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0-Edge Magic Labeling of Splitting Graph

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Abstract

In this paper the existence of *0*-edge magic labeling of $spl(P_n)$, $spl(C_n)$, $spl(K_{1,n})$, $spl(B_{m,n})$ and Tree are shown.

Keywords: splitting graph , $spl(P_n)$, $spl(C_n)$, $spl(K_{1,n})$, $spl(B_{m,n})$.

Introduction

A labeling of a graph G is an assignment f of labels to either the vertices or the edges or both subject to certain condition. In graph labeling where the vertices are assigned real value or subsets of a set subject to certain conditions, have often been motivated by practical problems, but they are also of interest in their own right. *0*-edge magic labeling was introduced by J. Jayapriya in the year 2012 [3]. A *0*-edge magic labeling of a simple graph G with vertex set V is a function f from V to $\{-1, 1\}$ such that every edge uv has the label $f(u) + f(v) = 0$. In this paper the existence of *0*-edge magic labeling of $spl(P_n)$, $spl(C_n)$, $spl(K_{1,n})$, $spl(B_{m,n})$ and Tree are shown.

Definition 1.1 [4] : For a graph G the splitting graph is obtained by adding to each vertex v, a new vertex v' such that v' is adjacent to each vertex that is adjacent to v in G. The resultant graph is $spl(G)$.

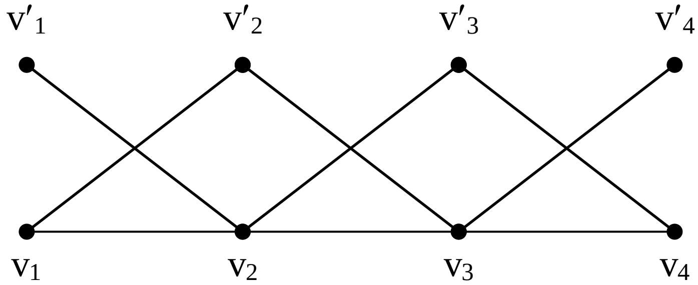

Figure 1.1: $spl(P_4)$

Main Result

Theorem 2.1: The graph $spl\ (P_n)$ admits 0-edge magic labeling.

Proof. The graph $G(V, E) = spl(P_n)$ has $2n$ vertices and $3n–3$ edges.
Let $V(G)= \{ v_i:1\leq i\leq n\}\cup\{ v_i\,':1\leq i\leq n\}$ and
$E(G)= \{ v_i\, v_{i+1}: 1\leq i\leq n–1\}\cup\{ v_i' v_{i+1}: 1\leq i\leq n–1 \}\cup\{ v_i' v_{i-1}: 2\leq i\leq n\}$.
Let $f: V\rightarrow \{1, –1\}$ such that $f(v_i) = f(v_i')=(–1)^i: 1\leq i\leq n$.
The edge weights are calculated as follows:
For $1\leq i\leq n–1;\ f(v_i)+f(v_{i+1})= 0.$
For $2\leq i\leq n;\ f(v_i')+f(v_{i-1})= 0.$
For $1\leq i\leq n–1;\ f(v_i')+f(v_{i+1})= 0.$

Thus all edge receives value 0. Hence the graph $spl(P_n)$ admits 0-edge magic labeling. 0-edge magic labeling of $spl(P_5)$ is shown in Figure1.2.

