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Mathematical Model of Fluid Filtration to Horizontal Well in Tight Heterogeneous Formation

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Abstract

The article proves the computational models of fluid filtration to the wells of a crosscut type in order to establish the degree of influence of deformation processes on the rate of change of fields productivity. On the basis of the finite element model the computational schemes for determination of the well productivity in the poroelastic heterogeneous medium have been obtained, at the same time the software package in the environment of object-oriented programming has been developed. On a basis of the numerical results of conducted calculations the method for study of features of filtrations fields and stressed state in a heterogeneous formation with horizontal wells is developed and justified. Obtained results and developed application package can be implemented for assessment of productivity of oil and gas formations, definition of directions of wells' drilling, definition of stressed state and solution of analogous filtrations problems.

Keywords: Filtration, Modeling, Oil, Well, Deformation, Finite Element Method, Mathematical Model.

Introduction

The development of oil fields with the complicated geological structure and physical-mechanical properties of fluids saturated in productive reservoirs, is currently carried out with the use of active methods of the contour impact, such as block, focal, pattern, step and curved water flooding [1-10]. Using these methods the selection of oil reserves is successfully conducted, but, nevertheless, eventually some areas with hard-to-recover reserves are identified.

Despite progress made in improving the efficiency of the impact, up to the present time the problems on assessment of improving the process of the production of hard-to-recover reserves in these recovery areas by injecting a certain amount of water into the formation through new injection wells created by drilling or shifting highly watered wells into the injection ones, have not been sufficiently solved. At the same time the operations on increasing well productivity are being carried out in existing wells. In addition, the choice of the recovery method and its validation in each separate case are of an important scientific and practical interest.

Timely detection of the causes of complications, manifested in a variety of geological and commercial conditions at the exploitation of multi-layered fields, research on the efficiency of technological methods aimed at improving the involved system of the development of multilayer oil fields, are relevant and problematic issues as any improvement of technological processes is associated with the introduction into the involved system of separate technological variations, which, in turn, do not emerge in full force during fluid filtration to the well, taking into account the deformation of the formation [11-21]. Whereas it is related to the complexity of the geological structure of the productive formation and physical properties of the fluids saturating them.

Statement of the Problem

Let the fluid is taken out of the infinitely long anisotropic array through the horizontal well (HW) located in the middle (Figure 1), while its cross sections curve due to the presence of the inclined plane of isotropy. Therefore, voltage, deformation and pressure depend on two coordinates – y and z – variable in cross-sectional planes. All anisotropic filtration coefficients also depend on two variables. Such fluid motion to the horizontal well in the tight array is called generalized plane filtration.

The stationary filtration equation in the formation with permeability coefficients in the plane of isotropy $k_{x'} = k_{y'}$ and in the direction perpendicular to this plane $k_{z'}$ is as follows:

$$\sum_{i,j=1}^{2} \frac{\partial}{\partial x_i}\left(\frac{k_{ij}(\varepsilon)}{\mu}\frac{\partial p}{\partial x_j}\right) = W_2, \tag{1}$$

$$W_2 = \begin{cases} \dfrac{Q_k(t)}{\pi r_c^2}\left(1 - \dfrac{r_k^2}{r_c}\right), & r_k \le r_c,\ k = \overline{1,m}, \\ 0, & r_k > r_c. \end{cases}$$

In this case, the filtration coefficients take the following values:

$$k_{yy} = \left(k_{y'}(\cos^2\varphi + 1) + k_{z'}\sin^2\varphi\right)\exp(-\alpha\varepsilon_y),$$
$$k_{zz} = \left(k_{y'}\sin^2\varphi + k_{z'}\cos^2\varphi\right)\exp(-\alpha\varepsilon_z),\quad (2)$$
$$k_{yz} = \left(k_{z'}\sin\varphi\cos\varphi - k_{y'}\sin^2\varphi\right)\exp(-\alpha\gamma_{yz}).$$

Then the generalized Hooke's law can be presented as follows:

$$\{\sigma\} = [D]\{\varepsilon\} + [I]p, \quad (3)$$

where

$\{\sigma\} = \{\sigma_y \quad \sigma_z \quad \tau_{yz} \quad \tau_{xz} \quad \tau_{xy}\}^T$; $\{\varepsilon\} = \{\varepsilon_y \quad \varepsilon_z \quad \gamma_{yz} \quad \gamma_{xz} \quad \gamma_{xy}\}^T$;
$[I] = diag\{1 \quad 1 \quad 0 \quad 0 \quad 0\}$.

