A mathematical model of a diesel engine for simulation modelling of the control system

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Abstract

A method for obtaining a mathematical model of an engine is proposed. The obtained model’s intended use is in the hardware-in-the-loop (HiL) real-time simulation modelling of the engine control system. The dynamic characteristics of the turbocharged diesel engine are described by differential equations of dynamic balance of the flows of mechanical energy and the gas mass in the main parts of the engine. In order to achieve high speed of calculations, the parameters requiring a long time for their calculation and the characteristics that represent the difference between the real and theoretical engine working cycles are predefined as some relatively simple functional dependencies. An example is considered of constructing a mathematical model for the locomotive diesel engine. The obtained results of HiL-simulation of the engine control system with the help of the developed mathematical model confirm the applicability of the proposed method. The presented results were obtained during research on the development of control systems and adaptation of sensors and actuators of fuel equipment with perspective technical parameters with the financial support of the Ministry of Education and Science of the Russian Federation in the form of subsidies from the federal budget (code of the lot 2015-14-579-0054).

Keywords: turbocharged diesel engine, mathematical model, functional dependencies, hardware-in-loop simulation.
Introduction

In connection with the need to accelerate the process of designing and adjusting the engines and their control systems, there arose a problem of developing HiL simulation methods and introducing them into practice. HiL simulation allows producing and debugging microelectronic devices of engine control system for the real engine working modes in parallel with the engine creation. Also, HiL simulation saves money spent on engine tests. In HiL simulation, the real hardware microelectronic devices are connected to digital implementation of the mathematical model of the studied engine which allows adjusting the software and control algorithms of the future power plant control system.

To implement HiL simulation, it is necessary to develop a set of software and hardware means combined in a single booth. The hardware part of the booth mainly comprises an electronic control unit (ECU) with a microprocessor at its core. If needed, the hardware part can be supplemented by actuators, sensors and other devices of the control system. The information exchange between controller and digital implementation of the mathematical model is performed by a special interface device which imitates the engine and some other analyzed parts of the power plant. In the interface device, the digital codes from the model are transformed into analog signals corresponding to real signals from physical sensors. These signals are transmitted to the control unit’s inputs. On the other hand, the interface device also transforms the electric signals from the control unit into digital codes corresponding to actuators’ positions and other parameters which affect the model. In the HiL simulation booth, the ECU interacts with the digital model as if it were the real engine.

A digital computer model for HiL simulation must meet some specific requirements. The algorithms of control systems, characteristics and engine operating modes are determined by the type of power plant in which the studied engine is used as a power source. That is why a digital implementation of the mathematical model for HiL simulation must cover not only the engine but also the main parts of the power plant. For example, a computer model of the vehicle power plant should contain equations which describe transmission and vehicle as a whole. The mathematical model should include all kinds of the engine operating modes, both static and dynamic.

The main requirement imposed on such models is the real-time character of the information exchange between the model and the control unit, appropriate for the cycle run time of ECU software control algorithms. A single tact of ECU microcontroller software cycle commonly falls in a range from few to fractions of milliseconds. Between two consecutive acts of information interchange, all variations must be performed, the values of the parameters of the power plant and engine working process in the model must be actualized and the signals from sensors and to actuators must be generated.

Thereby, a digital implementation of a mathematical model for HiL simulation must satisfy contradictory requirements: on one hand, it should imitate real dynamic processes with a sufficient accuracy, on the other hand, it should perform quick calculations of this processes enough to exchange the data between the model and ECU in real time. The experience of creating detailed engine models shows that the
calculations of operating mode parameters in such models are significantly slower than the real time scale [1,2,3,4,5]. In this connection, a problem has appeared of creating “fast” dynamic computer models for performing the HiL simulation. These models should imitate the engine and the main parts of the power plant with reliable calculations of working process parameters at high-speed timing (about fractions of milliseconds).

**Method of constructing a mathematical model of an engine**

In this paper, a method is proposed to solve the formulated problem as applied to a diesel engine.

