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On Some Labelings of Barbell Graph

P. Agasthi
Department of Mathematics Tagore Engineering College
Rathinamangalam, Vandalur, Chennai-600 048, Tamil Nadu, India.

N. Parvathi
Department of Mathematics, SRM University,
Kattankulathur--603203, Kanchipuram, Tamil Nadu, India.

K. Thirusangu
Department of Mathematics, S.I.V.E.T College,
Gowrivakkam, Chennai-600073, Tamil Nadu, India.

Abstract

The graphs considered here are finite, undirected and simple. A Barbell graph B (p, n) is the graph obtained by connecting n-copies of a complete graph K_p by a bridge. In this paper the ways to construct Square Sum, Square Difference, Strongly Multiplicative, Even Mean and Odd Mean labelings for Barbell graphs are reported.

Keywords: Barbell graph, Square Sum labeling, Square Difference labeling, Strongly Multiplicative labeling, Even Mean labeling and Odd Mean labeling.

AMS Classification: 05C78.

1. Introduction

The concept of graph labeling was introduced by A. Rosa in 1967. Let G (V, E) be a (p, q) graph. The Square Sum and Square Difference labelings were introduced by Ajitha, Arumugam and Germina [2009]. They proved that K_p ($p\leq5$) have Square-Sum and Square Difference labeling. The concept of Strongly Multiplicative graphs was introduced by Beineke and Hegde [2001]. They proved that K_p ($p\leq5$) is strongly Multiplicative graph. The concept of Mean labeling was introduced by Somasundaram and Ponraj [2003]. They studied the Mean labeling of Cycle, Path and some of their related graph like $C_m \bigcup P_n$, $P_m \times P_n$, $P_m \times C_n$ etc. In this paper we prove that the existence of Square sum, Square Difference, Strongly Multiplicative, Odd and Even Mean labelings of Barbell graph B (p, n) for p=5 and n≥2.

Definition 1.1:
Let G be a (p, q) graph. A one-one map f:V (G)→{0, …, p-1} is said to be a Square Sum labeling if the induced map $f^*(uv) = (f(u))^2 + (f(v))^2$ is injective. It is said to be a Square Difference labeling if the induced map $f^*(uv) = (f(u))^2 - (f(v))^2$ is injective.

Definition 1.2:
A (p, q) graph is said to be Strongly Multiplicative if there exist a one-one map f:V (G)→{1, …, p} such that the induced map f* (uv)=f (u)f (v) are distinct.

Definition 1.3:
A (p, q) graph is called Even Mean graph, if there exist a one-one map f:V (G)→{2, 4, 6, 8, …, 2q} such that the induced map f* (uv)= ((f (u)+f (v))/2) are distinct.

Definition 1.4:
A (p, q) graph is called Odd Mean graph, if there exist a one-one map f:V (G)→ {1, 3, 5, …, (2q-1)} such that the induced map f* (uv)= ((f (u)+f (v))/2) are distinct.

2. Main Results

Theorem 2.1:
The Barbell graph B (p, n) is a Square Sum graph for all 3≤p≤5 and for all n≥2.

Proof:
Let V be the vertex set and E be the edge set of B (p, n).
Denote the vertex set $V=\{v_1, v_2, \ldots, v_{np}\}$ and edge set as $E=\{e_1, e_2, \ldots, e_{(p^2-p+22n-22)/2}\}$.
Define a map f: V→{0, 1, 2, …, (np-1)} such that $f(v_i) = i-1$, $1 \le i \le np$.
Define the induced function on edges as $f^*: E \to N$ such that
$f^*(v_iv_j) = f(v_i)^2 + f(v_j)^2$
$= (i-1)^2 + (j-1)^2$
$f^*(v_iv_j) = (i-1)^2 + (j-1)^2$.
To prove f^* is injective we have to prove $f^*(v_iv_j) \neq f^*(v_{i+1}v_{j+1})$, when $i \neq j$.
Now $f^*(v_{i+1}v_{j+1}) = i^2 + j^2$
Therefore $(i-1)^2 + (j-1)^2 \neq i^2 + j^2$, when $i \neq j$.
Therefore, all edge labels are distinct.
Hence B (p, n) admits Square Sum labeling.

