Physical Model of A Heat Mass Transfer in Condensation Surfaces and Its Compliance to Skilled Data

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Abstract

Features of the device of gas condensation boilers are considered. The physical and mathematical model of heat exchange in condensation surfaces of heat generators is considered and analyzed. Effects of transfer of weight are considered. Profiles of temperature and the partial pressure of components in a stream of damp products of combustion at a condensation surface are investigated. The equations of convective heat and mass transfer by diffusion, on the basis of their balance equation are obtained. For physical and mathematical model criteria dependences are received generalized and private (for concrete designs of heat exchange devices). On the basis of the developed physical and mathematical model of heat exchange in condensation surfaces of boilers and on the received equation of thermal balance by means of the theory similarity criteria dependences are received private and generalized (for a certain type of fuel, for concrete designs of the heat exchanger). The received dependences are used for generalization of experimental data for various operating modes of condensation boilers of Baxi Duo Tec Compact and De Dietrich Innovers PRO MCA 45 working at natural gas and allowed to receive the private and generalizing dependences for determination of efficiency of heat exchange in condensation surfaces in the form of power functions. Calculations showed that this dependence can be used for determination of overall performance of condensation coppers. Quality and quantitative standards of influence of criteria on overall performance of condensation surfaces of heating of independent heat generators with power up to 100 kW are received.

**Keywords:** Condensing Boiler, Flue Gas, Mass Transfer, Products Of Combustion, Criteria Equations.
**Introduction**

At the moment, gas boilers are the most common heating equipment in the world. Gas condensing heat generators are commonly used in Europe and are gaining popularity in Russia. Currently, gas heat generators are the most widespread heating equipment in the world. Support by many countries at the state level of energy saving technologies and care of environmental protection promote active introduction of the innovative equipment meeting all standards. A priority task — increase of efficiency and improvement of ecological indicators of systems of heat supply. Thanks to it, in Europe the gas condensation coppers possessing demanded characteristics took the leading positions among the equipment for heating and hot water supply, and in some countries installation of any other gas coppers, except condensation, is forbidden. Condensing boilers can be single-circuit and dual-circuit, is used both for heating and for hot water systems. Wall-hung boilers have a capacity of 20 - 100 kW are widely used for household purposes, heating of country houses. For commercial or office use more powerful models are available in outdoor version. Condensing boilers have a number of advantages over traditional models of boilers. The main difference between the gas condensing boilers from the traditional gas-fired boilers is apparent that in addition to the heat generated by the combustion of gas, such boilers using vaporization latent heat in the condensation of water vapor from the moist combustion products. This allows more heat and can achieve savings of up to 11% fuel burned by increasing the efficiency of the heat generator [10]. The products of combustion of the gaseous fuel mixture is traditionally represented as "dry" flue gas (CO2; N2; excess air) and water vapor (from the combustion of hydrogen from the fuel moisture and air supplied for combustion). The volume fraction of the water vapor from the combustion of gaseous fuels in "wet" products of combustion can reach values $r_{H_2O} = 0.2$ [16].

The latter is important to note, as even a small admixture of non-condensable gases significantly reduces the heat transfer.

In condensing boilers installed two separate heat exchanger or one - two-step. Primary - functions as a heat exchanger and a traditional boiler. Combustion gases passing through it, a large part of the energy given and with a temperature above the dew point of leaving it. A typical element of condensing boilers is a secondary heat exchanger, also called condensation. The combustion products are cooled therein return water from the heating system to a temperature below the dew point [24]. Water absorbs heat from the combustion products, thereby cooling them. Upon reaching the dew point, contained in the flue gas water vapor condenses on the walls of the heat exchanger, it gives the latent energy of the coolant flowing into the primary heat exchanger for heating. Due to this process the combustion products coming from the boiler, have a very low temperature. The resulting condensate is chemically aggressive, so exchangers condensing boilers made of corrosion resistant materials such as stainless steel or aluminum-silicon alloy [6].

