\right)}\cdot$$
$$\cdot\ln\left(\frac{8\sin\frac{\pi}{l_1}\rho_1\sin\frac{\pi}{l_2}\nu_1\sin\frac{\pi}{l_3}\vartheta_1\sum_{i=1}^{N}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}\nu_i\sin\frac{\pi}{l_3}\vartheta_i}{l_1l_2l_3T_{set}}\right)$$

In a similar manner let us find a place of a control exposure actuation, which according to the tolerances specified above will be:

$$T(x,y,z,t)=\frac{8}{l_1l_2l_3}\times\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot$$

$$\cdot\sin\frac{\pi}{l_2}y\cdot\sin\frac{\pi}{l_3}z\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i$$

$$\sin\frac{\pi}{l_1}x=$$

$$=\frac{\frac{8}{l_1l_2l_3}\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot\sin\frac{\pi}{l_2}y\cdot\sin\frac{\pi}{l_3}z\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i}{T(x,y,z,t)}$$

$$x=\frac{l_1}{\pi}\arcsin\frac{\frac{8}{l_1l_2l_3}\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_2}y\cdot\sin\frac{\pi}{l_3}z\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i}{T(x,y,z,t)}.$$

Similarly:

$$y=\frac{l_2}{\pi}\cdot\arcsin\frac{\frac{8}{l_1l_2l_3}\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot\sin\frac{\pi}{l_3}z\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i}{T(x,y,z,t)}$$

$$z=\frac{l_3}{\pi}.\arcsin\frac{\frac{8}{l_1l_2l_3}\exp\left[-a^2\pi^2t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot\sin\frac{\pi}{l_2}y\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i}{T(x,y,z,t)}.$$

Using these equations we can calculate time and place temperature sensors actuation.
Let us evaluate an effectiveness of replacing of a heating method of solid heating elements with a permanent element to an impulse point elements. For this purpose, let us conduct an energy consumption research for silicon carbide rod heating, which in its turn will transmit heat throughout a chamber. Silicon carbide heater power calculation is carried out according to the formula:

*N=D*L*π*W,*

where *N* – heater power, W; *D* – heater work zone diameter, cm; *L* – length of heater work zone, cm; π – = 3.14; *W* – average specific power (W/cm^2).
Taking into account that a silicon carbide rod has a diameter of 25 x 400 x 1200 mm, *R* = 0.87 Ohm +10% at a temperature of 1070 °C, it is possible to calculate that this rod will have a power of:

N = 2.5 * 40 * 3.14 * 6 = 1884 W

Also, it is necessary to know an operation voltage, applied on rods. It will be calculated from the Ohm law by the formula

$$U=\sqrt{N*R},$$

where: *U* – voltage; *N* – heater power; *R* – resistance. It is noteworthy that within power calculation there was chosen a resistance, which corresponds to a temperature regime. For cases with higher temperature a resistance changes as well. Thus, for instance, for temperature of 1400 °C, a resistance is higher by 20% and is 1.04 Ohm.
Together with an inner resistance element watt load increases as well. A work load in a kiln is achieved at 1400°C, but it may be reduced by means of usage of other types of atmospheres. There is no minimal limit for elements' heating, however minimal set load for the whole areal surface of a rod is achieved at a temperature of 900 °C. For other cases it is possible to use the following formula for calculation:

*W = D*L*π*

For evaluation of effectiveness let us take a silicon carbide heating element with the following characteristics:

Size: 25 x 400 x 1200 mm.

Power at a temperature of 1070 (0.87 Ohm): 1884 W

Power at a temperature of 1400 (1.1 Ohm): 1800 W

Work mode: continuous.

Work mode: impulse (expected).

Heater power calculation result at set temperature regime is represented in the table 3. There is no any power gain, but it's worth noting that within an impulse heating some gain time of operation of a heating element. Let us calculate time of heating elements' inclusion. Start-up time will be calculated according to the formula:

$$\tau_1 = \frac{1}{a^2\pi^2\left(\frac{1}{l_1}+\frac{1}{l_2}+\frac{1}{l_3}\right)} \ln\left(\frac{8\sin\frac{\pi}{l_1}\rho_1\sin\frac{\pi}{l_2}\nu_1\sin\frac{\pi}{l_3}\vartheta_1\sum_{i=1}^{N}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}\nu_i\sin\frac{\pi}{l_3}\vartheta_i}{l_1 l_2 l_3 T_{set}}\right)$$