Example 2.1.1:
Square Sum labeling of B (5, 2), B (5, 3), B (4, 2) are given in the following figure1.

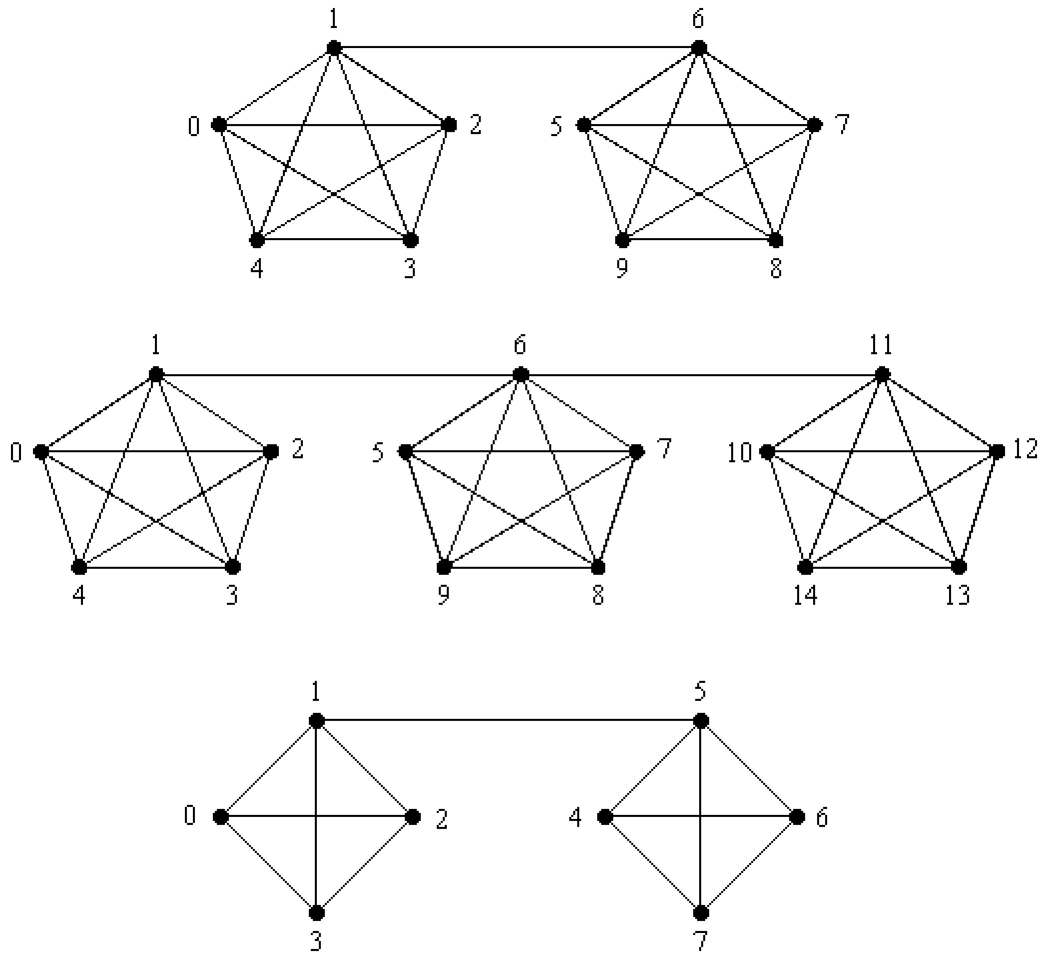

Figure 1: The Square-Sum labeling of B (5, 2), B (5, 3), B (4, 2)

Theorem 2.2:
The Barbell graph B (p, n) is a Square Difference graph for all $3 \leq p \leq 5$ and for all $n \geq 2$.