Condensation heating surface in most cases in the form of narrow channels with a smooth surface or edged tubes with high and thick edge. The size of the heat exchange channel is formed in both cases, the slit form with the width of the channel...
does not exceed 5-6 mm [18]. The cooling medium for condensing conditions - low temperature water – 25–45 °C.

Besides, condensing boilers are used tech full premixing burner which provides the preparation of gas-air mixture for optimum combustion mode specific proportions that minimizes the chances of incomplete combustion. As a result, in the exhaust gases is greatly reduced amount of harmful emissions, and low temperature flue gases, allows the use of chimneys from plastic [25].

Features of processes of heat transfer in surfaces of heating of heat generators of low power are connected both with geometrical factors, and with the operation modes in comparison with more powerful boilers. So, reduction of the geometrical sizes of a fire chamber leads to growth of a ratio of the area of its surface to volume in proportion to the characteristic size that leads to growth of specific thermal tension of surfaces of heating, under the same conditions of burning of fuel on the size of specific thermal tension of furnace volume [20]. More intense operating conditions of the heat perceiving surfaces of heating of boilers cause high requirements to quality of the heat carrier and hydraulic service conditions which significantly differ from characteristics of powerful boiler units [19].

**Method**

**A. Physical model**

Condensation of water vapor occurs on the cooling surface. Condensation occurs when the temperature of the cooling surface is less than the saturation temperature at the given pressure. On the surface, this appears when the condensate membrane with a thickness greater than the distance of the effective action of molecular forces [4].

Developing a physical model of heat and mass transfer in previously adopted under briefly describing the process can be for the quasi-stationary problem within the elemental plane layer \( dx \cdot dz \) moving in the channel of the combustion products can be considered the process of heat and mass transfer as one-dimensional at the surface of the condensate film to form a "fictitious" (conditional) of the boundary layer in the flow of products combustion [26].

On the surface of film condensation condenses water vapor and thus reducing their partial pressure \( P_{wv} \) within the "fictitious" boundary layer, and this correspondingly leads to an increase in the partial pressure of dry flue gas \( P_{dg} \), for causing the mutual opposite direction \( gradP_{wv} \) and \( gradP_{dg} \), and, accordingly, their counter-molecular and turbulent diffusion [8].

The flue gas temperature \( T_g \) in the general case, the process technology must be made with a higher \( T_{dp} \) fall within its "fictitious" boundary layer to the saturation temperature of the condensate membrane on the surface.
The analysis of process of a heat transfer when forming model allows for quasi-stationary conditions within an elementary layer \( dx \cdot dy \cdot l_z \) to consider a heat mass transfer only within a gas stream [14], locating it on a condensate membrane surface (figure 1). In the accepted designations \( l_z \) in fact is the characteristic size of the channel. Such assumption becomes in works of many authors in relation to processes of condensation of pure vapors [7]. Taking into account these assumptions for the accepted model, it is possible to write down the following equations of convective heat transfer on a condensate membrane surface element \( dx \cdot l_z \) [17]:

\[
dQ_{FK} = \alpha_k ((T_x - \frac{dT_x}{dx}) - T_{dp} f(x))dx \cdot l_z
\]

\( dQ_{FK} \) – heat convection, W;

\( \alpha_k \) – heat transfer coefficient, W/(m²·K).

The heat transfer coefficient \( \alpha_k \) is determined by three groups of factors. Firstly, geometric factors connected with a configuration of system of a convective heat transfer. Second, the hydrodynamic pressures caused primarily by the presence of two flow regimes - laminar (at low values of Re) and turbulence (for large values of Re). The heat exchange mechanism in two of these cases significantly differs. Besides, within each mode of a current there is a communication of coefficient of a heat transfer with stream speed qualitatively identical to both modes — at increase of speed of a stream the coefficient increases. However quantitative characteristics for the laminar and turbulent modes are various.
At last, the third group of factors is made by heat physical properties of the environment — density, an isobaric thermal capacity, viscosity and heat conductivity. They with difficulty influence heat transfer coefficient. With other things being equal for Wednesday with higher heat conductivity higher values of coefficient of a heat transfer are characteristic. Viscosity has indirect impact on intensity of a heat transfer: at smaller viscosity in a stream the speed profile, more favorable for increase of a heat transfer, is formed.