Table 3: Heater power calculation result

Current temperature °C	Calculated power according to specification W/cm^2	Heater power with usage of impulse heating method W/cm^2
0	1	8
100	1	8
200	1	8
300	2	8
400	2	8
500	3	8
600	3	8
700	4	8
800	5	8
900	6	8
1000	6	8
1100	6	8
1200	6	7
1300	5	7
1400	5	7
1400	5	7

According to the obtained results a dependence of a number of heating elements and time can be observed, for example, with 7 heating elements, a silicon carbide rod is warmed to a temperature of 1400 degrees Celsius, since heating elements are actuated every 10 seconds. Thus, at an average time of a heating element operation, that is 3000 hours in continuous mode, the use of this method of heating of a heater, operation time of an impulse heater is decreased up to 1500 hours. As a consequence, energy gain will occur. However, to heat a rod to a temperature of 1400 degrees

Celsius an impulse heater must takes more power in a pulsed mode, rather than at a constant heating.
Total average power at continuous heating is 3000*5= 15 000 W. And at pulse heating − 1200*8= 12 000 W. It is 80% of consumed resources.

$$\frac{1200*100}{15000}=80\%$$

Then, power gain using this method is 20% for an ideal kiln.

Investigation of the stability of distributed systems in the stabilization of the temperature field (check the adequacy of the mathematical model)

Investigate the stability of one, two, three-dimensional object management in the stabilization of the temperature field. For this purpose we construct Hodographs one-, two-, three-dimensional object management. For a one-dimensional object management will take the following initial conditions: l=10, k =10, d=9, $\zeta_1=x_1=1$, T_{set} = 0.3, $\xi_i \in \{1,2,3,4,5,6,7,8,9\}$, a^2=0.01.

$$W_n(s)=\frac{\exp[\beta_n x_{obs}]+\exp[-\beta_n x_{obs}]}{\exp[\beta_n l]+\exp[-\beta_n l]},$$

$$(n=\overline{1,\infty}),$$

where $\beta_n=\left(\frac{s}{a}+\phi_n^2\right)^{\frac{1}{2}}$, x_{obs} – observing point.

For a given object mutual location of the first spatial mode and Popov's line will have a form, represented in a figure 1.

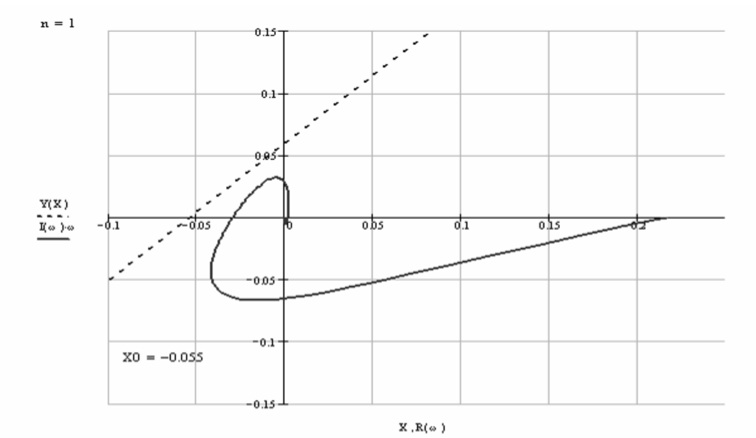

Figure 1: Mutual location of a hodograph and Popov's line at n=1.

For one-dimensional control object let us apply the following initial conditions: l = 0.5, k =10, d=9, $\zeta_1=x_1=1$, T_{set} = 0.3, $\xi_i \in \{1,2,3,4,5,6,7,8,9\}$, a^2=0.01.

$$W_n(s)=\frac{\exp[\beta_n x_{obs}]+\exp[-\beta_n x_{obs}]}{\exp[\beta_n l]+\exp[-\beta_n l]},\ (n=\overline{1,\infty}),$$

where $\beta_n=\left(\frac{s}{a}+\phi_n^2+\psi_n^2\right)^{\frac{1}{2}}$, x_{obs} – observing point.

For a given object mutual location of the first spatial mode and Popov's line will have a form, represented in a figure 2.