This goal is reached by combining the relations from the theory of turbocharged engine working process and the empirical data in the constructed model.

The problem is to find such compromise mixture of theoretical and empirical parts that will perform the calculations of dynamic processes with sufficient accuracy and speed.

In this approach, a number of physical processes in the engine are replaced by mathematical dependencies between the working process parameters obtained from the analysis of preliminary experimental or numerical studies.

The argument in favor of this approach is that in the detailed specification of the engine working process one should use the empirical data, but in the form of coefficients characterizing the difference between real and theoretical values in the fuel combustion processes and heat transfer equations. One should also use the values of internal energy loses in the engine.

It should be noted that the “half-empirical” approaches are successfully used in such research areas as the theory of regulation, statistics and neural network studies.

In the functional diagram (Fig. 1), a turbocharged diesel engine is represented as a combination of the following elements:

![Functional diagram of a turbocharged diesel engine](image)

**Figure 1:** Functional diagram of a turbocharged diesel engine

the engine itself – cylinders (CY), fuel equipment (FE), turbocharger (TC), the intake manifold (IM) and the exhaust manifold (EM). In the diagram, the arrows represent external effects and the interaction of the working process parameters.

Interaction between the parts of a diesel engine is realized mainly through the following parameters of the working process:

- angular velocity of the diesel engine’s shaft \( \omega_d \);
- angular velocity of the turbocharger rotor \( \omega_t \);
- fuel cyclic quantity \( g_c \);
- air pressure in the intake manifold $p_c$;
- air pressure in the exhaust manifold $p_g$.

Several parameters are considered as describing the external effects:
- the position of a pedal or lever which affects the fuel injection quantity $h$;
- the environment temperature $T_0$;
- the load torque $M_l$.

The dynamic characteristics of a turbocharged diesel engine are described by the equations of dynamic balance of the flows of mechanical energy and gas mass: some ordinary differential equations for its main elements. The left-hand side of each equation contains the time derivatives of one of the main output parameters (Fig. 2): angular speed of the engine shaft $\omega_d$, angular speed of the turbocharger rotor $\omega_t$, air pressure in the intake manifold $p_c$ and in the exhaust manifold $p_g$. The right-hand sides of equations contain torques and the gas flow rates.

**Structure of a Diesel Engine Model**

The structure of the proposed “fast” dynamic model of a diesel engine is provided on Fig. 2.

The parameters of the engine working process that are in the right-hand side of differential equations can be divided into three groups.

The first group contains the initial data: $\omega_{d0}, \omega_{t0}, p_c0, p_{g0}, T_0, h_0$.

The second group combines the parameters which are predefined as functional dependencies on the other parameters in order to achieve high speed of calculations.

This group contains coefficients which require a long time for their calculation and which characterize the difference between the real and theoretical values of the working process parameters: indicated efficiency factor of the engine $\eta_i$, charge ratio $\eta_v$, adiabatic efficiency of the compressor $\eta_{cad}$, efficiency factor of the turbine $\eta_t$, fuel cyclic quantity $g_c$, temperature of boost air $T_b$, temperature of exhaust gases in the exhaust manifold $T_g$, torque of internal losses $M_{loss}$, load torque $M_{load}$, gas flow through the compressor $G_c$, gas flow through the turbine $G_t$, gas expansion ratio in the turbine $\pi_t$.

The third group includes the parameters, whose values are evaluated by the working process formulas: boost air density $\rho$, excess air ratio $\alpha$, air flow rate through the engine $G_d$, fuel flow rate $G_f$, indicated torque of the engine $M_i$, compressor torque $M_c$, the turbine torque $M_t$.

**Figure 2:** Structure of a dynamic mathematical model of a turbocharged diesel engine.
A mathematical model of a diesel engine for simulation modelling

The right-hand sides of the differential equations and other necessary parameters of dynamical model are calculated from the initial data and the predefined functional dependencies. At each time step of numerical solution of differential equations, the current values of \( \omega_d, \omega_c, p_e, p_g \) are renewed for the further use in the right-hand sides of equations instead of the corresponding initial data values. 

