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Modeling the Heteroscedasticity in Data Distribution

Ayodele Abraham Agboluaje[1,2], Suzilah bt Ismail[1] and Chee Yin Yip[3]

*[1]School of Quantitative Sciences, College of Arts and Sciences,
Universiti Utara Malaysia, 06010 UUM Sintok, Kedah Darul Aman, Malaysia.*

*[2]Department of Mathematics and Computer Science, Faculty of Natural Sciences,
Ibrahim Badamasi Babangida University, Lapai, Nigeria.*

*[3]Department of Economics, Faculty of Business and Finance,
Universiti Tuanku Abdul Rahman, Kampar, Perak, 31900, Malaysia.*

Abstract

The main objective of this study is to provide a model that will uplift the weaknesses of the existing model for efficient estimation. Generalized autoregressive conditional heteroscedasticity (GARCH) family models weaknesses were overcome by the new Combine White Noise (CWN) model which proved to be more efficient. CWN estimation passed stability condition, stationary, serial correlation, the ARCH effect tests and it also passed the Levene's test of equal variances using both Australia (A.U.) and United States (U.S.) GDP data sets. The CWN estimation produced better results with minimum information criteria and high log likelihood values in both U.S. and A.U. data estimation. CWN has the minimum forecast errors which were better results when compare with the GARCH model dynamic evaluation forecast errors in both countries. The determinant of the residual of covariance matrix values revealed that CWN was efficient in the two countries, but A.U.was more efficient.Based on every result in the empirical analysis of the two countries, CWN was the more appropriate model.

Keywords: Combine White Noise, Efficient, Leverage, Log likelihood, Minimum forecast errors, Minimum information criteria

Introduction

In a stochastic time series, the error term mostly exhibit white noise errors or heteroscedastic errors. Vector Autoregressive model dealt completely with white noise errors, while several tests and series of family models have been developed to resolve the heteroscedastic errors and yet heteroscedasticity is still a challenge [1 - 8].To resolve this, the new model called combine white noise is implemented.

Harvey [9] revealed likelihood ratio test for heteroscedasticity. Breusch and Pagan [10] advocated an easy test for heteroscedasticity errors in a linear regression model by employing the general asymptotic properties of Langrangian multiplier test. White [11] obtained a direct test for heteroscedasticity. Later, all these tests could not withstand the pressures of the high frequency data in the stochastic time series for model estimation efficiency [3].

Engle [3] revealed the autoregressive conditional heteroscedasticity (ARCH) to model the conditional heteroscedasticity with mean zero, serially unrelated processes with unequal variances conditional on the earlier period, except constant unconditional variances to overcome the heteroscedastic challenges.The group errors were effectively handled by the ARCH models and it also accommodated the changes made by economic forecaster. The abnormalities like crashes, mergers, news effect or threshold effects in the financial and economic sector data analysis cannot be modeled properly by ARCH model. ARCH can only model fixed lag length.

Bollerslev [4] introduced generalized ARCH that was flexible to allow large lag length for efficient estimation and forecasting, and to uplift other ARCH weaknesses. Engle [11] introduced the dynamic conditional correlation (DCC) model which offered a reasonable approximation to series of time changing correlation procedures. The DCC outperformed the simple GARCH model, DCC proved most accurate among the varieties of estimators [12].GARCH cannot model efficiently the excess kurtosis and volatility persistence [13, 14].

Engle and Ng [15] integrated news impact curve into volatility estimates with many time varying volatility modeling being introduced. Different kinds of asymmetry were permitted in the impact of news on volatility by these models. To determine the news impact straight, a partially nonparametric model is introduced. The negative shocks brought in more volatility than positive shocks by the models and asymmetry was not adequately modeled.

Hentschel [6] described the behaviours of some GARCH family models as: The development of the conditional variance was illustrated by standard GARCH. The conditional standard deviation employed absolute GARCH and the natural logarithm of the conditional variance employed exponential GARCH. The popular GARCH models were nested by the family GARCH.

Threshold GARCH and exponential GARCH restrained the asymmetric effects of positive and negative shocks of the same dimension on conditional volatility in different ways [5, 6, 7, 8, 16, 17, 18, 19]. Leverage is a particular case of asymmetry .Leverage effects cannot be modeled by GARCH family model for the reason that any restriction imposed on it will be positivity restriction which has no leverage effect. The coefficient of variance equation must be negative for the existence of leverage[7, 8, 20].