During the condensation of steam, they are given to the heat of the phase transition. Furthermore, since the wall temperature is less than the surface temperature of the condensate, and given wall of the heat of condensation [12]. Condensate is overcooled to a temperature value which is between the temperature of the membrane surface and steam. In many cases the sub cooling is negligibly small compared with a heat of phase transition [5]. The heat transfer interfaced to a mass transfer at condensation is presented in a general view [9]:

\[ dQ_{\text{FM}} = r(-D_{\text{nd}} \nabla P_{\text{v}})dx \cdot dz \]

\( dQ_{\text{FM}} \) – thermolysis a mass transfer at condensation, W;

\( D_{\text{nd}} \) – provided coefficient of turbulent diffusion and mutual diffusion of steam and dry combustion gases, \( \text{m}^3/\text{s} \);

\( r \) – hidden heat of steam formation, kJ/kg.

Within this publication it should be noted:

– \( \nabla P_{\text{v}} \) for one-dimensional problem can be used as a local (within a layer), finite difference, and the partial pressure of steam saturation pressure at the surface of the condensate membrane:

\[ \nabla P_{\text{v}} = P_{\text{v}} - P_{\text{p}} \]

the provided coefficient of diffusion is considered as a total molar stream of the condensed steam \( \eta \), mol/s:

\[ \eta = D_{\text{nd}} \rho_s \frac{d\rho_s}{dy} + \eta' \rho_v \]

\( \eta' \) – molar flow of turbulent diffusion.

The assumption of laminar "fictitious" layer:

\[ \eta = \beta_{12} \ln \frac{1 - \rho_v^p}{1 - \rho_v^g} = \frac{D_{\text{nd}} \rho_g}{S} \ln \frac{1 - \rho_v^p}{1 - \rho_v^g} \]

\( \rho_v^p \) – molar share of steam on limit of the section of phases, mol/m³;

\( \rho_v^g \) – molar share of steam of gas, mol/m³;

\( \beta_{12} \) – the mass transfer coefficient connected with transfer of weight through a "fictitious" layer, m/s;
Thus, the total convection \( dQ \) by heat transfer \( dQ_{FK} \) and mass transfer by diffusion \( dQ_{FM} \) leads to a change in enthalpy of the elementary volume of wet combustion \( dJ \).

\[
dJ = dQ = dQ_{FK} + dQ_{FM},
\]

then in the accepted designations:

\[
dJ = dx \cdot dy \cdot lz \cdot c^s_p \cdot \rho^s \frac{dT_g}{dx},
\]

\( c^s_p \) – isobaric thermal capacity of products of combustion, kJ/(kg·K);

\( \rho^s \) – density of the combustion products, kg/m\(^3\).

and the balance equation in expanded form can be written [13]:

\[
dQ = dQ_{FK} + dQ_{FM} = \alpha_k (T_g - T_p) dx \cdot lz + r \cdot \beta_{12} (P_{wv} - P_p^w) dx \cdot lz = dx \cdot dy \cdot lz \cdot c^s_p \cdot \rho^s \frac{dT_g}{dx}
\]

For total heat transfer by convection \( \alpha_k \) and a mass transfer diffusion \( \alpha_m \), allowing possibility of use of the principle of additivity

\[
\alpha_s = \alpha_k + \alpha_m,
\]

Then the balance equation becomes:

\[
\alpha_s \cdot \Delta T = (\alpha_k + \alpha_m) \cdot (T_g - T_p) = dy \cdot c^s_p \cdot \rho^s \frac{dT_g}{dx}
\]

in which \( \alpha_m \) has final value:

\[
\alpha_m = r \cdot \beta_{12}.
\]

### B. Criteria equations

The analytical solution of the balance equation has to be made in a complex with the solution of the equations of the movement and continuity of a stream [1]. The exact analytical decision also assumes the accounting of change of the physical constants characterizing heat and physical and hydrodynamic properties of working figures from temperature and pressure (density, a thermal capacity, viscosity, etc.) [2].

