Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 1 (2016), pp. 391-400

http://www.ripublication.com

# On Bayes Estimates of Lindley Distribution under Linex Loss Function: Informative and Non Informative Priors

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**Abstract**

This paper focus on mathematical properties of Lindley distribution via Bayesian approach are derived under Linex loss functions. To this end, we explain the derivation of posterior distributions for the Lindley distribution under Linex loss functions using non-informative and informative priors (the extension of Jeffreys and Inverted Gamma priors) respectively. The comparison work are given using a Monte Carlo simulation on the performance of these estimators according to the mean square error (MSE).

**Keywords:** Elicitation of hyperparameters; Lindley distribution; loss function

## Introduction

In Bayesian analysis, the unknown parameter is regarded as being the value of a random variable from a given probability distribution, with the knowledge of some information about the value of parameter prior to observing the data $x_1, x_2 \dots, x_n$.

The Lindley distribution is one of the most popular distributions of failure time and life testing and reliability theory. It has been overlooked in the literature from 1958. Lindley distribution was originally developed by **Lindley (1958)** and some classical statistics properties are investigated by **Ghitany** et **al. (2008)**. **Krishna** and **Kumar (2011)** considered Lindley distribution under the progressive type **II** censoring for reliability estimation using maximum likelihood and Bayesian approach, however they did not considered it for complete data set using various loss functions. **Sankaran (1970)** introduced a discrete version of Lindley distribution known as discrete Poisson-Lindley distribution and **Ghitany** and **AlMutairi (2009)** described some estimation methods.

The distribution of zero-truncated Poisson-Lindley was introduced by **Ghitany** et **al. (2008)** who used the distribution for modeling count data in the case where the distribution has to be adjusted for the count of missing zeros. **Zamani** and **Ismail (2010)** introduced the Negative Binomial distribution as an alternative to zero-truncated Poisson-Lindley distribution. **Ghitany** et **al. (2011)** introduced a two parameter weighted Lindley distribution and pointed that Lindley distribution is particularly useful in modeling biological data from mortality studies. In addition, **Zeghdoudi** and **Nedjar (2015)** introduced a new distribution, named gamma Lindley distribution, based on mixtures of gamma $(2, \theta)$ and one-parameter Lindley distributions which is useful in modeling lifedata.

Recently, a study of the effect of some loss functions on Bayes Estimate and posterior risk for the lindley distribution are made by **Sajid Ali** et **al. (2013)**.

Let $x_1, x_2 \dots, x_n$ be independent and identically distributed lifetimes from Lindley distribution with an unknown parameter $\theta$. The probability density function is given by:

$$f(x;\theta) = \begin{cases} \dfrac{\theta^2\,(1+x)e^{-\theta x}}{1+\vartheta}, x, \theta > 0 \\ 0, otherwise \end{cases} \tag{1}$$

The likelihood function for a random sample $x_1, x_2 \dots, x_n$ which is taken from Lindley distribution is:

$$L(x,\theta) = \frac{\theta^{2n}}{(1+\theta)^n} \prod_{i=1}^{n} (1+x_i) e^{-\theta \sum_{i=1}^{n} x_i}, x, \theta > 0 \tag{2}$$

It is well known that, for Bayes estimators, the performance depend on the form of the prior distribution and the loss function assumed. In this study, we consider the Linex (linear-exponential) loss function, where the name Linex is justified by the fact that this asymmetric loss function rises approximately linearly on one side of zero and approximately exponentially on the other side.

The article is organized as follows, section 2 explains the derivation of posterior distributions for the Lindley distribution under Linex loss function using non-informative and informative priors (the extension of Jeffreys and Inverted Gamma priors) respectively. In section 3, comparison was made through a Monte Carlo simulation study on the performance of these estimators according to the mean square error (MSE). Results are summarized in tables and followed by conclusions with some remarks.

## Bayesian Estimation

To obtain Bayes estimators, we assume that $\theta$ is a real valued random variable with probability density function $\pi(\theta)$. The posterior distribution of $\theta$ i. e, $p(\theta|x)$ is the conditional probability density function of $\theta$ given the data. In this section we consider Bayes estimation of the unknown parameter $\theta$ based on the above mentioned priors and loss functions.