Proof:
Let V be the vertex set and E be the edge set of B (p, n).
Denote the vertex set $V = \{v_1, v_2, \ldots, v_{np}\}$ and edge set as $E = \{e_1, e_2, \ldots, e_{(p^2-p+22n-22)/2}\}$.
Define a map $f: V \rightarrow \{0, 1, 2, \ldots, (np-1)\}$ such that $f(v_i) = i-1$, $1 \leq i \leq np$.
Define the induced function on edges as $f^*: E \rightarrow Z$ such that
$f^*(v_iv_j) = f(v_i)^2 - f(v_j)^2$
$= (i-1)^2 - (j-1)^2$
$f^*(v_iv_j) = (i-1)^2 - (j-1)^2$
To prove f^* is injective we have to prove $f^*(v_iv_j) \neq f^*(v_{i+1}v_{j+1})$ where $i \neq j$.
Now $f^*(v_{i+1}v_{j+1}) = i^2 - j^2$
Therefore $(i-1)^2 - (j-1)^2 \neq i^2 - j^2$, when $i \neq j$.

Therefore, all edge labels are distinct.
Hence B (p, n) admits Square Difference labeling.

Example 2.2.1:
Square Difference labeling of B (4, 3), B (5, 2), B (5, 4) are shown in following figure 2.

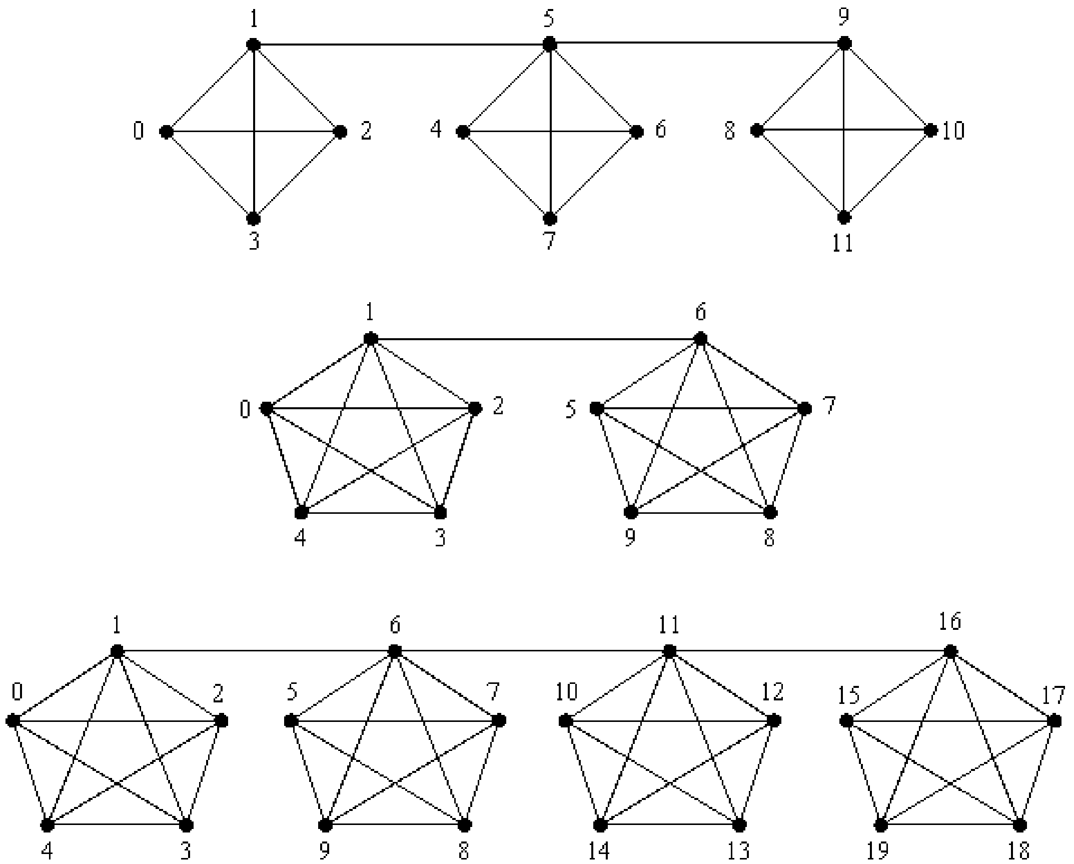

Figure 2: The Square Difference labeling of B (4, 3), B (5, 2) and B (5, 4)

Theorem 2.3:
The Barbell graph B (p, n) is a Strongly Multiplicative graph for all $3 \le p \le 5$ and for all $n \ge 2$.