A distinctive feature of our model is the use of the functional dependencies between the working process parameters (the second group of parameters in Fig. 2) which reduce the calculation time of the dynamic processes, thus providing a possibility of application of the model in the real-time HiL simulation.

To obtain the mentioned dependencies, an analysis of the turbocharged diesel engine working process was performed which resulted in establishing the relations between the defined parameters and their arguments.

The analysis was mainly based on the literature sources [6,7,8,9,10], which describe the theory of turbocharged diesel engine and provide the characteristics of certain engines.

Equations for the Elements of a Turbocharged Diesel Engine

The mathematical equations included into the considered “fast” dynamic model of turbocharged diesel engine are listed below.

A. Cylinders and fuel equipment

The dynamic balance of the mechanical energy of diesel and the energy consumer (the locomotive tractive motor) is described by the differential equation for the rotation of shaft, which characterizes the variation over time \( t \) of the angular velocity \( \omega_d \) of the diesel generator:

\[
\frac{d\omega_d}{dt} = \frac{1}{I_d} (M_i - M_{\text{loss}} - M_{\text{load}}),
\]

where \( I_d \) is the diesel generator’s moment of inertia, \( M_i \) is the indicated torque of the diesel, \( M_{\text{loss}} \) is the internal losses torque and \( M_{\text{load}} \) is the load torque.

The main indicator of the fuel combustion effectiveness in diesel engine cylinders is the indicated effectiveness factor \( \eta_i \).

The indicated effectiveness is affected by the excess air ratio \( \alpha \), gas expansion ratio of combustion \( \lambda \), degree of fuel distribution irregularity in the combustion chamber, the speed mode \( \omega_d \) and the density of air entering the cylinders \( \rho \).

Direct influence of density on the indicated effectiveness factor is insignificant and acts mainly through the changes in \( \alpha \) and \( \lambda \).

The expansion ratio \( \lambda \) of gases in the combustion process is one of major indicator of the working cycle dynamics. It is directly affected by the portion of heat released in the first two periods of combustion, and this portion is determined by the portion of fuel injected into the combustion chamber during the period of ignition lag.

The influence of the fuel distribution irregularity on \( \eta_i \) with a chosen method of mixing is realized through the quantities \( \omega_d \) and \( \alpha \).

The conducted analysis of the dependence of the indicated effectiveness on other
parameters of the working process shows that the biggest impact on $\eta_i$ is made by air excess ratio $\alpha$ and angular velocity of the engine shaft $\omega_d$. Therefore, in the calculations, the indicated effectiveness factor was set as a function $\eta_i(\alpha, \omega_d)$.

The charge ratio $\eta_v$ expresses the difference between the amount of air entering the engine and the amount of air which can fill cylinders with given $p_c$ and $T_c$ before intake valves. The value of $\eta_{nu}$ is significantly influenced by losses in the intake and exhaust manifolds and by the heat transfer from the warmed cylinder surface to air charge. With the increase of angular speed of engine shaft, the speed of intake air charge and exhaust gases also rise. This leads to the growth of losses and reduction of $\eta_v$. The increase of boost pressure $p_c$ results in the decrease of relative pressure losses at intake so the charge ratio grows. The influence of charge heating for different operating modes can be characterized by $\omega_d$ and $p_c$. Thus, the charge ratio can be specified as a function $\eta_v(\omega_d, p_c)$.

The model includes an assumption of small impact of the fuel equipment dynamic properties on the fuel injection process. Therefore, the cyclic fuel quantity can be specified as an algebraic function of position or opening time of fuel dosing lever $h$ and angular velocity of engine shaft $\omega_d$: $g_c(h, \omega_d)$.