**Bayes estimators under Linex loss function:**
The Linex loss function which is asymmetric, was introduced by **Varian (1975)**, **Rojo (1987)**, **Basu** and **Ebrahimi (1991)**, **Pandy (1997)**, **Soliman (2000)** and **Nassar** and **Eissa (2004)**.

It may be expressed as:

$$L(\hat{\theta},\theta) = \exp\left(a(\hat{\theta} - \theta)\right) - a(\hat{\theta} - \theta) - 1, \ a \neq 0$$

The sign and magnitude of the shape parameter $a$ reflects the direction and degree of asymmetry, respectively. (If $a > 0$), the overestimation is more serious than underestimation, and vice-versa). For $a$ closed to zero, the Linex loss is approximately squared error loss and therefore almost symmetric.
The posterior expectation of the Linex loss function equation is:

$$E[L(\hat{\theta},\theta)] \propto exp(a\hat{\theta})E[exp(-a\hat{\theta})] - a\left(\hat{\theta} - E(\theta)\right) - 1,$$

By result of **Zellner (1986)**, the Bayes estimator of $\theta$, denoted by $\hat{\theta}_{Lin}$ under the Linex loss is the value $\hat{\theta}$ which minimizes the above equation, is given by:

$$\hat{\theta}_{Lin} = -\frac{1}{a} ln\left[E[exp(-a\theta)]\right], \qquad (3)$$

When the expectation $E[exp(-a\theta)]$ exists and finite (see **Calabria** and **Pulcini (1996)**).
**Thompson** and **Basu (1996)** identified a family of loss functions $L(\Delta)$ where $\Delta$ is either the estimation error $(\hat{\theta} - \theta)^2$, such that

- $L(0) = 0$
- $L(\Delta) > (<) L(-\Delta) > 0$ for all $\Delta > 0$
- $L(.)$ is twice differentiable with $L'(0) = 0$ and $L''(\Delta) > 0$ for all $\Delta \neq 0$
-$0 < L'(\Delta) > (<) - L'(-\Delta) > 0$ for all $\Delta > 0$.

***Posterior distribution using the extension of Jeffreys prior:***
The extension of Jeffreys prior is assumed as non-informative prior for the parameter $\theta$. It was proposed by **Al-Kutubi, H. S.** and **Ibrahim, N. A. (2009)** and given as:

$$\pi(\theta) = k\frac{n^c}{\theta^{2c}}, \theta, c > 0 \text{ and } k \text{ is constant} \qquad (4)$$

Combining the extension of Jeffreys prior and the likelihood function of lindley, the posterior distribution for the parameter $\theta$ given the data $x_1, x_2 \dots, x_n$ is derived as follows:

$$p(\theta|x) = \frac{\prod_{i=1}^{n} L(x_i|\theta)\pi(\theta)}{\int_0^\infty \prod_{i=1}^{n} L(x_i|\theta)\pi(\theta)} = \frac{\frac{\theta^{2(n-c)}}{(1+\theta)^n} e^{-\theta\sum_{i=1}^{n} x_i}}{\int_0^\infty \frac{\theta^{2(n-c)}}{(1+\theta)^n} e^{-\theta\sum_{i=1}^{n} x_i} d\theta} \qquad (5)$$

Using the Linex loss function and assuming, the corresponding Bayes' estimator of the parameter $\theta$ is as follows:

$$\hat{\theta}_{Lin} = -\frac{1}{a} ln\left[E[exp(-a\theta)|x]\right]$$

$$E\left[e^{-a\theta}|x\right] = \int_0^{\infty} e^{-a\theta}\, p(\theta|x) d\theta = \frac{\int_0^{\infty} \frac{\theta^{2(n-c)}}{(1+\theta)^n} e^{-\theta\left(a+\sum_{i=1}^{n} x_i\right)} d\theta}{\int_0^{\infty} \frac{\theta^{2(n-c)}}{(1+\theta)^n} e^{-\theta \sum_{i=1}^{n} x_i} d\theta}$$