Proof:
Let G (V, E) be a graph.
Denote the vertex set V= $\{v_1, v_2, \ldots, v_{np}\}$ and edge set as E= $\{e_1, e_2, \ldots, e_{(p^2-p+22n-22)/2}\}$.
Define a map f: $V \rightarrow \{1, 2, \ldots, np\}$ such that $f(v_i) = i$, $1 \le i \le np$.
Define the induced function on edges as $f^*: E \rightarrow N$ such that
$f^*(v_i v_j) = ij$

To prove f^* is injective we have to prove $f^*(v_i v_j) \neq f^*(v_{i+1} v_{j+1})$ when $i \neq j$.
Now $f^*(v_{i+1} v_{j+1}) = (i+1)(j+1)$
Therefore $ij \neq (i+1)(j+1)$ when $i \neq j$.
Therefore all edge labels are distinct.
Hence B (p, n) admits Strongly Multiplicative labeling.

Example 2.3.1:
Strongly Multiplicative labeling of B (5, 2) and B (5, 3) are given in following figure 3.

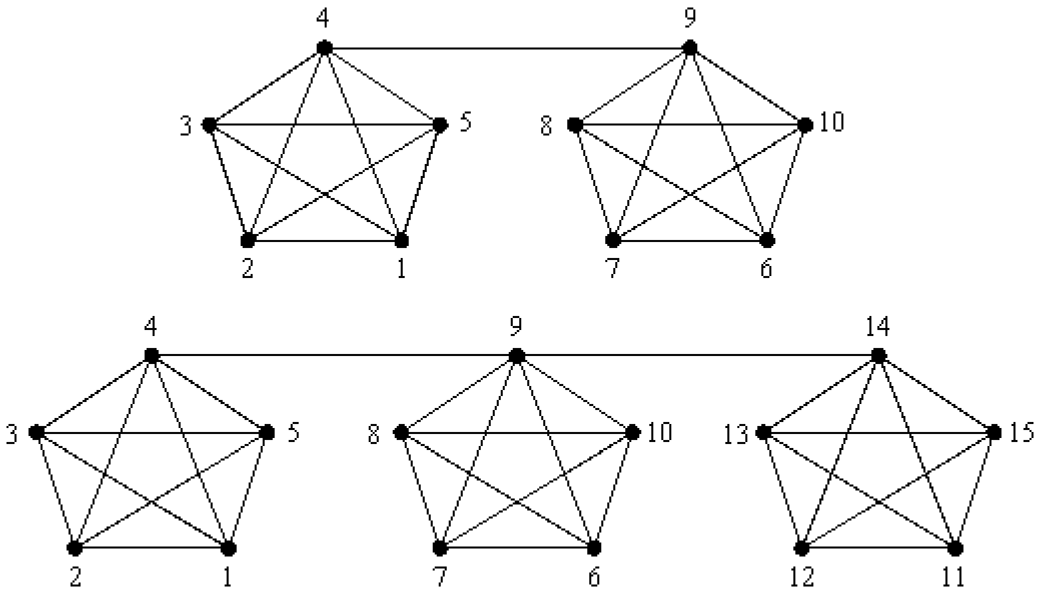

Figure 3: The Strongly Multiplicative labeling of B (5, 2) and B (5, 3)

Theorem 2.4:
The Barbell graph B (p, n) admits Even-Mean labeling for p=5 and for all $n \geq 2$.