The value of exhaust gases temperature $T_g$ depends on many factors, primarily, on the indicated effectiveness factor $\eta_i$, air excess ratio $\alpha$, parameters of air entering the engine $p_c$ and $T_a$, speed mode $\omega_d$ and the coefficient of total heat losses per cycle. An increase of air excess ratio leads to a decrease of the average gas temperature in the expansion and exhaust processes and an increase of the indicated effectiveness factor. A decrease of gas temperature leads to a decrease of heat transfer in the expansion process. In its turn, an increase of indicated effectiveness factor leads to a decrease of temperature of the gases leaving cylinders and, as a consequence, to a decrease of heat transfer in the exhaust process. The total impact of all factors leads to a noticeable change of total losses per cycle.

As the engine shaft angular velocity grows, the exhaust gases temperature also grows because of the change of heat release process in cylinder and reduced heat transfer in the exhaust process. An increase of the boost air temperature which depends on combination of $p_c$ and $\alpha$ results in the rise of the amount of heat contributed by air into the engine cylinders. As a result, the gas temperature and total losses per cycle are rising. Taking into account possible reduction of the number of working process parameters influencing the exhaust gas temperature, we used a dependence in the form $T_g(\omega_d, \alpha, p_k)$.

The internal losses of mechanical energy in the engine consist of the friction losses and the energy consumed on combustion gases expelling, new air charge filling, on fuel, grease and cooling equipment driving [11]. The friction losses get higher with the growth of the speed operating mode. Also friction rises when air boost increases because of the increased pressure on moving parts. The energy consumption for the organization of the gas exchange processes depends on the ratio of the input $p_c$ and output $p_g$ pressures. An analysis of the engine characteristics shows that a connection exists between the values of $p_c$ and $p_g$. Therefore, it is preferable to use a function $M_{loss}(n_d, p_c)$ for the losses torque.
Other parameters needed for computation of the right-hand sides of differential equations are determined by the formulas of the theory of the diesel engine working process with the use of the functions given above. Considering the equation of state for the intake air as an ideal gas, we get

\[ \rho = \frac{p_c}{(R_a T_a)}, \]

where \( R_a = 287 \frac{J}{mol \cdot K} \) is the air gas constant.

The air flow rate through the engine

\[ G_d = \rho i V \left( \frac{n_d}{120} \right) \eta_v, \]

where \( i \) is number of cylinders, \( V \) is the working volume of cylinder, \( n_d/120 \) is the number of cycles in 1 second, \( n_d = 30 \omega_d/\pi \).

The fuel flow rate

\[ G_f = i g_c n_d/120. \]

The air excess ratio

\[ \alpha = G_d/(14.3 G_f). \]

The indicated engine torque

\[ M_i = H_u G_f \eta_i/\omega_d, \]

where \( H_u = 42500 \frac{kJ}{kg} \) is the net heating value of the diesel fuel.

**B. Turbocharger**

The dynamic balance of mechanical energy of turbine and compressor is described by the equation of turbocharger rotor rotation

\[ \frac{d\omega_t}{dt} = \frac{1}{I_t} (M_t - M_c), \]

where \( I_t \) is the turbocharger rotor inertia, \( M_t \) is the turbine torque, \( M_c \) is the compressor torque.

The ratio of work spent on compressor driving considering losses of hydraulic flow resistance and heat exchange and adiabatic work of compression process is given by adiabatic effectiveness of compressor \( \eta_{cad} \).

The parameters that define the compressor operation mode are interconnected. For some value of rotor angular velocity \( \omega_t \), the impact of the diesel engine on the compressor is manifested through the value of air pressure in the intake manifold \( p_c \). Other parameters of working process such as \( G_c \) and \( T_c \) are determined by the combination of \( \omega_t \) and \( p_c \). Therefore, in this model the functions \( \eta_{cad}(\omega_t, p_c) \) and \( G_c(\omega_t, p_c) \) are used. The temperature at the compressor output

\[ T_c = T_0 \left[ 1 + \frac{\pi_c^{k-1} - 1}{\eta_{cad}} \right], \]

where \( k = 1.4 \) is the adiabatic index, \( \pi_c = p_c/p_0 \) is the ratio of pressures in compressor, \( p_0 \) is the environment pressure.
Adiabatic work of compressing 1 kg of air in the compressor
\[ L_{cad} = \frac{k}{k-1} R \alpha T_0 \left( \frac{k-1}{\pi_c} - 1 \right). \]