As soon as the data size increased with high frequency data, the traditional models cannot have efficient and accurate results because of the behaviours of error terms that were not recognizable in the stochastic volatility time series [7, 8]. The new approach of Combine White Noise uplifted the traditional models weaknesses to model the error terms for appropriate estimations and to have reasonable outputs.

Materials and Methods

The data set of U.S. Gross Domestic Product (GDP) quarterly data from 1960Q1 to 2014Q4 and A.U. Gross Domestic Product (GDP) quarterly data from 1960Q3 to 2015Q2 were retrieved from the DataStream of Universiti Utara Malaysia library for this study.

Consider the auto regression model

$$y_t = \phi y_{t-1} + \varepsilon_t, \tag{2.1}$$

Permit the stochastic approach of a real-valued time to be ε_t, and the complete information through t time is I. The GARCH model is

$$\varepsilon_t \mid \mathrm{I}_{t-1} \sim \mathrm{N}(0, h_t), \tag{2.2}$$

$$\begin{aligned} h_t &= \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \\ &= \omega + A(L)\varepsilon_t^2 + B(L)h_t \end{aligned} \tag{2.3}$$

The EGARCH specification is

$$\log h_t = \alpha + \beta \mid z_{t-1} \mid + \delta z_{t-1} + \gamma \log h_{t-1}, \quad \mid \gamma \mid < 1 \tag{2.4}$$

where $z_t = \varepsilon_t / \sqrt{h_t}$ is the standardized shocks, $z_t \sim iid(0, \alpha)$. $\mid \gamma \mid < 1$ is when there is stability. The impact is asymmetric if $\delta \neq 0,$ although, there is existence of leverage if $\delta < 0$ *and* $\delta < \beta < -\delta.$ While both β and δ must be positive which the variances of two stochastic processes are, then, modeling leverage effect is not possible [7, 8].

The unequal variances (heteroscedastic errors) behaviours in the process of estimation being exhibited by GARCH models can be simplified into Combine White Noise models. The standardized residuals of GARCH errors which are unequal variances are decomposed into equal variances (white noise) in series to deal with the heteroscedasticity. The regression model is employed to transform each equal variances series to model.

Moving average process is employed for the estimation of these white noise series and called Combine White Noise.

$$Y_1 = \varepsilon_{1t} + \theta_{11}\varepsilon_{1,t-1} + \theta_{12}\varepsilon_{1,t-2} + ..\theta_{jq}\varepsilon_{j,t-q}$$

$$Y_2 = \varepsilon_{2t} + \Phi_{21}\varepsilon_{2,t-1} + \Phi_{22}\varepsilon_{2,t-2} + ..\Phi_{jq}\varepsilon_{j,t-q}$$

$$\vdots .$$

$$Yj = \varepsilon_{jt} + \phi_{j1}\varepsilon_{j,t-1} + \phi_{j2}\varepsilon_{j,t-2} + ..\phi_{jq}\varepsilon_{j,t-q}$$

$$\begin{aligned} Y_{jt} &= \sum_{j=1}^{q} \theta_j \varepsilon_{j,t-q} + \sum_{j=1}^{q} \Phi_j \varepsilon_{j,t-q} + ...\sum_{j=1}^{q} \phi_j \varepsilon_{j,t-q} \\ &= A(L)\varepsilon_t + B(L)\varepsilon_t + ... \end{aligned} \tag{2.5}$$

$$= \varepsilon_t [A(L) + B(L) + \ldots] \tag{2.6}$$

$$= Q\varepsilon_t \tag{2.7}$$

$$= U_t \,,$$

It can be written as

$$Y_t = U_t \,,$$

$$U_t \sim \mathrm{N}(0, \sigma_c^2) \tag{2.8}$$

where A(L) + B(L) + … = Q which are the matrix polynomial, U_t is the error term of combine white noise model and σ_c^2 is the combination of equal variances.

The combine variances of the combine white noise is

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 + \ldots \tag{2.9}$$

Considering the best two variances in the best two models produced by the Bayesian model averaging output. The combine variance follows:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 \tag{3.0}$$

The variance of errors, σ_c^2 in the combine white noise can be written:

$$\sigma_c^2 = W^2 \sigma_1^2 + (1-W)^2 \sigma_2^2 + 2\rho W \sigma_1 (1-W) \sigma_2 \tag{3.1}$$

where the balanced weight specified for the model is *W*. The least of σ_c^2 appearing, when the equation is differentiated with respect to *W* and equate to zero, obtaining:

$$W = \frac{\sigma_c^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{3.2}$$

Where ρ is the correlation; intra-class correlation coefficient is used for a reliable measurement.