It may be noted here that the posterior distribution $p(\theta|x)$ takes a ratio form that involves an integration in the denominator and cannot be reduced to a closed form. Hence, the evaluation of the posterior expectation for obtaining the Bayes estimator of $\theta$ will be tedious. Among the various methods suggested to approximate the ratio of integrals of the above form, perhaps the simplest one is **Lindley**'s **(1980)** approximation method, which approaches the ratio of the integrals as a whole and produces a single numerical result. Thus, we propose the use of **Lindley**′s **(1980)** approximation for obtaining the Bayes estimator of $\theta$. Many authors have used this approximation for obtaining the Bayes estimators for some lifetime distributio-ns; see among others, **Howlader** and **Hossain (2002)** and **Jaheen (2005)**.

If $n$ is sufficiently large, according to **Lindley (1980)**, any ratio of the integral of the form

$$I(x) = E[\mu(\theta)] = \frac{\int_{\theta} \mu(\theta) exp[L(\theta,x) + g(\theta)] d\theta}{\int_{\theta} exp[L(\theta,x) + g(\theta)] d\theta}, \theta > 0 \quad (6)$$

where $\mu(\theta)$ = function of $\theta$ only,

$L(\theta,x)$ = log of likelihood,

$g(\theta)$ = log of prior of $\theta$.

Thus,

$$I(x) = \mu(\hat{\theta}) + 0.5[(\hat{\mu}_{\theta\theta} + 2\hat{\mu}_{\theta}\hat{p}_{\theta})\hat{\sigma}_{\theta\theta}] + 0.5\left[(\hat{\mu}_{\theta}\hat{\sigma}_{\theta\theta})(\hat{L}_{\theta\theta\theta}\hat{\sigma}_{\theta\theta})\right] \quad (7)$$

Where $\hat{\theta}$ =MLE of $\theta = \frac{-(\bar{x}-1)+\sqrt{(\bar{x}-1)^2+8\bar{x}}}{2\bar{x}}, \bar{x} > 0.$

$\hat{\mu}_{\theta} = \frac{\partial\mu(\hat{\theta})}{\partial\hat{\theta}}$,

$\hat{\mu}_{\theta\theta} = \frac{\partial^2\mu(\hat{\theta})}{\partial\hat{\theta}^2}$,

$\hat{p}_{\theta} = \frac{\partial g(\hat{\theta})}{\partial\hat{\theta}}$,

$\hat{L}_{\theta\theta} = \frac{\partial^2 L(\hat{\theta})}{\partial\hat{\theta}^2}$,

$\hat{\sigma}_{\theta\theta} = -\frac{1}{\hat{L}_{\theta\theta}}$,

$\hat{L}_{\theta\theta\theta} = \frac{\partial^3 L(\hat{\theta})}{\partial\hat{\theta}^3}$,

$$= \frac{\int_0^\infty \mu(\theta) exp[L(\theta,x) + g(\theta)]d\theta}{\int_0^\infty exp[L(\theta,x) + g(\theta)]d\theta}, \theta > 0$$

Following the steps explained above, we have

$\mu(\theta) = e^{-a\theta}$,

$$L(\theta, x) = 2nlog\theta - nlog(1+\theta) - \theta \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} log(1 + x_i),$$

$g(\theta) = clogn - 2clog\theta,$

$\hat{\mu}_\theta = -ae^{-a\theta},$

$\hat{\mu}_{\theta\theta} = -a^2 e^{-a\theta},$

$$\hat{p}_\theta = -\frac{2c}{\theta},$$

$$\hat{L}_{\theta\theta} = -\frac{2n}{\theta^2} + \frac{n}{(1+\theta)^2},$$

$$\hat{\sigma}_{\theta\theta} = \frac{\theta^2(1+\theta)^2}{2n(1+\theta)^2 - n\theta^2},$$

$$\hat{L}_{\theta\theta\theta} = \frac{4n}{\theta^3} - \frac{2n}{(1+\theta)^3},$$