Actual compression work considering hydraulic resistance and heat exchange
\[ L_c = \frac{L_{cad}}{\eta_{cad}} \]

The power used for compressor driving (when referring the mechanical losses to turbine)
\[ N_c = G_c \frac{L_{cad}}{\eta_{cad}} \]

The torque needed for compressor driving
\[ M_c = \frac{N_c}{\omega_t} \]

To take into account the effectiveness of transformation of the available heat drop into mechanical energy, it is necessary to set a turbine effectiveness factor \( \eta_t \) dependency on operating mode. The parameters of turbine operating mode are \( \omega_t, \pi_t, T_g, G_t \), where \( \pi_t = p_g/p_{t0} \) is the ratio of pressure drop between gas entering the turbine \( p_g \) and at the output of turbine \( p_{t0} \). The data analysis for the V16 diesel engine tests shows that the ratio of pressure drop in turbine depends mostly on \( p_g \). So in the model it is set as a function \( \pi_t(p_g) \).

Considering the laws of similarity theory, operating mode of turbine can be set as a combination of pressure drop ratio \( \pi_t \) and relative angular velocity of rotor \( \omega_{tr} = \omega_t/\sqrt{T_g} \). These primary parameters are used as arguments for dependencies \( \eta_t(\omega_{tr}, \pi_t) \) and \( G_{tr}(\omega_{tr}, \pi_t) \), where \( G_{tr} = G_t \sqrt{T_g/p_g} \) is the relative gas flow rate through the turbine.

The obtained characteristics of relative gas flow rate through the turbine of V16 diesel engine shows that this parameter can be defined as \( G_{tr}(\omega_{tr}) \).

Other parameters are determined by the formulas of the turbine working process.

Adiabatic work of 1 kg of gas in the turbine
\[ L_{tad} = \frac{k_g}{k_g - 1} R_g T_g \left( 1 - \pi_t^{1-k_g} \right), \]

where \( k_g = 1.35 \) is the adiabatic index of the exhaust gases and \( R_g = 286 J/(mol \cdot K) \) is the gas constant of exhaust gases.

Effective part of adiabatic work is transformed into mechanical work of gases considering losses of flow moving in turbine and friction in turbocharger \( L_t = L_{tad} \eta_t \).

The power of turbine rotor
\[ N_t = G_t L_t \]

The torque of turbine rotor
\[ M_t = \frac{N_t}{\omega_t} \]
C. Intake and exhaust manifolds of a diesel engine

Variation of air mass in the intake manifold of diesel $dm_a$ during the elementary period of time $dt$

$$dm_a = G_c dt - G_d dt$$

From the equation of state of the air as an ideal gas

$$m_a = \frac{V_{in} p_c}{R_a T_a},$$

where $V_{in}$ is the volume of the intake manifold.

The formula for the air pressure variation in the intake manifold is

$$\frac{dp_c}{dt} = \frac{R_a T_a}{V_{in}} (G_c - G_d).$$

Variation of gas mass in the intake manifold of diesel engine $dm_g$ during the elementary period of time $dt$

$$dm_g = G_d dt + G_f dt - G_t dt$$

Under the applicability assumption of ideal gas state equation to the exhaust gas, we have

$$m_g = \frac{p_{out} V_{out}}{R_g T_g},$$

where $V_{out}$ is the volume of the outlet manifold.

Equation of gas pressure variation in the exhaust manifold

$$\frac{dp_g}{dt} = \frac{R_g T_g}{V_{out}} (G_d + G_f - G_t).$$

Functional Dependencies of the Model

The most difficult part of obtaining a “fast” dynamic model of the turbocharged diesel engine is to determine the dependencies between the diesel working process parameters. Commonly in HiL simulation, the characteristics are given in two ways: as lookup tables and as functions. Initial data with variations and combinations of primary working process parameters corresponding to the field of possible dynamic modes must be used to obtain the characteristics.