In general the analytical decision without the whole complex of assumptions is impossible, and otherwise can be considered as unreasonable and doesn't maintain criticism without experimental confirmation.

In this research on the basis of the developed physical and mathematical model possibly by methods of the theory of similarity to receive criteria dependences of a general view and private (for concrete designs of heat exchange devices) for processing and generalization of experimental data [23].

Within this publication, at a material summary, for the developed physical and mathematical model of process and the differential balance equation with use of methods of the theory of similarity it is possible to receive the generalizing criteria equation:
Generalized number Stanton evaluates the efficiency of the condensing boiler and shows the ratio of heat flow, the perceived heating surfaces and the maximum possible under the condition of cooling flue gas to a temperature of heat perceiving surface:

\[
St^* = \frac{Nu^*}{Re^f \cdot Pr} = \frac{\alpha_s}{\omega_f \cdot c_p^g \cdot \rho^g} = \frac{I_s - I_g'}{I_g - I_s'};
\]

\[St^* \] - generalized Stanton's number;

\[Re^f = \frac{\omega_f \cdot d_e}{\gamma} \] - Reynolds's criterion, on the speed carried to a heating surface;

\[Pr = \frac{\gamma}{\alpha} \] - Prandtl number;

\[Nu^* \] - generalized Nusselt number;

\[\omega_f \] - speed of a stream of flue gases correlated to a heating surface, m/s;

\[\gamma \] - kinematic viscosity, m²/s;

\[\alpha \] - thermal diffusivity, m²/s;

\[I_s', I_g', I_g \] - enthalpy of flue gases at a temperature on an entrance to a convective surface, at the exit from it and at a temperature of the cooled wall, kJ/kg.

Experiments were made on gas wall condensation coppers of Baxi Duo-Tec Compact and De Dietrich Innovens PRO MCA 45. As fuel natural gas of the following structure was used: \(\text{CH}_4 = 94,1\%; \text{C}_2\text{H}_6 = 3,2\%; \text{C}_3\text{H}_8 = 0,95\%; \text{C}_4\text{H}_{10} = 0,32\%; \text{C}_5\text{H}_{12} = 0,36\%; \text{H}_2 = 0,02\%; \text{N}_2 = 1,15\%; \text{CO}_2 = 0,2\% [22].\)

The received general criteria equation when burning concrete gaseous fuel for a concrete design with identical geometrical and hydrodynamic characteristics [27] of conditions of course of processes, can be simplified to the look allowing to carry out an assessment of influence of regime parameters of work of condensation surfaces of heating:

\[
St^* = f(Re^f; Pr; \frac{T_s - T_{dp}}{T_s}, \frac{h_d}{d_e}),
\]

The dependence obtained is used to generalize the experimental data for different modes of operation of condensing heat generators Baxi Duo-Tec Compact and De Dietrich Innovens PRO MCA 45 allows us to derive private dependence, and on the basis of their relationship generalizes to determine the efficiency of heat exchange in the condensing surfaces of heat generators in the form of functions:

\[
St^* = A \cdot Re^m \cdot Pr^n \cdot \left(\frac{T_s - T_{dp}}{T_s}\right)^t \cdot \left(\frac{h_d}{d_e}\right)^i.
\]
Results
For the purpose of obtaining values of indicators of degrees of private dependences of m, n, k, l and coefficient A the data obtained as a result of experiments by determination of overall performance of condensation part of heat generators (tail part of the heat exchanger) at various capacities and water temperatures on an entrance to this surface i.e. were attracted, under various conditions of heat exchange [21]. Thus, at the first stage, processing and generalization of experimental data allowed to receive the generalized Stanton’s numbers $S_{t*}$ from temperature criterion $\frac{T_g - T_{dp}}{T_s}$ (figure 2) [3]. In processing groups of skilled data for which with an accuracy of ±5% requirements of relative constancy of other criteria are fulfilled were used: 1st group of experiences. De Dietrich Innovens PRO MCA 45.