Results and Discussions

The time plot of the data for both U.S. GDP and A.U. GDP showed upward trend which were behavior of non-stationary series.

The data of U.S. and A.U. GDP were transformed in returns series to examine the volatility clustering, long tail skewness and excess kurtosis which were the characteristics of heteroscedasticity. The volatility with unequal variances was revealed in the graphs.

The report in table 1 with U.S. data revealed that there was left long tail skewness, excess kurtosis and Jarque-Bera test was significant which indicated non-normality with standard deviation less than one. In table 1 also A.U. data reported that there was right tail skewness, excess kurtosis and Jarque-Bera test was significant with non-normality and standard deviation greater than one.

The standard deviation of A.U. data distribution was greater than one, while the standard deviation of U.S. data distribution was less than one. U.S. data distribution was skewness to the left while A.U. data distribution was to the right. The excess kurtosis was higher in data distribution of U.S.

Table 1 revealed that the ARCH LM tests for the effect of heteroscedasticity in the data series for F-Statistic and Obs*R-squared was not significant whichsuggested ARCH presence in the data in U.S., while it was significant in A.U. data distribution which means, there was no ARCH presence in the data.

Table 1: Histogram-normality and ARCH tests for U.S. and A. U. data

Coefficient/value U.S. data	probability	Coefficient/value	probability A.U. data	
Normal test				
Standard deviation	0.840452		1.055567	
Skewness	-0.320441		0.364743	
Kurtosis	4.515921		3.949680	
Jarque-Bera	24.71731	0.000004	13.08565	0.001440
ARCH tests				
F-Statistic	1.372665	0.2427	4.908379	0.0003
Obs*R-squared	1.376645	0.2407	22.57658	0.0004

Tables 2A and 2B showed that the AIC, BIC and HQ minimum information criteria with log-likelihood that were used to select the appropriate model between ARCH and GARCH models. EGARCH model was choosing because it had minimum values of AIC, BIC and HQ with high log-likelihood values. U.S. data estimation had minimum information criteria and high log likelihood, when compared with A.U. data estimation for ARCH and GARCH estimation.

In tables 2A and 2B the CWN had the minimum information criteria with high log likelihood. The CWN estimation gave better results with minimum information criteria and high log likelihood when compared with GARCH estimation. The CWN in table 2A for U.S. data had minimum information criteria and high log likelihood when compared with A.U. data estimation in table 2B.

Table 2A: U.S. data ARCH, GARCH and CWN models coefficients, information criteria and log likelihood values

	α	β	δ	γ	AIC	BIC	HQ	LL
ARCH	0.37700	0.14103			2.30379	2.42799	2.35396	-243.11
	(0.0000)	(0.0000)						
EGARCH	0.32771	0.32056	-0.0656	0.89149	2.26776	2.37644	2.35396	-240.19
	(0.0000)	(0.0160)	(0.3970)	(0.0000)				
CWN					-0.5235	-0.4306		63.32035

Note: α is the coefficient of the mean equation, β and δ are the coefficients of the variance equations, while γ is the coefficient of the log of variance equation. In the parentheses is the Probability Value (PV)

Table 2B: A.U. data ARCH, GARCH, and CWN models coefficients, information criteria and log likelihood values

	α	β	δ	γ	AIC	BIC	HQ	LL
ARCH	0.13645 (0.000)	0.31623 (0.006)			2.90733	2.96942	2.93241	-312.89
EGARCH	-0.0462 (0.448)	-0.0157 (0.811)	0.02031 (0.422)	1.0106 (0.000)	2.65324	2.76191	2.69713	-282.20
CWN					11.1777	11.3635		-1211.97

Note: α is the coefficient of the mean equation, β and δ are the coefficients of the variance equations, while γ is the coefficient of the log of variance equation. In the parentheses are the probability values (PV).

Leverage is not possible using GARCH family model, since any restriction imposed will be positivity restriction which has no leverage effect [7, 8]. No Statistical procedure removed heteroscedasticity completely [2, 21, 22].

Therefore, the standardized residuals graph of the EGARCH model (EGARCH errors) with unequal variances and zero mean were decomposed into equal variances series (white noise series) to overcome the EGARCH weaknesses. The graphs of equal variances (white noise series) with mean zero were acquired from the standardized residuals graph of the EGARCH. The white noise series were fit into regression model to make the white noise series models.