The formation is entered by the horizontal well of a crosscut type (see Figure 1). On the contour of the well hole the following pressure has been set:

$$p\big|_s = p_s \quad (4)$$

On the external boundaries the following pressure has been set:

$$p\big|_{ABCD} = p_1, \quad (5)$$

$$p\big|_{A'B'C'D'} = p_2, \quad (6)$$

$$\left.\frac{\partial p}{\partial n}\right|_{AA'DD'} = \left.\frac{\partial p}{\partial n}\right|_{BB'CC'} = 0, \quad (7)$$

$$\left.\frac{\partial p}{\partial n}\right|_{AA'DD'} = \left.\frac{\partial p}{\partial n}\right|_{BB'CC'} = 0. \quad (8)$$

As can be seen from Figure 1, the hole of the horizontal well is located across the isotropic plane of the formation. Therefore, HW is of a mining crosscut type. Due to the transversal isotropy of the elastic array properties the cross-sectional area of a crosscut HW curves, but due to the homogeneity of these properties along the extended HW hole it remains unchanged. The equilibrium equation is given in the form of the static elasticity problem. While the generalized Hooke's law equation in the transversely isotropic formation for a crosscut HW, connecting full voltage with the deformation and pressure, has the following form (3). The elements of the symmetric matrix [D] from (3) are presented as follows:

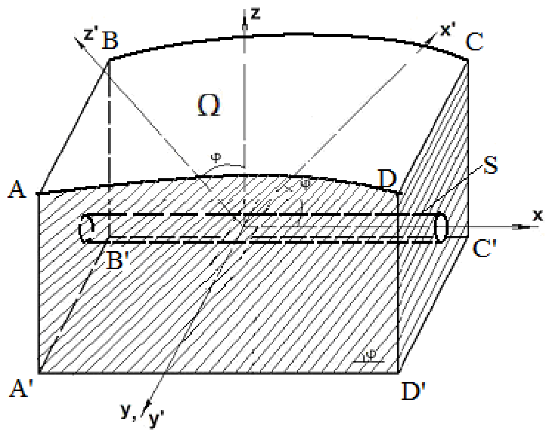

Figure 1: The scheme for the computational domain

$$d_{1,1} = \frac{E_1(E_1 - E_2\nu_2^2)}{(1+\nu_1)(E_1(1-\nu_1)-2E_2\nu_2^2)}, \tag{9}$$

$$d_{1,2} = \frac{E_1\left(E_1\nu_1 + E_2\nu_2^2\right)}{(1+\nu_1)(E_1(1-\nu_1)-2E_2\nu_2^2)}\sin^2\varphi + \frac{E_1E_2\nu_2}{E_1(1-\nu_1)-2E_2\nu_2^2}\cos^2\varphi,$$

$$d_{1,3} = \frac{E_1[(\nu_2-1)E_2\nu_2 + (E_1 - E_2\nu_2)\nu_1]}{2(1+\nu_1)(E_1(1-\nu_1)-2E_2\nu_2^2)}\sin 2\varphi,$$

$$d_{2,2} = \frac{E_1\left(E_1 - E_2\nu_2^2\right)}{(1+\nu_1)(E_1(1-\nu_1)-2E_2\nu_2^2)}\sin^4\varphi + \frac{E_1E_2(1-\nu_1)}{E_1(1-\nu_1)-2E_2\nu_2^2}\cos^4\varphi +$$

$$+\frac{1}{4}\left[\frac{E_1E_2\nu_2}{E_1(1-\nu_1)-2E_2\nu_2^2} + 2G_2\right]\sin^2 2\varphi,$$

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$$d_{4,5} = \frac{1}{2}\left[\frac{E_1}{2(1+\nu_1)} - G_2\right]\sin 2\varphi,\quad d_{5,5} = G_2\sin^2\varphi + \frac{E_1}{2(1+\nu_1)}\cos^2\varphi.$$

Spatial problem for fluid filtration in the deformable inclined transversely isotropic porous medium with the horizontal well of a crosscut type (1)-(9) is represented as a task of generalized plane filtration and generalized plane deformation.

Numerical Implementation

The finite element method (FEM) is used for numerical implementation. Without going into details on the theoretical study of the FEM peculiarities, we shall briefly mention some of distinctive features of this method.