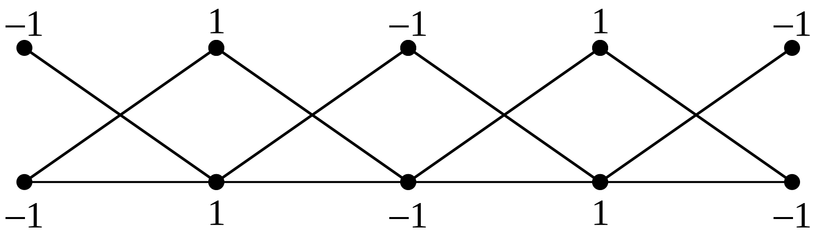

Figure 1.2: 0-Edge Magic Labeling of $spl(P_5)$

Theorem 2.2: The graph $spl(C_n)$ when $n \equiv 0\ (mod\ 2)$ admits 0-edge magic labeling.
Proof. The graph $G(V, E) = spl(C_n)$ has $2n$ vertices and $3n$ edges.
Let $V(G)= \{ v_i:1\leq i\leq n\}\cup\{ v_i\,':1\leq i\leq n\}$ and
$E(G)= \{ v_i\, v_{i+1}: 1\leq i\leq n–1\}\cup\{ v_i' v_{i+1}: 1\leq i\leq n–1 \}\cup\{ v_i' v_{i-1}: 2\leq i\leq n\} \cup\{v_1v_n\ , v_n' v_1\ , v_1' v_n\}$.
Let $f: V\rightarrow \{1, –1\}$ such that $f(v_i) = f(v_i')=(–1)^i: 1\leq i\leq n$.
The edge weights are calculated as follows:
For $1\leq i\leq n–1;\ f(v_i)+f(v_{i+1})= 0.$
For $2\leq i\leq n;\ f(v_i')+f(v_{i-1})= 0.$
For $1\leq i\leq n–1;\ f(v_i')+f(v_{i+1}) = 0.$
Also $f(v_1) +f(v_n) = 0,$
$f(v_n')+ f(v_1) = 0.$

Thus all edge receives value 0. Hence the graph $spl(C_n)$ admits 0-edge magic labeling. Figure 1.3 shows 0-edge magic labeling of $spl(C_6)$.

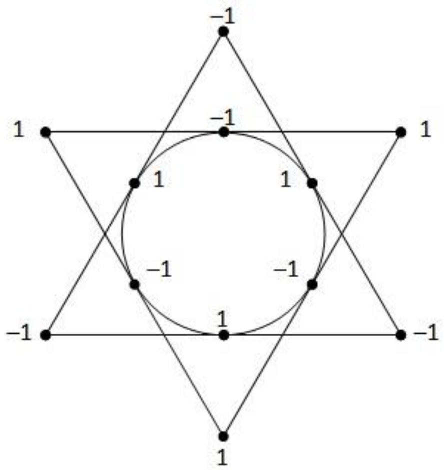

Figure 1. 3: *0*-Edge Magic Labeling of $spl(C_6)$

Theorem 2.3: The graph $spl(K_{1,n})$ admits *0*-edge magic labeling.

Proof. The graph $G(V, E)= spl(K_{1,n})$ has $2n+2$ vertices and $3n$ edges.
Let $V(G)= \{ v_i:1\le i\le n+1\}\cup\{ v_i{}' :1\le i\le n+1\}$ and
$E(G)= \{ v_1 v_i: 2\le i\le n+1\}\cup\{ v_1{}' v_i: 2\le i\le n+1 \}\cup\{ v_i{}' v_1: 2\le i\le n+1\}$.
Let $f:V\to\{1,-1\}$ such that $f(v_i) = f(v_i{}')=(-1)^i: 1\le i\le n$.
The edge weights are calculated as follows:
For $2\le i\le n+1;\ f(v_1)+f(v_i)= 0.$
For $2\le i\le n;\ f(v_1{}')+f(v_i)= 0.$
For $1\le i\le n-1;\ f(v_i{}')+f(v_i)= 0.$

Thus each edge receives value *0*. Hence the graph $spl(K_{1,n})$ admits *0*-edge magic labeling.
0-edge magic labeling of $spl(K_{1,4})$ is shown in Figure 1.4.

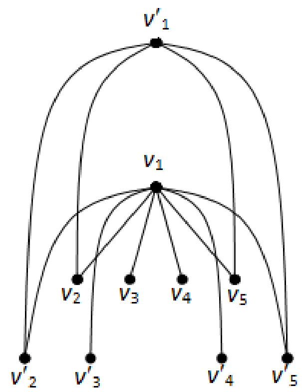

Figure 1.4: *0*-Edge Magic Labeling of $spl(K_{1,4})$

Theorem 2.4: The graph $spl(B_{m,n})$ $m,n \geq 2$ admits 0-edge magic labeling.