$$E[exp(-a\theta)|x] = e^{-a\hat{\theta}} + 0.5\left[ae^{-a\hat{\theta}}\left(a + \frac{4c}{\hat{\theta}}\right)\frac{\hat{\theta}^2(1+\hat{\theta})^2}{2n(1+\hat{\theta})^2 - n\hat{\theta}^2}\right] - 0.5\left[ae^{-a\hat{\theta}}\left(\frac{4n}{\hat{\theta}^3} - \frac{2n}{(1+\hat{\theta})^3}\right)\left(\frac{\hat{\theta}^2(1+\hat{\theta})^2}{2n(1+\hat{\theta})^2 - n\hat{\theta}^2}\right)^2\right]$$

$$\hat{\theta}_{Lin} = -\frac{1}{a}log\left[e^{-a\hat{\theta}} + 0.5\left[ae^{-a\hat{\theta}}\left(a + \frac{4c}{\hat{\theta}}\right)\frac{\hat{\theta}^2(1+\hat{\theta})^2}{2n(1+\hat{\theta})^2 - n\hat{\theta}^2}\right] - 0.5\left[ae^{-a\hat{\theta}}\left(\frac{4n}{\hat{\theta}^3} - \frac{2n}{(1+\hat{\theta})^3}\right)\left(\frac{\hat{\theta}^2(1+\hat{\theta})^2}{2n(1+\hat{\theta})^2 - n\hat{\theta}^2}\right)^2\right]\right]. \quad (8)$$

***Posterior distribution using the inverted gamma prior (IG):***

The inverted gamma prior is a good life distribution model which represents the reciprocal of a variable distributed according to the gamma distribution. It is observed that if $\theta$ has an inverted gamma$(\alpha, \beta)$ distribution then $\frac{1}{\theta}$ has a gamma$(\alpha, \beta)$ distribution.

It is given as:

$$\pi(\theta) = \frac{\alpha^\beta}{\Gamma(\beta)}\frac{1}{\theta^{\beta+1}}e^{-\frac{\alpha}{\theta}}, \alpha, \beta, \theta > 0 \quad (9)$$

The posterior distribution of parameter $\theta$ for the given data using Eq. (2) and the inverted gamma prior is:

$$p(\theta|x) = \frac{\prod_{i=1}^{n} L(x_i|\theta)\pi(\theta)}{\int_0^\infty \prod_{i=1}^{n} L(x_i|\theta)\pi(\theta)} = \frac{\frac{\theta^{2n-\beta-1}}{(1+\theta)^n} e^{-\frac{\alpha}{\theta}-\theta\sum_{i=1}^{n} x_i}}{\int_0^\infty \frac{\theta^{2n-\beta-1}}{(1+\theta)^n} e^{-\frac{\alpha}{\theta}-\theta\sum_{i=1}^{n} x_i} d\theta} \tag{10}$$

Now, the corresponding Bayes' estimator for the parameter $\theta$ under the Linex loss function is

$$\hat{\theta}_{Lin} = -\frac{1}{a} log\left[E[exp(-\theta)|x]\right]$$

$$E\left[e^{-a\theta}|x\right] = \int_0^\infty e^{-a\theta}\, p(\theta|x)d\theta = \frac{\int_0^\infty \frac{\theta^{2n-\beta-1}}{(1+\theta)^n} e^{-\frac{\alpha}{\theta}-\theta\left(a+\sum_{i=1}^{n} x_i\right)} d\theta}{\int_0^\infty \frac{\theta^{2n-\beta-1}}{(1+\theta)^n} e^{-\frac{\alpha}{\theta}-\theta\sum_{i=1}^{n} x_i} d\theta}$$

