Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 1 (2016), pp. 309-312
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The Middle Edge Domination of Jump Graph

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Abstract

The middle edge domination of jump graph M_{ed} (J(G)) of a graph J(G)=(V, E) is graph with the vertex set E∪S where S is the set of all minimal edge dominating set G and with two vertices u, v∈ E ∪ S adjacent if u∈ E and v=F is a minimal edge dominating set of G containing u or u, v are not disjoint minimal edge dominating sets in S. In this paper, a characterization of J(G) for which M_{ed} (J(G)) is discussed.

Keywords: Graph, Jump Graph, Edge dominating Graph, Middle Edge dominating Graph.

Ams Mathematics subject Classification: 05C69, 05C78.

Introduction

All graphs considered here are finite, undirected without loops, isolated vertices or multiple edges. Any undefined term in this paper may found in Kulli [1]. Let G=(V, E) be a graph with $|V|$=p≥2 and $|E| = q$. A set D $\subseteq$ V(J(G)) is dominating set of jump graph, if every vertex not in D is adjacent to a vertex in D. The domination number of the jump graph is the minimum cardinality of dominating set of jump graph J(G). A set F$\subseteq$E of edges in E-F is adjacent to at least one edge in F. An edge dominating set F of G is a minimal edge dominating set if for every e in F, F-e is not an edge dominating set of G. The edge domination number γ'(G) of G is the minimum cardinality of an edge dominating set of G.

Let A be a finite set. Let $F=\{A_1, A_2,..., A_n\}$ be a partition of A. Then the intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F and in which two vertices A_i and $A_j$are adjacent if and only if $A_i \cap A_j=\Phi$. The minimal edge dominating graph $MD_e(G)$ of a graph G is the intersection graph defined on the family of all minimal edge dominating sets of G. This concept was introduced in [6]. Some other dominating graphs are studied, for example, in [2] [3] [4] [5].
The edge dominating graph D_e (G) of a graph G is the graph with the vertex set $E \cup S$ where S is the set of all minimal edge dominating sets of G and with two vertices u, v in $E \cup S$ adjacent if $u \epsilon E$ and v=F is a minimal edge dominating set of G containing u. This concept was introduced by kulli [2].
The middle edge dominating graph $M_{ed}(G)$ of a graph G=(V, E) is the graph with the vertex set $E \cup S$ where S is the set of all minimal edge dominating sets of G and with two vertices u, v ϵ $E \cup S$ adjacent if $u \epsilon E$ and V=F is a minimal edge dominating set of G containing u or u, v are not disjoint minimal edge dominating sets in G.
In Figure 1, a graph G and its middle edge dominating graph $M_{ed}(G)$ are shown

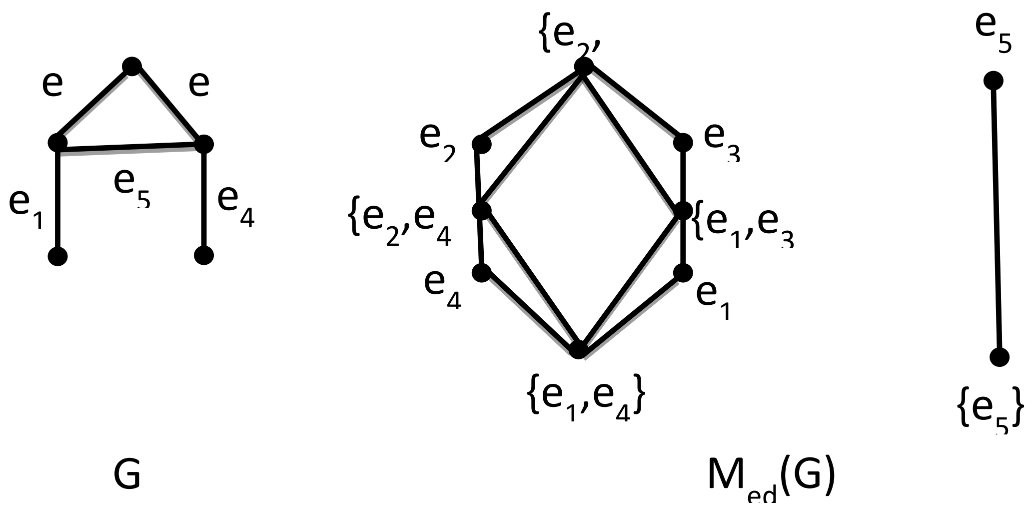

Figure-1

We note that the middle edge dominating graph $M_{ed}(G)$ is defined only if G has not isolated vertices. As usual P_n, C_n and K_n are respectively, the path, cycle and complete graph of order n, $K_{r, s}$ is the complete bipartite graph with two partite sets containing r and s vertices Cl_n is the circular ladder graph is visualized as two concentric n-cycles in which each of the n pairs of corresponding vertices is joined by an edgefound in [8]. Any undefined term or notation in this paper can be found in [7].