Proof. The graph $G(V, E) = spl(B_{m,n})$ has $2m+2n+4$ vertices and $3m+3n+3$ edges. Let $V(G)= \{ v_i ,v_i ':1\leq i\leq m+1\}\cup\{ u_i , u_i ':1\leq i\leq n+1\}$ and
$E(G)= \{v_1 v_i: 2\leq i\leq m+1\}\cup\{ u_1 u_i: 2\leq i\leq n+1\}\cup\{ v_1 ' v_i: 2\leq i\leq m+1 \}\cup$
$\{ u_1 ' u_i: 2\leq i\leq n+1 \}\cup\{ u_1 v_1\}\cup\{ u_1 v_1 '\}\cup\{ u_1 ' v_1\}\cup$
$\{ v_i ' v_1: 2\leq i\leq m+1\}\cup\{ u_i ' u_1: 2\leq i\leq n+1\}$.
Let $f:V\rightarrow\{1, -1\}$ such that $f(u_i) = f(u_i ')= -1 : 2\leq i\leq n+1,$
$f(v_i) = f(v_i ')=1 ; 2\leq i\leq m+1, f(u_1 ') = f(u_1)=1, f(v_1 ') = f(v_1)= -1.$
The edge weights are calculated as follows:
For $2 \leq i \leq m+1; f(v_1)+f(v_i)= 0.$
For $2 \leq i \leq m+1; f(u_1)+f(u_i)= 0.$
For $2 \leq i \leq m+1;f(v_1 ')+f(v_i)= 0.$
For $2 \leq i \leq n+1; f(u_1)+f(u_i)= 0.$
For $2 \leq i \leq m+1; f(v_i ')+f(v_1)= 0.$
For $2 \leq i \leq n+1; f(u_i ')+f(u_1)= 0.$
Also $f(u_1)+f(v_1)=0,$
$f(u_1)+f(v_1 ')=0,$
$f(u_1 ')+f(v_1)=0,$
Thus all edge receives value 0. Hence the graph $spl(B_{m,n})$ admits 0-edge magic labeling.
Figure 1.5 shows 0-edge magic labeling of $spl(B_{3,6})$.

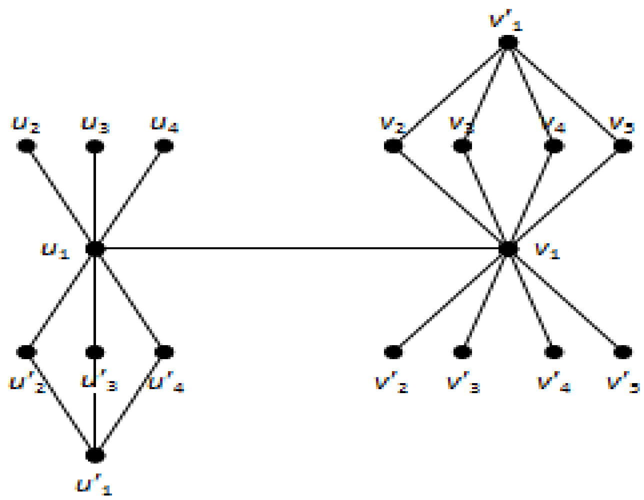

Figure 1. 5: 0-Edge Magic Labeling of $spl(B_{3,6})$

Theorem 2.5 : Splitting graph of any tree is *0*-edge magic.

Proof. Case (i): Let x be an arbitrary vertex in the tree T, such that $deg(x) = k, k > 1$.
We choose x has k successors, $v_1, v_2, \ldots, v_k$ where all v_i's are pendent then by adding a new vertex x' and joining we get $x'v_1, x'v_2, x'v_3, \ldots, x'v_k$ which form $(k\text{-}1)\ C_4$ cycle namely $\{xv_1x'v_2x,\ xv_2x'v_3x,\ xv_3'v_4x, \ldots\ldots,\ xv_{k\text{-}1}x'v_kx\}$. Next we insert $v_1', v_2', \ldots, v_k'$ and join each v_i' to the vertex x. we get k –new pendent edge $xv_1', xv_2', \ldots, xv_k'$. Assume $f(x) = f(x') = 1(say)$, then $f(v_i') = f(v_i) = -f(x)$. Thus all edge weights receives *0*.
0-edge magic labeling of *spl(T)* with arbitrary vertex is shown in Figure 1.6.