In accordance with FEM the computational domain is divided into a number of subdomains – finite elements. Then each finite element is equipped with a system of basic functions (shape function). An approximate solution of the original problem is sought as a linear combination of the basic functions from the conditions that formally coincide with the equations of the Galerkin or Ritz method.

Using the shape function obtained with the exact solution of the equation system under consideration, is the main difference of FEM from other numerical methods in terms of its software implementation and application. An approximate solution is sought as a linear span of the system of basic functions. Finite-dimensional problem (the system of linear equations for the nodal values of the finite element) is formally obtained in all cases from the condition of orthogonality of the residual of an approximate solution with some finite-dimensional subspace.

The finite element method is significantly different in terms of the theoretical description. This is due to the fact that the basic functions generally used in FEM have approximating properties in the space, in which a solution of the original problem is sought. In FEM basis functions are calculated as solutions of the auxiliary problems for the equation under consideration in each finite element and set directly. However, this FEM feature is not a problem in terms of the theory, as it turns out that approximating properties shall specify the boundary conditions in the calculation of basic functions.

Thus, the original boundary-value problem can be replaced by its equivalent problem for the evaluation of unknowns through nodal values of finite elements. In accordance with the approach described, FEM can be considered as a projection method for solving equations, that allows their use in the study of the well-known theory of projection methods.

As a result, in terms of theory the scheme for construction of FEM approximations is as follows. First, the computational domain is divided into a certain number of disjoint subdomains tightly covering this domain – finite elements.

Note also that it is possible to use the combined approach to problems of non-stationary filtration, according to which for approximation of the problem both FEM and finite difference methods are used.

The reduction in the combined approach is necessary only for construction of the computational scheme and theoretical study of the method, that allows to use this approach in the validation, discussed in the first section.

Along with the filtration characteristics of the finite element of the transtropic medium for determination of the stress-deformation state we use FEM ratios. We do not dwell on the obvious recording of FEM ratios for elasticity problems [6-10], but give the necessary changes in the numerical implementation of filtration problems in the deformable transtropic medium to the horizontal well.

Let us consider the impact of pressure as follows:

$$\{f^e\} = -\int_{\partial e} \begin{bmatrix} N_i & 0 \\ N_j & 0 \\ N_k & 0 \\ 0 & N_i \\ 0 & N_j \\ 0 & N_k \end{bmatrix} \begin{Bmatrix} p_y \\ p_z \end{Bmatrix} ds .$$

Deformation components are calculated as follows:

$$\varepsilon_y = \frac{\partial u}{\partial x} = N'_{i_y} u_i + N'_{j_y} u_j + N'_{k_y} u_k ,$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = N'_{i_z} w_i + N'_{j_z} w_j + N'_{k_z} w_k ,$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial u}{\partial z} = N'_{i_y} w_i + N'_{i_z} u_i + N'_{j_y} w_j + N'_{j_z} u_j + N'_{k_y} w_k + N'_{k_z} u_k$$

or in the matrix form they can be presented as follows:

$$\{\varepsilon\} = [B]\{\delta^e\},$$

where the elements of the gradient matrix appear as follows:

$$[B] = \begin{bmatrix} b_i & b_j & b_k & 0 & 0 & 0 \\ 0 & 0 & 0 & c_i & c_j & c_k \\ c_i & c_j & c_k & b_i & b_j & b_k \end{bmatrix}$$

Taking into account the external loads $\{f^e\}$, the following values shall be obtained in the finite element:

$$[k^e]\{\delta^e\} = \{f^e\}$$

or

$$[k^e]\begin{Bmatrix} u_i & u_j & u_k & w_i & w_j & w_k \end{Bmatrix}^T =$$

$$= \begin{Bmatrix} f_{y_i}{}^e & f_{y_j}{}^e & f_{y_k}{}^e & f_{z_i}{}^e & f_{z_j}{}^e & f_{z_k}{}^e \end{Bmatrix}^T$$

where $[k^e]$ - means the element stiffness matrix (ESM)

$$[k^e] = \frac{1}{2\Delta}[B]^T[D][B]$$

In their turn, the element stiffness matrices make up the system stiffness matrix (SSM) of the following order: 2Nx2N (*N* means a number of nodal points of the computational domain),

$$[K] = \sum_e [k^e]$$

it means that ESM forms the left side of the system of linear equations (SLE):