Definition1. 1: The line graph L(G) of G has the edges of G as its vertices which are adjacent in L(G) if and only if the corresponding edges are adjacent in G. We call the complement of line graph L(G) as the jump graph J(G) of G, found in [9]. The jump graph J(G) of a graph G is the graph defined on E(G) and in which two vertices are adjacent if and only if they are not adjacent in G. Since both L(G) and J(G) are defined on the edge set of a graph G.

Remark1. 1: The isolated vertices of G (if G has) play no role in line graph and jump graph transformation. Here we assume that the graph G under consideration is non-empty and has no isolated vertices found in [9].

Definition1. 2: A set $D \subseteq V(J(G))$ is said to be dominating set of J(G), if every vertex not in D is adjacent to a vertex in D. The domination number of jump graph denoted by $\gamma(J(G))$, is the minimum cardinality of a dominating set in J(G). For any graph G, with $p \leq 4$, the jump graph J(G) of G, is disconnected. Since we study only the connected jumpgraph, we choose $p>4$ [10].

Results

Theorem2. 1: M_{ed} $(J(G))=PK_{1,2}$ if and only if $J(G)=C_p$ with $p \geq 5$

Proof: Suppose M_{ed} $(J(G))=PK_{1,2}$, $p \geq 5$. Assume $J(G) \neq C_p$. Then there exists at least one minimal edge dominating set S Containing two or more edge of G. By definition, S will form a subgraph P_3 in M_{ed} (J(G)). Which is a contradiction.
Conversely suppose $J(G)=C_p$. Then each edge e_i of J(G) forms a minimal edge dominating set $\{e_i\}$. Thus e and $\{e_i\}$ are adjacent vertices in M_{ed} (J(G)). Since each minimal edge dominating set $\{e_i\}$ contains only one edge, no two vertices of G are adjacent in M_{ed} (J(G)) and no two corresponding vertices of minimal edge dominating sets are adjacent in M_{ed} (J(G)). Thus M_{ed} $(J(G))=PK_{1,2}$

Theorem 2. 2: M_{ed} $(J(G))=(p-2)K_{1,2}$ if and only if $J(G)=P_p$with $p \geq 5$

Proof: Suppose M_{ed} $(J(G))=(p-2)K_{1,2}$, $p \geq 5$. Assume $J(G) \neq P_p$. Then there exists at least one minimal edge dominating set S containing two or more edges of G. By definition, S will form a subgraph in M_{ed} (J(G)). Which is a contradiction
Conversely suppose $J(G)=P_p$. Then each edge e_i of J(G) forms a minimal edge dominating set $\{e_i\}$. Thus e and $\{e_i\}$ are adjacent vertices in M_{ed} (J(G)). Since each minimal edge dominating set $\{e_i\}$ contains only one edge, no two vertices of G are adjacent in M_{ed} (J(G)) and no twocorresponding vertices of minimal edge dominating sets are adjacent in M_{ed} (J(G)). Thus M_{ed} $(J(G))=(p-2)$ $K_{1,2}$

Theorem2. 3: M_{ed} $(J(G))=PK_{1,2}$ if and only if $J(G)=K_{2,p}$ with $p>2$

Proof: Suppose M_{ed} $(J(G))=PK_{1,2}$, $p>2$ Assume $J(G) \neq K_{2,p}$. Then there exists at least one minimal edge dominating set S containing two or more edges of G. By definition, S will form a subgraph in M_{ed} (J(G)). Which is a contradiction
Conversely suppose $J(G)=K_{2,p}$. Then each edge e_i of J(G) forms a minimal edge dominating set $\{e_i\}$. Thus e and $\{e_i\}$ are adjacent vertices in M_{ed} (J(G)). Since each minimal edge dominating set $\{e_i\}$ contains only one edge, no two vertices of G are

adjacent in M_{ed} (J(G)) and no two corresponding vertices of minimal edge dominating sets are adjacent in M_{ed} (J(G)). Thus M_{ed} (J(G))=P $K_{1, 2}$

Corollary 2. 1: M_{ed} (J(G))≥3$K_{1, 3}$ if and only if J(G)=K_p with p≥5

Theorem2. 4: M_{ed} (J(G))=3(n+3)$K_{1, 3}$ if and only if J(G)=$Cl_p$with p≥4

Proof: Suppose M_{ed} (J(G))=3(n+3)$K_{1, 3}$, p≥4Assume J(G)≠Cl_pThen there exists at least one minimal edge dominating set S containing two or more edges of G. By definition, S will form a subgraph in M_{ed} (J(G)). Which is a contradiction
Conversely suppose J(G)=Cl_pThen each edge e_i of J(G) forms a minimal edge dominating set {e_i}. Thus e and {e_i} are adjacent vertices in M_{ed} (J(G)). Since each minimal edge dominating set {e_i} contains only one edge, no two vertices of G are adjacent in M_{ed} (J(G)) and no two corresponding vertices of minimal edge dominating sets are adjacent in M_{ed} (J(G)). Thus M_{ed} (J(G))=3(n+3)$K_{1, 3}$

Conclusion

Thus we conclude the middle edge domination number of jump graph for K_p, C_p, P_p, Cl_p and $K_{m, n}$

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