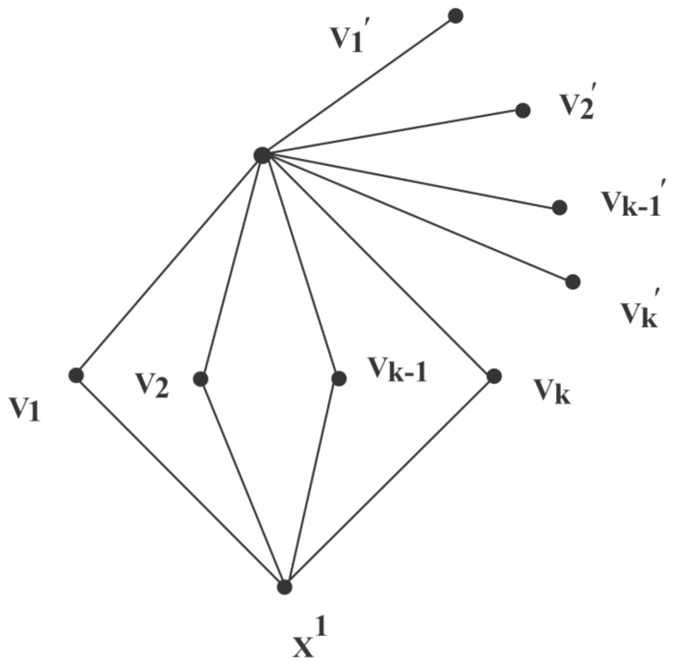

Figure 1.6: *0*-Edge Magic Labeling of *spl(T)* with Arbitrary Vertex

Case (ii): If v_i is an interior vertex then $deg(v_i) = r$ (say).
Let Nbh $(v_i) = \{ v_1, v_2, \ldots, v_r\}$. Next we insert v_i' and join v_i' to the vertex adjacent with v_i. We get new edges $v_i'v_1,\ v_i'v_2,\ v_i'v_3, \ldots,\ v_i'v_r$. Therefore we get r-C_4 cycles namely $\{ v_iv_1\ v_i'v_2v_i\ ,\ v_iv_2\ v_i'v_3v_i\ ,\ v_iv_3\ v_i'v_4v_i\ , \ldots,\ v_iv_{r\text{-}1}v_i'v_rv_i\}$. Assume $f(v_i) = (-1)^i$ and $f(v_i') = f(v_i)$. Thus all edge receives *0*.
Figure 1.7 shows *0*-edge magic labeling of *spl(T)* with interior vertex.

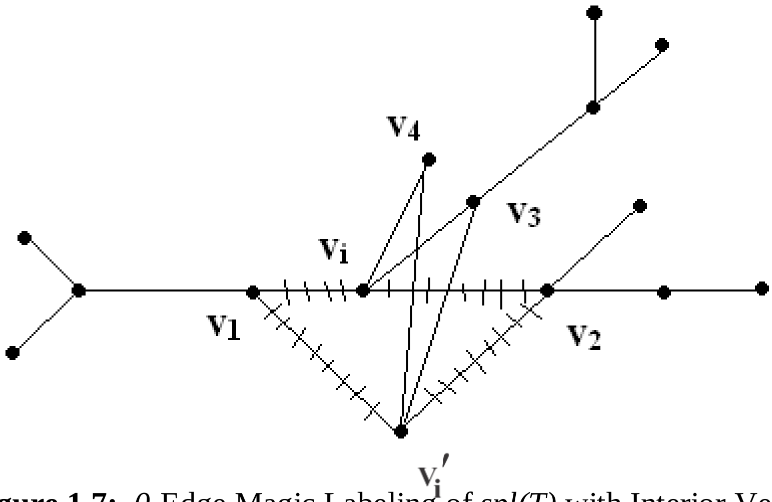

Figure 1.7: *0*-Edge Magic Labeling of *spl(T)* with Interior Vertex

Case (iii): If v_j is a pendent vertex incident with v_k , it produces pendent edge $v_k v_j'$. Assume $f(v_j) = f(v_j') = 1(say)$, then $f(v_k) = -1$. Therefore $f(v_k v_j') = 0$. Thus all edge receives value 0. Hence split graph of any tree is *0*-edge magic.

Conclusion

Thus *0*-edge magic labeling for some class of graphs namely $spl(P_n)$, $spl(C_n)$, $spl(K_{1,n})$, $spl(B_{m,n})$ and for Tree are shown in this paper.

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