Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 1 (2016), pp. 401-404
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Around Multivariate Credibility: Properties of the Covariance Matrix

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Abstract

The credibility formula of Frees (2003) considers the data from both the claim number and claim amounts processes. In this paper we will add some properties of the covariance matrix which has a significant role theoretically and practically.

Keywords: Credibility theory, Irreducibility of a sequence of random variables, Positive definite matrix, Frees formula.

2000 MSC: Primary 62E17; Secondary 62P05

Introduction

According to Rodermund (1989), the concept of credibility has been the casualty actuaries' most important and enduring contribution to casualty actuarial science."

In this sense, credibility theory is used to determine the expected claims experience of an individual risk when those risks are not homogeneous, given that the individual risk belongs to a heterogeneous collective. The main objective of the credibility theory is to calculate the weight which should be assigned to the individual risk data to determine a fair premium to be charged, for recent detailed introductions to credibility theory, see Norberg (2004), Bühlmann and Gisler (2005).

We define the credibility theory as"cornerstone" of the field of actuarial science (Hick-man and Heacox (1999) based on the concept of limiting the estimator of individual premium to the class of estimators that are linear with respect to all observations of the portfolio.

In the work of Frees (2003), we add the conditions for some matrices to use correctly. Firstly, we talk about the variance-covariance matrix, where is defining as:

$$V_i = VarY_i = \begin{pmatrix} Var(N_i) & Cov(N_i, S_i) \\ Cov(N_i, S_i) & Var(S_i) \end{pmatrix} \quad (1)$$

(See Goulet et al.(2005)) . In frees work this matrix will be reversed. But this inversion is not correct or valid if the condition of inversion is verified. So, it is sufficient that the matrix V is positive definite.

The paper is organized as follows: Section 2 is devoted to irreducible of sequence of random variables. Section 3 provides the main contribution of this paper; i.e., modification of the principal model. Finally, a small simulation is carried out to illustrate the theoretical conclusions and some remarks.

Irreducible of Sequence of Random Variables

Definition 1 *Let $(\Omega, , Pr)$ be a probability space and $L^2 = L^2(\Omega, , Pr)$ the Hilbert space of square-integral random variables. A sequence of random variables $X = (X_1, \ldots, X_n)$ is called (linearly) reducible with respect to the probability measure if there exist real coefficients α_0 and $\alpha = (\alpha_1, \ldots, \alpha_n) \neq 0$ such that*

$$lin - reduc(\sum_{i=1}^{n} \alpha_i X_i = \alpha_0) = 1. \quad (2)$$

In the context of -square-integral random variables the irreducibility of a sequence of random variables can be characterized as follows:

Proposition 2 A sequence $X = (X_1, \ldots, X_n)$of -square-integral random variables are irreducible if and only if their -covariance matrix

$$cov - matr(X) := \left((X_i, X_j)\right)_{1 \le i,j \le n} \quad (3)$$

is positive definite.

Proof. Let $X = (X_1, \ldots, X_n) \in (L^2\ ())^n$. .

The matrix (X) not being positive definite is equivalent to having 0 as spectral value. The later is equivalent to $\alpha^{tr}(X)\alpha = (\sum_i \alpha_i X_i) = 0$ for some $\alpha = (\alpha_1, \ldots, \alpha_n) \neq 0$. But the vanishing of the -variance is again equivalent to $\sum_i \alpha_i X_i = \alpha_0$, i.e. the reducibility of$X$.

The inversion notion of following random variables on Frees (2003) assumptions is added as follows:

Modification Model

The inversion notion of following random variables on Frees (2003) assumptions is added as follows:

- The data available are disaggregated by risk class i (town), over time t (year) and by line of business j.
- For each observationitj, the responses consist of claim amount S_{itj} and claim number N_{itj} and we have the exposure e_{itj}

- The claims amounts follow an aggregate loss model, so that
  $S_{itj} = \sum_{k=1}^{N_{itj}} C_{itj,k}$. (4)

Here, $\{C_{itj,k}\ ,k\ =\ 1,2,\ldots,\}$ are claims resulting from individual losses from the same distribution.

- Conditional on α_i , the claims are independent, where α_i is a structure variable.
- Conditional on α_i , the claims numbers N_{itj} are independent of the claims amounts $\{C_{itj,k}\ ,k\ =\ 1,2,\ldots,\}$.
- The random variables $\{N_{itj}\ ,S_{itj}\ ,t\ =\ 1,\ldots,T_i\ ,\}$ are independent among risk classes.