Taking into account the low gas inertia of turbocharger, for determination of the efficiency factors $\eta_{cad}, \eta_t$ and flow rates $G_c, G_t$ we can use the empirical universal characteristics of compressor and turbine. The working process of diesel is determined by the combination of engine and turbocharger parameters. At dynamic operation modes, equilibrium of the energy and gas flows common to static modes is disturbed due to mechanical inertia of the cylinder block of engine and turbocharger. Thereby, the default static speed or load characteristics do not cover all possible combinations of engine working process parameters.

It is advisable to carry out special experimental studies with independent variations of primary working process parameters such as fuel quantity and boost air pressure in order to obtain a detailed description of diesel engine dynamics. Implementation of these experimental studies is a very complicated problem, because it demands stand-alone source of boost air instead of default turbocharger [12,13]. Extended static
characteristics of diesel engine can also be obtained by computations with the use of modern software for engine working process simulation [14,15,16,17].

When using lookup tables, the values of parameters in the second group are given at the nodes of a grid, the indices of which are variations of primary parameters. The parameter value at every calculated mode can be obtained by linear interpolation method. A functional form of working process parameters dependency representation can be specified as a polynomial with coefficients determined by the least square method. Selection of the polynomial type should be performed on the basis of the condition of accord between the described dependencies and the diesel working process behavior.

The use of polynomial dependencies between the parameters in “fast” dynamic model allows simulating dynamic modes of transient processes both inside original modes field (interpolation) and outside of it (extrapolation). The experience of numerical studies shows that in some models it is expedient to use a combination of representation of the dependency between the parameters of the engine working process: both as lookup tables and in a functional form.

Example of Development of A Diesel Engine Model

A. Specifics of functional dependencies selection for the model

The proposed method was applied for obtaining computer models of diesel engines and power plants for different types of vehicles: locomotive and heavy-duty trucks. The developed models were used in the studies and the formation of initial settings of control systems.

To illustrate the specifics of the proposed method of mathematical model construction, we present the results of development of “fast” model of a diesel engine for the locomotive power plant. The turbocharged diesel engine of the considered locomotive has 16 cylinders with diameter and stroke of 260 millimeters.

The load characteristics of diesel engine and results of the special studies of transient modes imitation on single cylinder test stand were used as initial data for the development of mathematical model of the cylinder block [13].

The results of experimental studies of transient modes imitation confirm the assumption that the indicated effectiveness factor \( \eta_i \) can be set as a dependence on air excess factor \( \alpha \) and engine shaft angular velocity \( \omega_d \) as it was obtained from the engine working process theory analysis.

The experience shows that the experimental data usually get noised and may contain large random error of measuring. In this case, the use of different interpolation and approximation methods may not give the correct representation of the functional dependency. Therefore, some statistical treatment should be used in the preliminary data processing or in obtaining the approximate expression. The latter lies in the foundation of a widely used technique of approximation, the regression analysis.

The most common method of model parameters estimation is the method of least squares which is reduced to finding the regression function with the surface lying inside the “cloud” of data points that provide minimal sum of the squared deviations. In the current mathematical theory for multidimensional Gaussian points [18], i.e. for normally distributed random variables, this approach is quite well developed. The
application of other statistical techniques (such as non-parametrical ones [19]) is more complicated and may lead to unreliable results. Because of this behavior, the methods oriented on Gaussian points are applied sometimes to the data with unknown distribution.

The first step of solving the regression problem for the considered dependencies between selected variables is to suggest a possible type of function [20]. Linear combinations of exponential functions with real exponent indices were considered. Such choice is due to simplicity of this form and suitability for practical calculations, on one hand, and the possibility of graphical analysis of initial data, on the other hand. Performing of linear regression in general form confirms assumption of dependency type already at beginning step.