Using the same steps mentioned above, we find

$\mu(\theta) = e^{-a\theta}$,

$$L(\theta, x) = 2nlog\theta - nlog(1+\theta) - \theta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} log(1+x_i),$$

$$g(\theta) = \beta log\alpha - log(\Gamma(\beta)) - (\beta+1)log\theta - \frac{\alpha}{\theta},$$

$$\hat{\mu}_\theta = -ae^{-a\theta},$$

$$\hat{\mu}_{\theta\theta} = -a^2e^{-a\theta},$$

$$\hat{p}_\theta = \frac{\alpha}{\theta^2} - \frac{\beta+1}{\theta},$$

$$\hat{L}_{\theta\theta} = -\frac{2n}{\theta^2} + \frac{n}{(1+\theta)^2},$$

$$\hat{\sigma}_{\theta\theta} = \frac{\theta^2(1+\theta)^2}{2n(1+\theta)^2 - n\theta^2},$$

$$\hat{L}_{\theta\theta\theta} = \frac{4n}{\theta^3} - \frac{2n}{(1+\theta)^3},$$

$$E[exp(-a\theta)|x] = e^{-a\hat{\theta}} + 0.5\left[a(a-2)e^{-a\hat{\theta}}\left(\frac{\alpha}{\hat{\theta}^2} - \frac{\beta+1}{\theta}\right)\frac{\hat{\theta}^2(1+\hat{\theta})^2}{2n(1+\hat{\theta})^2 - n\hat{\theta}^2}\right] -$$
$$0.5\left[ae^{-a\hat{\theta}}\left(\frac{4n}{\hat{\theta}^3} - \frac{2n}{(1+\hat{\theta})^3}\right)\left(\frac{\hat{\theta}^2(1+\hat{\theta})^2}{2n(1+\hat{\theta})^2 - n\hat{\theta}^2}\right)^2\right]$$

$$\hat{\theta}_{Lin} = -\frac{1}{a} log \left[ e^{-a\hat{\theta}} + 0.5 \left[ a(a-2)e^{-a\hat{\theta}} \left( \frac{\alpha}{\hat{\theta}^2} - \frac{\beta+1}{\theta} \right) \frac{\hat{\theta}^2 (1+\hat{\theta})^2}{2n(1+\hat{\theta})^2 - n\hat{\theta}^2} \right] - 0.5 \left[ ae^{-a\hat{\theta}} \left( \frac{4n}{\hat{\theta}^3} - \frac{2n}{(1+\hat{\theta})^3} \right) \left( \frac{\hat{\theta}^2 (1+\hat{\theta})^2}{2n(1+\hat{\theta})^2 - n\hat{\theta}^2} \right)^2 \right] \right]. \quad (11)$$

**Elicitation of hyperparameter (s) :**

According to **Garthwaite** et **al. (2004)** elicitation is the process of formulating a person's knowledge and beliefs about one or more uncertain quantities into a (joint) probability distribution for those quantities. In the context of Bayesian statistical analysis, it arises most usually as a method for specifying the prior distribution for one or more unknown parameters of a statistical model. It is a difficult task because we first have to identify prior distribution and then its hyperparameters.

## Simulation Study

In this section, Monte Carlo simulation study is performed to compare the methods of estimation by using mean square Errors (MSE. s) as follows:

$$MSE(\hat{\theta}) = \frac{\sum_{i=1}^{N} (\hat{\theta}_i - \theta)^2}{N},$$

where N is the number of replications. We generated 10000 samples of size n= 20, 40, 60, 80 and 100 to represent small, moderate and large sample sizes from Lindley distribution with three values of $\theta$ ($\theta$ = 0. 1, 1, 3).

In order to compare the Bayes' estimators obtained in the above section under two different loss functions and two priors, we choose the values of the extensive Jeffreys constants; (c = 1, 2. 5) and for the Inverted Gamma prior, the following pairs of values of the hyper parameters $\alpha$ and $\beta$ are chosen ($\alpha$, $\beta$) = $\{(1, 1.5), (1.5, 2)\}$, with two values of loss symmetry a $(a = \pm 1)$.