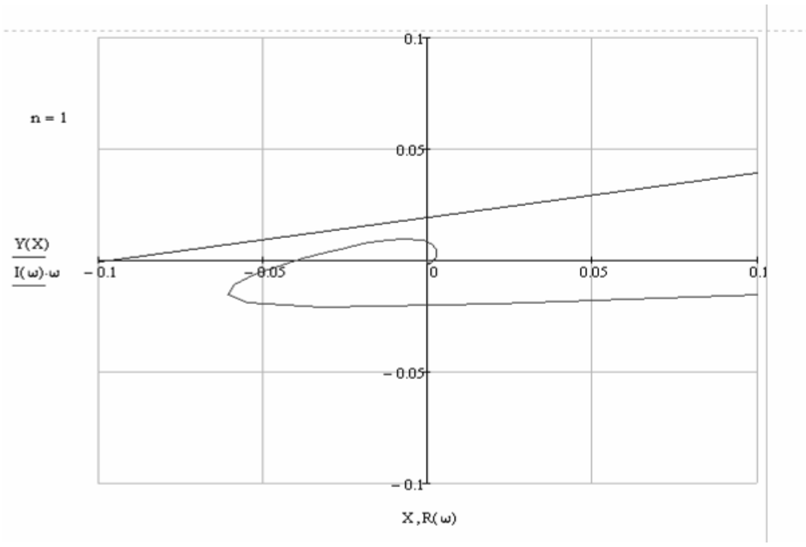

Figure 2: Mutual location of a hodograph and Popov's line at *n*=1.

For two-dimensional control object let us apply the following initial conditions l_1=10, l_2=10, l_3=10, a^2=0.01, *k*=10, *d*=9, τ=3, $x_1=y_1=z_1=p_1=v_1=\vartheta_1=1$, *t*=1..500, $\xi_i, p_1, v_1, \vartheta_1 \in \{1,2,3,4,5,6,7,8,9\}$.

$$W_{0,\eta,y}(s)=\frac{\exp\left[\beta_{\eta,y}*\overset{*}{z}\right]+\exp\left[-\beta_{\eta,y}*\overset{*}{z}\right]}{\exp\left[\beta_{\eta,y}*z_L\right]+\exp\left[-\beta_{\eta,y}*z_L\right]},$$

$(\eta, y=\overline{1,\infty})$,

$$\beta_{\eta,y}=\left(\frac{s}{a^2}+\psi_n^2+\phi_y^2+\alpha_y^2\right)^{\frac{1}{2}},$$

$$\psi_\eta=\pi*\frac{\eta}{x_L},$$

$$\phi_y=\pi*\frac{y}{y_L},$$

$$\alpha_\eta=\pi*\frac{\eta}{z_L}$$

For a given object mutual location of the first spatial mode and Popov’s line will have a form, represented in a figure 3.

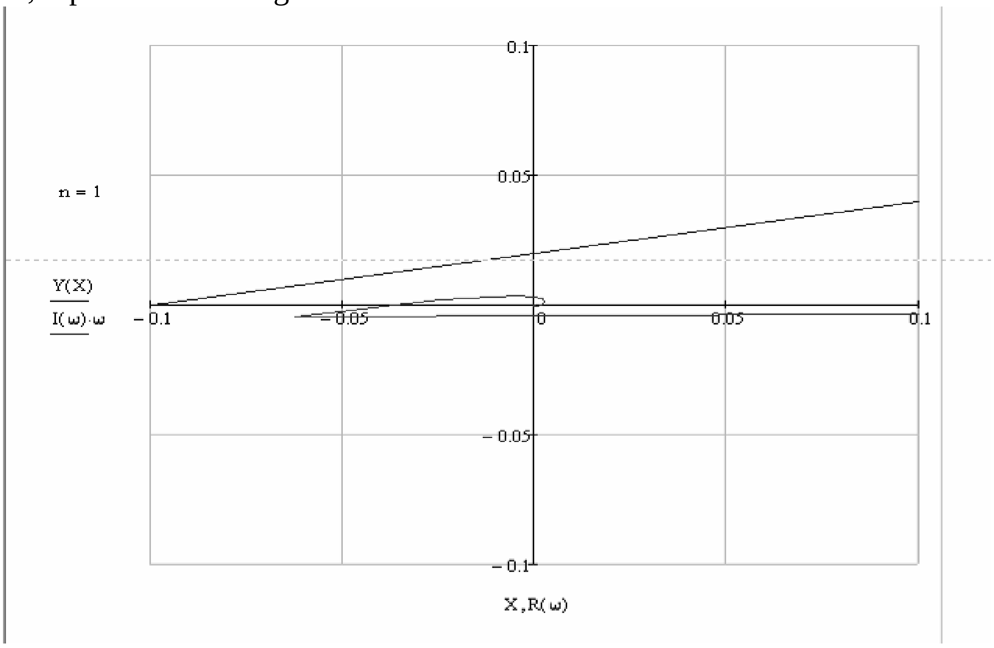

Figure 3: Mutual location of a hodograph and Popov’s line at *n*=1.