$$Re_f = 1,555\div 1,572; \ Pr = 0,696\div 0,698; \ \frac{h_d}{d_e} = 42,5;$$

2nd group of experiences. Baxi Duo-Tec Compact

$$Re_f = 7,124\div 7,209; \ Pr = 0,695\div 0,696; \ \frac{h_d}{d_e} = 13,9;$$

3rd group of experiences. De Dietrich Innovens PRO MCA 45.

$$Re_f = 2,187\div 2,229; \ Pr = 0,689\div 0,693; \ \frac{h_d}{d_e} = 42,5.$$

Figure 2: Dependence of the generalized Stanton’s number on the temperature criterion $\frac{T_g - T_{dp}}{T_s}$.
The received dependence shows that with increase in a temperature factor on an entrance to condensation part of the heat exchanger of a boiler [11], the generalized Stanton's criterion, i.e. efficiency of heat exchange increases. Results of generalization are expressed by private dependence:

\[ St^* = A \cdot \left( \frac{T_g - T_{dp}}{T_p} \right)^{0.24}. \]

Having received value of an exponent of \( k \) for private dependence on a temperature factor \( \frac{T_g - T_{dp}}{T_p} \), it is possible to process skilled data and to receive dependence of a complex \( St^* / \left( \frac{T_g - T_{dp}}{T_p} \right)^{0.24} \) on criterion of Re and to define an exponent (figure 3).

\[ \lg Re_f \]

Figure 3: Dependence of the complex \( St^* / \left( \frac{T_g - T_p}{T_p} \right)^{0.24} \) on Reynolds's criterion

Figure shows a summary of experimental data for which an accuracy of \( \pm 5\% \) requirements are fulfilled: 1st group of experiences. Baxi Duo-Tec Compact

\[ \text{Pr} = 0.689\pm0.698; \quad \frac{h_{ad}}{d_e} = 13.9; \]

2nd group of experiences. De Dietrich Innovens PRO MCA 45.

\[ \text{Pr} = 0.689\pm0.698; \quad \frac{h_{ad}}{d_e} = 42.5. \]

Dependence shows that with increase in intensity of transfer of impulses at a heating surface (growth of criterion of Reynolds \( Re_f \)), value of a complex \( St^* / \left( \frac{T_g - T_p}{T_p} \right)^{0.24} \) increases, i.e. heat transfer increases. Results are generalized by the private dependence reflecting influence of aerodynamic factors:
$St^* = A \cdot Re_f^{0.55} \cdot \left(\frac{T_g - T_p}{T_s}\right)^{0.24}$. 

The exponent at criterion of heat physical properties of products of combustion (Pr) is defined on the basis of skilled data depending on the complex $\frac{St^*}{Re_f^{0.55} \left(\frac{T_g - T_p}{T_s}\right)^{0.24}}$ received above (figure 4).

**Figure 4:** Dependence of the complex $\frac{St^*}{Re_f^{0.55} \left(\frac{T_g - T_p}{T_s}\right)^{0.24}}$ on Prandtl criterion

Figure shows a summary of experimental data, for which the following requirement: 1st group of experiences. De Dietrich Innovens PRO MCA 45. 

$$\frac{h}{d} = 42.5;$$

2nd group of experiences. Baxi Duo-Tec Compact.

$$\frac{h}{d} = 13.9.$$ 

The received dependence shows that with reduction of criterion of Prandtl (Pr) which characterizes heat physical properties of combustion gases [15] on a thermolysis, the complex $\frac{St^*}{Re_f^{0.55} \left(\frac{T_g - T_p}{T_s}\right)^{0.24}}$ increases.

Results of processing of skilled data allow to receive private dependence of the generalized Stanton's number on the listed criteria:

$St^* = A'' \cdot Re_f^{0.55} \cdot Pr^{-0.47} \cdot \left(\frac{T_g - T_p}{T_s}\right)^{0.24}.$
The ratio $\frac{h_d}{d_e}$ is a structural characteristic of the heat exchanger, so for each boiler $\frac{h_d}{d_e} = \text{const}$. To determine the exponent in the dependence generalizing on the basis of experimental data can reveal a complex $\frac{St^*}{\text{Re}_f^{0.55} \text{Pr}^{-0.47} \left( \frac{T_g - T_p}{T_s} \right)^{0.24}}$ dependence on the ratio $\frac{h_d}{d_e}$ (figure 5).