The Bayesian model averaging (BMA) procedure output revealed two best models from the first best models out of five best models released by BMA [23]. For confirmations, fit linear regression with autoregressive errors having zero mean and variance one [24]. The best two models were the white noise models obtained from BMA.

Tables 3A and 3B indicated that independent samples test were conducted to test whether data set of the two white noise models have equal variances or not. The test in Tables 3A and 3B for both U.S. and A.U. data revealed that the variability in the distribution of the two data sets was no significantly different values which were greater than the p-value 0.05. Thus, the two models had equal variances in both countries. Table 3B results for A.U. data revealed more reasonable equal variances [25, 26, 27].

Table 3A : Levene's test for equal variances for U.S. data

Independent samples test

Levene's test for equality of variances	t-test for equality of means							95% Confidence interval of the difference	
					Sig.	Mean	Std.Error		
	F	Sig.	t	df	(2-tailed)	difference	difference	Lower	Upper
B Equal variances assumed	1.414	0.235	2.159	438	0.031	0.05909	0.02737	0.0053	0.11288
Equal variances not assumed	2.159	255.236	0.032	0.05909	0.027370.005190.11299				

Table 3B: Levene's test for equal variances for A.U. data

Independent samples test

Levene's test for equality of variances	t-test for equality of means							95% Confidence interval of the difference	
					Sig.	Mean	Std.Error		
	F	Sig.	t	df	(2-tailed)	difference	difference	Lower	Upper
B Equal variances assumed	.045	.833	-2.993	438	.003	-.01409	.00471	02334	-.0048
Equal variances not assumed			-2.993	424.759	003	-.01409	.00471 .	-.02335	-.0048

Table 4 revealed for both U.S. and A.U. data estimation that CWN emerged as suitable model for estimation and forecasting in comparison with EGARCH models. In A.U. the CWN and EGARCH had minimum forecast errors values, except that the mean absolute percentage error (MAPE) was higher in EGARCH when comparing with U.S. forecast errors. Forecasting was better using A.U. GDP data compare to employing U.S. GDP data.

Table 4: The summary of GARCH and CWN models estimation and forecasting evaluation for U.S. and A.U. data set

	CWN	GARCH	CWN	GARCH
	U.S. data		A.U. data	
Estimation residual diagnostic				
Stability Test (Lag structure)	Stable	Stable	Stable	Stable
Correlogram (square) residual	covariance stationary	Stationary	covariance stationary	Stationary
Portmanteau Tests	No autocorrelation	No autocorrelation	No autocorrelation	No autocorrelation
Histogram-Normality Tests	Not normal	Not Normal	Not normal	Appear normal
ARCH Test	No ARCH effect	No ARCH effect	No ARCH effect	No ARCH effect
Dynamic forecast evaluation				
RMSE	0.482821	627.8018	0.0333325	0.489917
MAE	0.113995	439.1633	0.007404	0.366493
MAPE	1.387052	2.98032	1.233974	107.6098
Residual diagnostic				
Correlogram (square) residual	Stationary	Stationary	Stationary	Stationary
Histogram-Normality Tests	Not normal	Not normal	Not normal	Appear normal
Serial Correlation LM Tests	No serial correlation	No serial correlation	No serial correlation	No serial correlation
Heteroscedasticity Test	No ARCH effect	No ARCH effect	No ARCH effect	No ARCH effect
Stability diagnostic				
Ramsey reset tests	Stable	Stable	Stable	Stable
Determinant residual covariance	0.001923		5.75E-06	

Conclusion

GARCH family weaknesses were overcome by the CWN which also proved to be more efficient. CWN estimation passed stability condition, stationary, serial correlation, the ARCH effect tests and it also passed the Levene's test of equal variances using U.S. data. CWN estimation passed stability condition, stationary, serial correlation, the ARCH effect tests and passed the Levene's test of equal variances using A.U. data with a promise.

The CWN estimation produced better results with minimum information criteria and high log likelihood values in both U.S. and A.U. data estimation. CWN had the minimum forecast errors which were better results when compare with the GARCH model dynamic evaluation forecast errors in both U.S. and A.U. data [28, 29, 30]. The determinant of the residual of covariance matrix values revealed that CWN was efficient in the two countries, but A.U. was more efficient.

Based on every result in the empirical analysis of the two countries, CWN was the more appropriate model. For this reason, CWN is recommended for modeling the data that exhibits conditional heteroscedasticity and leverage effect.

The contribution of this study to the scientific community is that the CWN uplifts the weaknesses of the existing models and improve the forecast accuracy. CWN forecast output is more reasonable for effective policy making. This will boost the economy nations.

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