Other methods of nonlinear regression in solving the general form of the problem were also considered for a wide range of function types. Use of nonlinear models does not give any significant results. Therefore, the main emphasis in the work was made on improving the quality of linear regression models.

Exponent functions with positive real indices as the basic functions allow describing the available data in a limited but wide interval of values. The problem was solved by insertion of the independent variables with real exponent indices multiplications into linear combination. Besides, the quality of model’s predictions concerning the empirical data has greatly improved. The obtained degree of compliance of the regression model to the initial data in terms of the Chaddok scale can be classified as very high.

**B. Illustration of functional dependencies**

Selection of the type of polynomials was performed according to the criterion of high precision of approximation with the simplest possible structure of polynomials. The graphical polynomial representation as a surface and its cuts, in fact, are a graphical view of the engine processes and dependencies and, therefore, should match the real physics of the working process.

Performing a substantial preliminary work on polynomial type selection resulted in recommendations for polynomial composition. Polynomials should include members with both positive and negative exponent indices with the values primarily in range from −3 to +3. Full composition of such polynomials contains many terms even for two independent variables. It took much time to investigate different compositions of terms before making final choice of the structure of polynomials. The terms that have minor impact on approximation quality were removed from the polynomial structure. The chosen polynomials contain minimum possible set of terms with the high enough approximation quality.

Polynomials should correctly describe dependencies between the working process parameters not only inside the field of initial data but also outside of it. Analysis of surfaces which graphically represent polynomials results in the statement that, in several cases, the increase in the polynomial’s degree leads to an abrupt change of values on the boundary of the initial data field. This does not match the real physics of the processes in the engine. To avoid this, sharp edges of the allowed parameters variation area should be outside the field of possible dynamic operating modes of the
In accordance with the proposed method, the following functional dependencies between the engine working process parameters were obtained for the “fast” dynamic model: indicated effectiveness factor \( \eta_i(\omega_d, \alpha) \), charge ratio \( \eta_c(\omega_d, p_c) \), cyclic fuel quantity \( g_c(\omega_d, h) \), exhaust gases temperature \( T_g(\omega_d, \alpha, p_c) \), torque of internal losses \( M_{\text{loss}}(\omega_d, N) \) (\( N \) is the setting of the consumer), adiabatic effectiveness factor of compressor \( \eta_{\text{cad}}(\omega_t, \pi_c) \), effectiveness factor of the turbine \( \eta_t(\omega_{tr}, \pi_t) \), air flow rate through compressor \( G_c(\omega_t, \pi_t) \), relative flow rate of exhaust gases through turbine \( G_{tr}(\omega_{tr}, \pi_t) \).

As an example, there are presented in Fig. 3 the surfaces for several polynomials which describe functional dependencies between the working process parameters: \( \eta_i(\omega_d, \alpha) \), \( \eta_c(\omega_d, p_c) \), \( \eta_{\text{cad}}(\omega_t, \pi_c) \), \( \eta_t(\omega_t, \pi_t) \), \( G_c(\omega_t, \pi_t) \), \( G_{tr}(\omega_{tr}, \pi_t) \). The surfaces are marked with the initial data points, corresponding to both the steady and unsteady regimes of the diesel engine working. The figure shows good matching of polynomial surfaces and the initial data in the range of possible variations of the engine working process parameters.

**Figure 3:** Surfaces of some polynomial functions

The gained experience allows describing a wide range of dependencies between the working process parameters needed for the creation of dynamic models of other diesel engines and power plants.

**An example of application of the developed model of a diesel engine**

To perform HiL simulation of the transient modes of the locomotive power plant, a special stand was built with a computer part containing a model of V16 diesel engine and power plant with electrical transmission of energy from engine to the locomotive wheelsets [21]. Electrical transmission of the locomotive consists of synchronous traction generator, rectifier and traction DC electric motors [22,23].