$$[K]\{U\} = \{F\}$$

Here $\{U\}^T = \{u_1 \quad u_2 \quad ... \quad u_N \quad w_1 \quad w_2 \quad ... \quad w_N\}$

$$\{F\}^T = \{F_{y_1} \quad F_{y_2} \quad ... \quad F_{y_N} \quad F_{z_1} \quad F_{z_2} \quad ... \quad F_{z_N}\}$$

$$F_{y_i} = \sum_e f_{y_i}^e, \quad F_{z_i} = \sum_e f_{z_i}^e .$$

Thus the systems of linear equations (SLE) are formed. In determination of pressure, FEM ratios can be written with the account of the permeability coefficient of the transversely isotropic medium with planes of isotropy inclined at an angle. For each element the pressure approximation is carried out as follows:

$$\{p^e\} = \{N_i \quad N_j \quad N_k\} \begin{Bmatrix} p_i \\ p_j \\ p_k \end{Bmatrix}$$

where $N_\alpha(\alpha = i, j, k)$ - means the shape function.

ESM components shall be rewritten as follows:

$$[B] = \begin{bmatrix} b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix},$$

$$[D] = \begin{bmatrix} k_{yy} & k_{yz} \\ k_{yz} & k_{zz} \end{bmatrix}$$

here $b_\alpha, c_\alpha(\alpha = i, j, k)$ and $k_{ii}(i = y, z)$ shall be determined from (2) respectively.

Let us construct the FEM algorithm for the problem of fluid filtration in the elastically deformable transtropic medium to the horizontal well, i.e. the part of the horizontal well with selected fluid (perforation interval) passed along the extension of the rocks.

The algorithm for solving the problem is divided into 2 (two) stages. The first stage is devoted to determining the stress-deformation state (SDS) of the transversely isotropic medium with inclined planes of isotropy taking into account the original values of pressure. At the second stage, for each time interval, taking into account the deformation, the following values of pressure are determined. At the next stage the procedure for SDS determination is repeated, then the pressure is calculated.

Let us analyze the impact of the stress-deformation state on the flow rate of the horizontal well of a crosscut type based on numerical calculations. In particular, let us carry out the quantitative analysis of the impact of the angle of the formation inclination (isotropy plane inclination), distribution of stress and displacement components in the formation, that determines, in its turn, the efficiency of the horizontal well of a crosscut type.

The calculations have been performed for the isotropic siltstone, elastic and filtration parameters have been taken in the following order:

E=0.62 kg/cm^2, ν=0.2, G_2=0.258 kg/cm^2, k=0.106 Darcy.

Geometrical parameters are defined in the same way as in the first section.

Multivariate calculations for determination of the flow rate of the horizontal well have been made in the following parameters of the anisotropic (with isotropy planes) formation.

$$\varphi = 0, 30^0, 45^0, 60^0, 90^0,$$

$$E_1 = E, \quad \frac{E_2}{E_1} = 0.2, 0.5, 1, 2, 5;$$

$$k_{y'} = k, \quad \frac{k_{z'}}{k_{y'}} = 0.1, 0.2, 0.5, 1, 2, 5, 10;$$

The calculation of the flow rate of the horizontal well indicates a significant impact of the angle of the isotropy plane on the stress-deformation state of the formation with regard to the fluid filtered in it (see Table 1-2). As can be seen on Figure 2 the flow rate of the horizontal well of a crosscut type (t/day) with $\frac{k_{z'}}{k_{y'}} = 10$ has its maximum value. In case when $\frac{k_{z'}}{k_{y'}} = 0.1, 0.2, 0.5$ and 1 with an arbitrary value of φ, no significant changes in the flow rate Q can be observed, i.e. with these values of the filtration coefficient the flow rate of the horizontal well remains almost unchanged. Further, Q reaches its highest value in case when φ=0 and $\frac{k_{z'}}{k_{y'}} = 10$. Consequently when the vertical filtration coefficient is increased ten times, the productivity of the horizontal fine-layered formation reaches its maximum value. Subsequent deviations from the maximum flow rate of the horizontal well can be noticeable with the following values subject to the inclination angle of the plane of isotropy $\varphi = 90^0, 30^0,\ 60^0$ and 45^0. An impact of elastic parameters on the flow rate Q is especially noticeable when $\frac{E_2}{E_1} = 0.2$ and φ=0.