According to these results we can draw a conclusion about stability of a temperature processes behavior in one, two or three-dimensional control object with distributed parameters. Having analyzed a synthesized system of an automatic control of temperature fields of conveyor type tunnel kilns for stability it is possible to make the following conclusions.

1. Similar processes of heat treatment, derived from the Green's functions, described by a mathematical model $f(x,y,..,N,t)=$ $\frac{\partial Q(x,y,......,P,t)}{\partial t}-a^2\left[\frac{\partial^2 Q(x,y,......,P,t)}{\partial x^2}+\frac{\partial^2 Q(x,y,......,P,t)}{\partial y^2}+....+\frac{\partial^2 Q(x,y,......,N,t)}{\partial N^2}\right]$ are stable.
2. Synthesized three-dimensional mathematical model of an oven for bread baking is spatially distributed and nonlinear due to a pulse control principle.

Discussion

The relevance of performed research is determined by a complexity of an implementation of objects’ nonlinear control systems with distributed parameters. Controlled values of such systems depend not only on time but also on distribution on spatial area, occupied by an object. In this connection, class of control actions is essentially expanding, primarily due to a possibility of inclusion a number of space-time controls, described by the functions of several variables − time and space coordinates.

Peculiarities of systems with distributed parameters require a creation of an apparatus for analysis and synthesis on the basis of non-traditional, for classical control theory, mathematical tools. There are various forms of description of models of systems with distributed parameters in the form of differential equations in partial derivative; structural representation of systems with distributed parameters, which relies on a fundamental solution of a boundary problem; representation of distributed objects in a form of complex transfer coefficients by their own vectors of an operator of an object.
For an analysis of control objects described by nonlinear equations in partial derivatives, approximation methods are most commonly used. However, it should be noted that to date a method of approximation of distributed systems of specially chosen centered system has not been developed, thus, in many problems an approximation process is unstable with respect to errors of intermediate calculations.
Models of the considered systems and methods of synthesis have recently been developed by a large number of authors due to an absolute relevance and high demand for technical solutions in practice. However, many works end with stages of systems' modeling, suggesting further parametric synthesis, usage of which is associated with a solution of some problems. Examined work differs favorably in that fact that it is brought to its logical conclusion - control algorithms obtained.
Scientific results, obtained in this paper are based on analytical solutions of a boundary value problem in a form of Green's functions for an analysis and synthesis of nonlinear distributed systems.
Reliability of the main conclusions and results is determined by using a common original nonlinear mathematical model in solving a problem of analysis and synthesis of control systems of an object temperature field. Offered control laws are obtained by applying mathematically correct procedures, and their performance is confirmed by a large number of computer experiments.

To the most considerable results of this work should be referred:

- development of a methodology for calculating of an optimal number of heating sources, depending on values of a temperature field;
- conduct a study of temperature fields' behavior in objects of different geometry;
- generalization of results to a class of systems with distributed parameters of fundamental solution of a boundary problem.

A practical significance of the work lies in the fact that the developed method for calculation of an installation location of heating elements, depending on a temperature field, allows us to consider a possibility of installation of sectional pulse heaters in electrical tunnel kilns of conveyor type. Analysis of the results showed:

1) Ability to achieve a desired kiln temperature range through the use of pulsed heating elements.
2) Ability to stabilize a temperature field within an acceptable range. Dependence of temperature on a length of a section.

In addition, there were obtained formulas for a calculation of time and place of temperature sources actuation.

Conclusion

The above shown technique is considering a possibility of replacing of solid heating elements for impulse elements. Novelty and technical peculiarity of this article is as follows:

1. Using an innovative approach to heating hexagonal silicon carbide structures is an important task, since rods made of this alloy are used for firing ceramics, bricks and other products.
2. This method is not designed solely for hexagonal silicon carbide structures. It has a general form, which can be easily adapted to other alloys.
3. The Offered technique will reduce a final cost of a product by saving energy consumption of a company.
4. This method, together with software and hardware systems for a stabilization of a temperature field of conveyor type tunnel kilns will allow solving a wide range of tasks required by modern industry [1].

Thus, the developed method can be generalized to a class of systems for which there exists a fundamental solution (Green's function). At the same time, a complexity of the expression of the Green function naturally causes an increase of a computing process cost. However, if you compare costs, which are now being spent on a low efficiency of heating elements, a use of mathematical modeling to calculate a location of heating elements is justified [6, 7, 8-20].

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