A computer program corresponding to the developed mathematical models of the considered diesel engine and locomotive power plant was created using the modern computer simulation software.
A real locomotive electronic control unit with a microprocessor controller which is the core of hardware part of the booth was connected to computer model of power plant through an interface device. To verify the developed mathematical models and to examine control system, HiL simulation of power plant transient process was performed for the typical operating modes of locomotive. The obtained results of HiL simulation was then compared to similar experimental processes from the tests of the real locomotive at the same conditions.

During HiL simulation, the operational control of power plant operating mode and locomotive motion was performed by the means of computer. The transient processes of changing the basic parameters of the diesel engine and power plant working process were displayed on the computer screen in real time. The results of simulation were saved in the computer memory and then printed as transient processes plots.

Changing the modes during HiL simulation was performed by changing the position of the locomotive control lever (operator controller). The operator controller had 15 positions, each of them corresponding to different diesel operating modes in accordance to the locomotive diesel power characteristics. The algorithm of controller work provides support of diesel power at the value which is set by the operator control lever. The value of the set power is maintained by two regulators of control system: regulator of diesel shaft angular velocity and regulator of traction generator driving system.

As an example, the transient process with consecutive shifts of control lever position from 1 to 12 with subsequent switching to 1 position after 10 seconds of work on each shift is presented in the Fig. 4 (experiment) and Fig. 5. (HiL simulation).

![Figure 4: Experiment](image1)

![Figure 5: HiL simulation](image2)
In the experimental and simulated processes, the following parameters characterizing the working of power plant were fixed: diesel generator shaft angular velocity \( n_d \), minute\(^{-1} \); turbocharger rotor angular velocity \( n_t \), minute\(^{-1} \); position of fuel quantity control lever \( h \), mm; limitation of fuel quantity control lever position that is settled by regulator for different locomotive operator controller lever positions \([h]\), mm; excessive pressure of boost air \( p_c \), kPa. In HiL simulation, the current of tractive generator \( I_g \), A, and voltage \( U_g \), V, are added to those parameters.

Analysis of HiL simulation results and comparison of the calculated processes with the analogical experimental processes shows that the simulated processes correctly describe the variation of the primary locomotive power plant parameters with the predicted error of imitation of dynamic processes less than 10%.

The obtained results confirm applicability of the developed booth and verify mathematical models on which it based upon. This fact allows using the booth for HiL simulation of dynamic processes of diesel and locomotive power plant in a wide range of operating modes and control system settings. Fig. 6 shows HiL simulation results of controlling process with abrupt shifting of locomotive operator controller position in sequence of 1-8-15-8-1.

HiL simulation of the transient processes of the locomotive control system yields the results confirming possibility of applying the proposed method in the development of complicated models. These models can be used for imitation of the static and dynamic processes in diesel engines and power plants in real time.

HiL simulation is a very effective means of designing and debugging the engine control systems. It is advisable to use HiL simulation with proper models of industrial facilities to adjust and optimize operating modes and working processes.

**Conclusion**

1. A method of obtaining a mathematical model of a turbocharged diesel engine is proposed for HiL simulation of the engine control systems. In this method, the equations of the theory of engine working process along with the functions based on empirical data are used for calculating the engine parameters in real time.

2. A turbocharged diesel engine is considered as a set of interacting parts: cylinders, turbocharger, the intake and exhaust manifolds. Variations of the output parameters (angular velocities of engine shaft and turbocharger rotor as well as intake air and exhaust gas pressures) are determined by differential
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Equations of dynamic balances of the flows of mechanical energy and gas mass. The variables included in the differential equations are divided into three groups: initial data, variables defined as functional dependencies and variables calculated by the formulas of working process theory.

3. An example is considered of developing a mathematical model of a diesel engine according to the proposed methods.

4. The presented results of HiL simulation of electronic control system of the locomotive diesel power plant have confirmed applicability of the proposed method for the development of mathematical models, which imitate in real time both the static and dynamic modes of the diesel engines and power plants.

References