Table 1: Changes of the flow rate of the horizontal well when $\frac{k_{z'}}{k_{y'}} = 1$

Φ (degrees)	**E_1/E_2**				
	0.2	**0.5**	**1**	**2**	**5**
0	1.878	1.803	**1.792**	1.786	1.778
30	1.814	1.797	**1.792**	1.791	1.813
45	1.711	1.786	**1.792**	1.794	1.81
60	1.666	1.77	**1.792**	1.799	1.807
90	1.623	1.743	**1.792**	1.811	1.822

Table 2: Changes of the flow rate of the horizontal well when $\frac{E_1}{E_2} = 1$

$k_{z'}/k_{y'}$	**φ (degrees)**				
	0	**30**	**45**	**60**	**90**
0.1	2.483	1.122	0.986	2.001	5.094
0.2	1.968	1.107	1.075	1.863	3.858
0.5	1.384	1.244	1.344	1.694	2.329
1	**1.792**	**1.792**	**1.792**	**1.792**	**1.792**
2	4.658	3.389	2.688	2.487	2.768
5	19.292	9.316	5.376	5.536	9.841
10	50.938	20.011	9.856	11.219	24.828

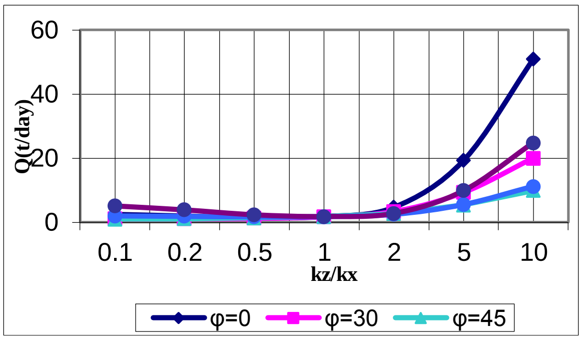

Figure 2: The graph of the function $Q = Q\left(\frac{k_{z'}}{k_{y'}}\right)$ of the flow rate of the horizontal well with the elasticity E=0.2 t/m^2

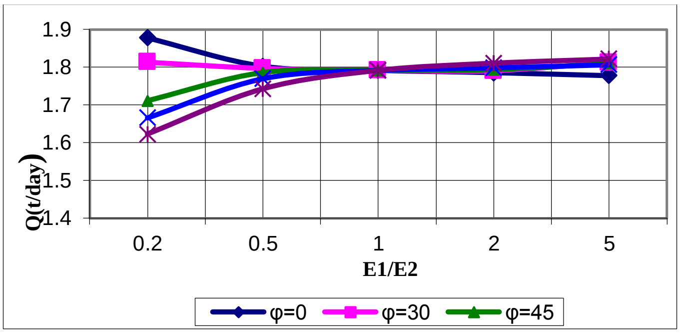

Figure 3: The graph of the function $Q = Q\left(\frac{E_1}{E_2}\right)$ of the flow rate of the horizontal well with the permeability k=0.106 Darcy.

In all of the multivariate calculations in case of the isotropic formation the value of the flow rate Q = 1.792 t/day remains unchanged. As can be seen from the results obtained, the calculation of the flow rate confirms the isotropic case in particular.

Conclusions

The formulation and numerical implementation of the problem of determining the flow rate of the horizontal well of a crosscut type have been considered in the present article, with due regard to the filtration and elastic characteristics of the deformable anisotropic medium. Spatial problem of the fluid filtration in the deformable inclined transversely isotropic porous medium to the horizontal well has been represented as a problem of generalized plane filtration and generalized plane deformation. In addition, the possibilities of modeling of fluid filtration in the medium with the doubly periodic slits system have been shown. Such medium includes the system of parallel fractures or weak layers located at the same distance from each other. Fractures may be horizontally inclined at an angle φ. The direction of the anisotropy axes of such fractured medium is constant for the entire array and coincides with the direction of the extension and dip of the formations. The corresponding flow rate of the well in such medium can be numerically determined using the developed algorithms and software system.

On the basis of the results of the study the act certifying implementation of the results of the research project "Increase of oil production by means of definition of deformed state of bottomhole zone of producing formation" was issued by "Buzachi neft'" ltd and "CaspiOilGas" ltd., which are located in Mangystau Region of the Republic of Kazakhstan; the results of the study allowed to improve pore space of reservoir and increase flow rate up to 17.0 ton per day. An increase of flow rate was equal to 30.8 % from initial value. So far, the archived effect still takes place.

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