We will work on one line of business. We make the following model assumption: $\tau\ =\ 1,2$. Moreover, on a non-negligible set the pair $(S_{i,s}\ ,N_{i,s}\ )$ is assumed to be $Pr(\cdot\ |\Theta\ i\ )$-irreducible:

$$cond\ -\ irreducPr_{\Theta}\{\ \omega\ /\ (S_{i,s}\ ,N_{i,s}\ )\ is\ Pr(\cdot\ |\ \Theta_i\ (\omega))\ -irreducible\}\ >\ 0. \qquad (5)$$

We use the following notations:

$$y_i' = (N_i, S_i\ ) \text{ and } y\ =\ (y_1\ ,\ldots, y_n\ )'. \qquad (6)$$

With $\gamma_1\ =\ E[C|\alpha]$, $\lambda_1\ =\ E[N|\alpha]$., and $E(y_i\ )\ =\begin{pmatrix} E(N_i) \\ E(S_i) \end{pmatrix}$. (7)

For second moments, we require the matrix $V_i\ =\ V\,ar\,y_i$ i- the matrix is given by:

$$V\ =\ V\,ar(N_i\ ,S_i\ )\ =\begin{pmatrix} V\,ar(\overline{N_i}\ ) & Cov(\overline{N_i}, \overline{S_i}) \\ Cov(\overline{N_i}, \overline{S_i}) & V\,ar(\overline{S_i}\ ) \end{pmatrix} \qquad (8)$$

The best linear (in y) unbiased predictor of W where W is a random variable:

$$W^*\ =\ E(W)\ +\ Cov(W,y)V^{-1}\,[y\ -\ E(y)]. \qquad (9)$$

Under the assumption of irreducibility, the covariance matrix V is positive definite, so reversible. Also, we can easily demonstrate that the denominator of the ratio estimator credibility of Frees (2003) is different of zero. By [4] the general formula of credibility in [3] with only one insurance contract and no weights (or volumes) is :

$$\alpha\bar{S}\ +\ \beta\bar{N}\gamma_1\ +\ \eta\gamma_1\,\lambda_1 \qquad (10)$$

Where

$$\alpha := \frac{Cov(E(S|\Theta),\bar{S})\ -Cov(E(S|\Theta),\bar{N})\gamma_1}{V\,ar(\bar{S})-V\,ar(\bar{N})(\gamma_1)^2} \qquad (11)$$

$$\beta := \frac{Cov(E(S|\Theta),\bar{N})}{V\,ar(\bar{N})\gamma_1} - \alpha \qquad (12)$$

$$\eta := 1\ -\ \alpha\ -\ \beta \qquad (13)$$

A first result from the assumptions made is the following proposition:

Proposition 3 Under the model assumptions, we have

$$V\,ar(\bar{S})\ -\ V\,ar(\bar{N})(\gamma_1\,)^2\ \ >\ 0\ cond\ -\ det\ -\ 1\ -\ B. \qquad (14)$$

Proof. To show (14), we know the sequence of random variables (S,N) is Pr-irreducible, and so is the pair vector $(\bar{S},\bar{N})$. Proposition 2 tells us that their covariance matrix is positive definite:

$$Pr(\bar{S},\bar{N}) = \begin{pmatrix} Var(\bar{N}) & Cov(\bar{N},\bar{S}) \\ Cov(\bar{N},\bar{S}) & Var(\bar{S}) \end{pmatrix} = \begin{pmatrix} Var(\bar{N}) & Var(\bar{N})\gamma_1 \\ Var(\bar{N})\gamma_1 & Var(\bar{S}) \end{pmatrix}$$

This implies that the determinant $Var(\bar{S}) - Var(\bar{N})(\gamma_1)^2$ 2 of the last matrix is strictly positive which the statement is in (14).

Remark 4 If we look at the following matrices found in Frees (2003) $(T_i^{-1}\overline{E_{2,i}}C_2 + C_2)$ in equation (19) and $(T_i^{-1}\overline{E_{3,i}}\Sigma_2 + \Sigma_\lambda)$ in equation(20) and C_2 , $E_{2,i}$ in page Number (24) and $\bar{E}_{3,i}$ define in proposition (3) are all reverse without certain conditions for the inverted, it is therefore necessary:

- $det(T_i^{-1}\overline{E_{2,i}}C_2 + C_2) \neq 0$;
- $\det(T_i^{-1}\overline{E_{3,i}}\Sigma_2 + \Sigma_\lambda) \neq 0$;
- $det(C_2) \neq 0$;
- $det(E_{2,i}) \neq 0$;
- $det(E_{3,i}) \neq 0$;

where $det(X)$ signified the determinant of X matrix.

Conclusion

The paper describes variance-covariance matrix leading to credibility formulae of Frees (2003), where the condition of the irreducibility of variables (claim amount and number) can be computed. Also, the credibility formulae of Frees (2003) can be derived.

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