The results are summarized and tabulated in the following tables

**Table 1:** Bayes Estimates and respective MSE. s under Linex loss function ($\alpha = 1$, $\beta = 1.5, a = -1, c = 1$)

| $\theta$ | **0. 1** | **1. 0** | **3. 0** |
|---|---|---|---|
| **n** | **Extension of the Jeffrey prior** | | |
| **20** | 0. 1019706<br>(3. 37851e-05) | 1. 009012<br>(0. 00132743) | 3. 008401<br>(0. 0001022) |
| **40** | 0. 100922<br>(7. 77294 e-06) | 1. 006399<br>(0. 00039593) | 3. 015483<br>(2. 7655 e-06) |
| **60** | 0. 1010106 | 1. 001558 | 3. 004253 |

| | | | |
|---|---|---|---|
| | (5. 05305 e-06) | (0. 00028968) | (2. 1286 e-06) |
| **80** | 0. 1009269<br>(3. 42731 e-06) | 1. 000983<br>(0. 00015828) | 3. 001069<br>(5. 2229 e-07) |
| **100** | 0. 1003229<br>(1. 11661 e-06) | 1. 007224<br>(5. 8767 e-05) | 3. 009347<br>(5. 0728 e-07) |
| **n** | **Inverted Gamma Prior** | | |
| **20** | 0. 108032<br>(0. 009999979) | 1. 007571<br>(3. 12592 e-05) | 3. 01398<br>(0. 000809630) |
| **40** | 0. 1082453<br>(0. 000186651) | 1. 003947<br>(1. 63539 e-05) | 3. 004458<br>(0. 000299220) |
| **60** | 0. 1068127<br>(8. 97865 e-06) | 1. 003767<br>(9. 59760 e-06) | 3. 00645<br>(3. 78963 e-05) |
| **80** | 0. 1061052<br>(6. 12503 e-06) | 1. 004543<br>(8. 66125 e-06) | 3. 00494<br>(4. 3692 e-05) |
| **100** | 0. 1013089<br>(5. 73806 e-07) | 1. 001351<br>(8. 93025 e-07) | 3. 002272<br>(6. 85087 e-05) |

**Table 2:** Bayes Estimates and respective MSE. s under Linex loss function ($\alpha = 1.5$, $\beta = 2, a = +1, c = 2.5$)

| $\theta$ | **0. 1** | **1. 0** | **3. 0** |
|---|---|---|---|
| **n** | **Extension of the Jeffrey prior** | | |
| **20** | 0. 1185545<br>(0. 0001469819) | 1. 002763<br>(0. 0008518226) | 3. 00289<br>(0. 04930709) |
| **40** | 0. 1042461<br>(0. 0001170025) | 1. 000948<br>(0. 0002172971) | 3. 017564<br>(0. 01775183) |
| **60** | 0. 1054167<br>(3. 81163 e-05) | 1. 002078<br>(0. 0001614424) | 3. 009071<br>(0. 00440219) |
| **80** | 0. 1061721<br>(3. 353811 e-05) | 1. 003871<br>(0. 0001555009) | 3. 000159<br>(0. 00278309) |
| **100** | 0. 1044127<br>(9. 491945 e-06) | 1. 004317<br>(0. 000125149) | 3. 001924<br>(0. 002123919) |
| **n** | **Inverted Gamma Prior** | | |
| **20** | 0. 1070039<br>(0. 0007547608) | 1. 005795<br>(0. 01096389) | 3. 017463<br>(0. 2006986) |
| **40** | 0. 1020788<br>(0. 0002157109) | 0. 9939864<br>(0. 002843982) | 3. 007924<br>(0. 162567) |
| **60** | 0. 1056784<br>(8. 098614 e-05) | 1. 000983<br>(6. 493136 e-05) | 3. 004392<br>(0. 1457605) |
| **80** | 0. 10927702<br>(2. 592846 e-05) | 1. 007546<br>(5. 599529 e-05) | 3. 009507<br>(0. 01140389) |
| **100** | 0. 1015657<br>(2. 363515 e-05) | 1. 002627<br>(2. 43957 e-05) | 3. 004382<br>(0. 004297851) |

## Conclusion

We consider the Bayesian analysis of the Lindley model via informative and non-informative priors under Linex loss functions. Based on posterior distribution, different properties, we conclude that informative priors (CP, GP) performance approximately equal and have smaller posterior risk as compared to non informative priors. Also, Jeffreys prior results are more precise than Inverted Gamma prior. For future studies, we can explain the derivation of posterior distributions for the Lindley distribution under squared error using non-informative and informative priors(the extension of Jeffreys and Inverted Gamma priors) respectively. In addition, this work can be extended using censored data.

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