**Figure 5:** Dependence of the complex $\frac{St^*}{\text{Re}_f^{0.55} \text{Pr}^{-0.47} \left( \frac{T_g - T_p}{T_s} \right)^{0.24}}$ on ratio $\frac{h_d}{d_e}$

As follows from the dependencies, with an increase in the structural characteristics of the heat exchanger $\frac{h_d}{d_e}$, the value of complex $\frac{St^*}{\text{Re}_f^{0.55} \text{Pr}^{-0.47} \left( \frac{T_g - T_p}{T_s} \right)^{0.24}}$ for the condensation surface of a boiler increases.

The received private dependences allow unifying them for all volume of skilled data the sedate dependence generalizing results of experiments:

$$St^* = A'''' \cdot \text{Re}_f^{0.55} \cdot \text{Pr}^{-0.47} \cdot \left( \frac{T_g - T_p}{T_s} \right)^{0.24} \cdot \left( \frac{h_d}{d_e} \right)^{0.57}.$$  

The coefficient $A''''$ can be determined by graphic dependence of a complex $\text{Re}_f^{0.55} \cdot \text{Pr}^{-0.47} \cdot \left( \frac{T_g - T_p}{T_s} \right)^{0.24} \cdot \left( \frac{h_d}{d_e} \right)^{0.57}$ from the generalized Stanton's number.
(figure 6).

![Graph showing dependence of the complex $R_{ij}^{0.55}$ and $Pr^{-0.47}$ on $St$ *.

Figure 6: Dependence of the complex $Re_i^{0.55}Pr^{-0.47}\left(\frac{T_g-T_{dp}}{T_s}\right)^{0.24}\left(\frac{h_d}{d_e}\right)^{0.57}$ on the generalized Stanton's number

The results are expressed as a private dependence:

$$St^* = 0.0414 \cdot Re_i^{0.55} \cdot Pr^{-0.47} \cdot \left(\frac{T_g-T_{dp}}{T_s}\right)^{0.24} \cdot \left(\frac{h_d}{d_e}\right)^{0.57}.$$

**Conclusion**

To assess the reliability of the obtained dependence compared to the calculated values of the generalized Stanton, and the experimental data (figure 7).

![Graph showing dependence of the generalized Stanton's number on complex $0,0414 \cdot Re_i^{0.55} \cdot Pr^{-0.47} \cdot \left(\frac{T_g-T_{dp}}{T_s}\right)^{0.24} \cdot \left(\frac{h_d}{d_e}\right)^{0.57}$.

Figure 7: Dependence of the generalized Stanton's number on complex $0,0414 \cdot Re_i^{0.55} \cdot Pr^{-0.47} \cdot \left(\frac{T_g-T_{dp}}{T_s}\right)^{0.24} \cdot \left(\frac{h_d}{d_e}\right)^{0.57}$.
Being set by confidential probability of 95% and counting total value of a mean square error the size of a confidential interval is defined:

\[ \Delta \delta \% = \sqrt{\sum_{i=1}^{16} \Delta \delta_i^2} = \sqrt{103,28} = 10,16\%. \]

That is, at values: \( \text{Re}_f = 1,554 \pm 8,080; \text{Pr} = 0,6891 \pm 0,6983; T_s = 34,3 \pm 52,8 \text{ C}^\circ; \frac{h_u}{d_e} = 13,9 \pm 42,5 \text{, received as a result of experiences, with probability of 95\% of a deviation of a calculated value of the generalized Stanton's criterion of skilled data don't go beyond a confidential interval of } \pm 10,16\%. \)

Thus, in the investigated range of geometric changes \( \frac{h_u}{d_e} \), restricted \( \text{Re}_f \); \( \frac{T_e - T_{dp}}{T_s} \) and thermal \( \text{Pr} \) parameters obtained qualitative and quantitative assessment of the impact of these factors on the reduced efficiency of condensing heating surfaces autonomous heat generators up to 100 kW.

**References**