Proof:
Let V be the vertex set and E be the edge set of B (p, n)
Denote the vertex set $V = \{v_1^i, v_2^i, \ldots, v_p^i\}$ and edge set as $E = \{e_1, e_2, \ldots, e_{(p^2-p+22n-22)/2}\}$.
Define $f: V \rightarrow \{2, 4, 6, \ldots, 2q\}$ as follows.
Label the vertices $v_1^1, v_2^1, v_3^1, v_4^1, v_5^1$ as follows.
$f(v_1^1) = 2$
$f(v_2^1) = 4$
$f(v_3^1) = 16$
$f(v_4^1) = 6$
$f(v_5^1) = 10$
To label the remaining vertices of Barbell graph namely $v_1^2, v_2^2, v_3^2, v_4^2, v_5^2, \ldots, v_1^n, v_2^n, \ldots, v_5^n$ consider the following cases.

Case 1:
When n ≡ 0 (mod 2)
The vertices are labeled by the formula,
$f(v_j^n) = f(v_j^{n-1})+n+(n-1)PC_2$ for $n \geq 2, 1 \leq j \leq 5$

Case 2:
When n≡ 1 (mod 2)
The vertices are labeled by the formula,
$f(v_j^n) = f(v_j^{n-1})+(n-1)+PC_2(n-1)$ for $n \geq 3, 1 \leq j \leq 5$
Define the induced function on edges as $f^*: E \rightarrow N$ such that

$$f^*(v_i^n v_j^m) = \frac{f(v_i^n)+f(v_j^m)}{2}$$

To prove f^* is injective we have to prove $f^*(v_i^n v_j^m) \neq f^*(v_{i+1}^n v_{j+1}^m)$, when m=n, m=n+1 and i≠j
When m= n;

Now $f^*(v_{i+1}^n v_{j+1}^m) = \frac{f(v_{i+1}^n)+f(v_{j+1}^m)}{2}$

Therefore $\frac{f(v_i^n)+f(v_j^m)}{2} \neq \frac{f(v_{i+1}^n)+f(v_{j+1}^m)}{2}$ when i≠j

When m= n+1;

Now $f^*(v_{i+1}^n v_{j+1}^{n+1}) = \frac{f(v_{i+1}^n)+f(v_{j+1}^{n+1})}{2}$

Therefore $\frac{f(v_i^n)+f(v_j^{n+1})}{2} \neq \frac{f(v_{i+1}^n)+f(v_{j+1}^{n+1})}{2}$ when i≠j

Therefore all edge labels are distinct.
Hence f is Even Mean labeling.

Example 2.4.1:
Even Mean labeling of B (5, 3) is shown in the following figure 4.

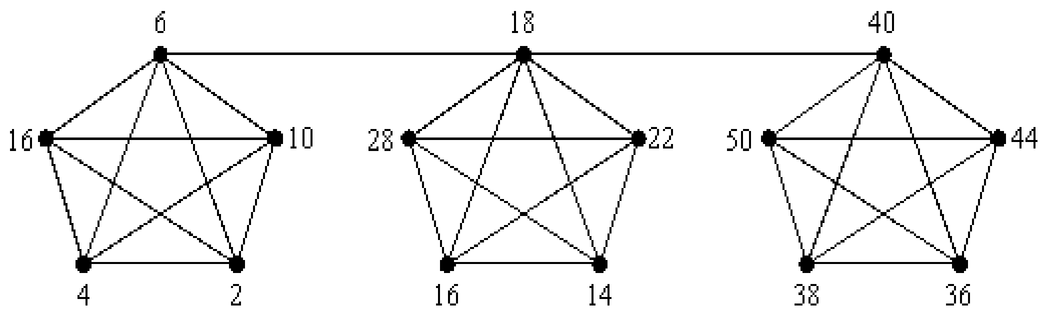

Figure 4: The Even Mean labeling of B (5, 3)

Theorem 2.5:
The Barbell graph B (p, n) admits Odd Mean labeling for p=5 and for all $n \geq 2$.

Proof:
Let V be the vertex set and E be the edge set of B (p, n)
Denote the vertex set V= $\{v_1^i, v_2^i, \ldots, v_p^i\}$ and edge set as E= $\{e_1, e_2, \ldots, e_{(p^2-p+22n-22)/2}\}$.
Define f: V→ {1, 3, 5, …, (2q-1)} as follows.
Label the vertices v_1^1, v_2^1, v_3^1, v_4^1, v_5^1 as follows.
$f(v_1^1) = 1$
$f(v_2^1) = 3$
$f(v_3^1) = 15$
$f(v_4^1) = 5$
$f(v_5^1) = 9$
To label the remaining vertices of Barbell graph namely v_1^2, v_2^2, v_3^2, v_4^2, v_5^2, …, v_1^n, v_2^n, …, v_5^n consider the following cases.

Case 1:
When $n \equiv 0 \pmod 2$
The vertices are labeled by the formula,
$f(v_j^n) = f(v_j^{n-1}) + n + (n-1)PC_2$ for $n \geq 2$, $1 \leq j \leq 5$

Case 2:
When $n \equiv 1 \pmod 2$
The vertices are labeled by the formula,
$f(v_j^n) = f(v_j^{n-1}) + (n-1) + PC_2(n-1)$ for $n \geq 3$, $1 \leq j \leq 5$
Define the induced function on edges as $f^*: E \rightarrow N$ such that

$$f^*(v_i^n v_j^m) = \frac{f(v_i^n) + f(v_j^m)}{2}$$

To prove f^* is injective we have to prove $f^*(v_i^n v_j^m) \neq f^*(v_{i+1}^n v_{j+1}^m)$, when m=n, m=n+1 and $i \neq j$
When m= n;

$$\text{Now } f^*(v_{i+1}^n v_{j+1}^m) = \frac{f(v_{i+1}^n) + f(v_{j+1}^m)}{2}$$

$$\text{Therefore } \frac{f(v_i^n) + f(v_j^m)}{2} \neq \frac{f(v_{i+1}^n) + f(v_{j+1}^m)}{2} \text{ when } i \neq j$$

When m= n+1;

$$\text{Now } f^*(v_{i+1}^n v_{j+1}^{n+1}) = \frac{f(v_{i+1}^n) + f(v_{j+1}^{n+1})}{2}$$

$$\text{Therefore } \frac{f(v_i^n) + f(v_j^{n+1})}{2} \neq \frac{f(v_{i+1}^n) + f(v_{j+1}^{n+1})}{2} \text{ when } i \neq j$$

Therefore all edge labels are distinct.
Hence f is Odd Mean labeling.

Example 2.5.1:
Odd Mean labeling of B (5, 3) is shown in the following figure 5.

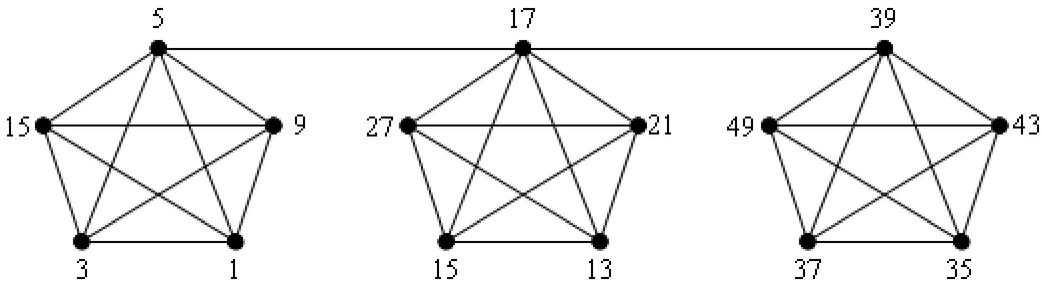

Figure 5: The Odd Mean labeling of B (5, 3)

Conclusion

In this paper we have examined the existence of Square sum, Square difference, Strongly Multiplicative and Even Mean, Odd Mean labelings for Barbell graph for all n≥ 2. Further investigation can be done to obtain the above labeling for some class of graphs.

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