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Chemical Reaction, Radiation and Dufour Effects on Casson Magneto Hydro Dynamics Fluid Flow over A Vertical Plate with Heat Source/Sink

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Abstract

In this article it is examined Chemical reaction, Radiation and Dufour effects on casson MHD fluid flow over a vertical plate with heat source/sink. This Problem is solved numerically using the perturbation technique for the velocity, the temperature and the concentration species. The skin friction, Nusselt number and Sherwood number are also obtained and are shown in tabular form. The effects of various physical parameters like as Chemical reaction parameter, Dufour parameter, Radiation parameter, Heat parameter, Casson parameter, Schmidt number, Grashof number, Prandtl number, Hartmann Parameter, and modified Grashof number has been discussed in detailed.

Keywords: Casson fluid, Dufour effect, MHD, Chemical reaction.

Introduction

The non-Newtonian fluids plays a vital role in various fields such as pharmaceutical, biological, chemical, food, and personal care processing industries. complex fluids(non-linear) are power-law fluid, nano fluid, casson fluid [tomato sauce, honey, concentrated fruit juices, Human blood, etc.], micro polar fluid etc. These include mixtures of fluids, blood fluids and bio-fluids etc. Eldate.N.T.M. [1] studied Heat Transfer of MHD non-newtoniancasson fluid flow between two rotating cylinder. Mustafa.M., et al [2] analysed Stagnation-point flow and Heat Transfer of a casson fluid towards a stretching sheet.Rama SubbareddyGorla.,et al., [3] discussed Mixed Convection in Non-Newtonian Fluids along a Vertical Plate in Porous Media with Constant Surface Heat Flux. KerehalliVinayaka Prasad., et al [4] studied Non-Newtonian Power-Law Fluid Flow and Heat Transfer over a Non-Linearly Stretching Surface. Kenneth walters., [5] used non-Newtonian fluid Mechanics. RobensonCherizol.,et al [6] introduced Review of Non-Newtonian Mathematical Models for Rheological Characteristics of Viscoelastic Composites .Schowalter.w.r. [7] Mechanics of non-newtonian Fluids.Acrivosa.,shah.M.J. [8] explained Momentum and Heat Transfer in Laminar Boundary Layer Flows of Non-Newtonian Fluids Past External Surfaces.

Authors [9-14] have been analysed on Non-Newtonian Fluids and other aspects. In this article it is examined Chemical reaction, Radiation and Dufour effects on Casson MHD fluid flow over a vertical plate with heat source/sink. This Problem is solved numerically using the perturbation technique for the velocity, the temperature and the concentration species. The skin friction, Nusselt number and Sherwood number are also obtained and are shown in tabular form. The results are made in this article are good agreement with previous work[15].

Formulation of the problem

MHD Casson fluid of incompressible, viscous, electrically- conducting fluid over a vertical plate moving with constant velocity with radiation and chemical reaction in the presence of Dufour effect is considered. The rheological equation of state for an isotropic and incompressible flow of Casson fluid [1,2] is

$$\tau_{ij} = \begin{cases} \left(\mu_B + p_y \big/ \sqrt{2\pi}\right) 2e_{ij}, \; \pi > \pi_c \\ \left(\mu_B + p_y \big/ \sqrt{2\pi_c}\right) 2e_{ij}, \; \pi < \pi_c \end{cases} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (A)$$

Where μ_B is plastic dynamic viscosity, p_y is yield stress, π_c is critical value of π ,and π is the product of the component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i, j)th component of deformation rate. The x - axis is taken along the plate in the vertical upward direction and the y - axis is taken normal to the plate. The surface temperature of the plate oscillates with small amplitude about a non-uniform mean temperature. The fluid is assumed to have constant properties except for the influence of the density variations with temperature and concentration which are considered only in the body force term. The temperature of the plate oscillates with little amplitude about a non-uniform temperature. By usual Boussinesq's approximation, the flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = \left(1+\frac{1}{\beta}\right)\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T'-T'_\infty\right)$$
$$+g\beta^*\left(C'-C'_\infty\right) - \frac{\sigma}{\rho}B_0^2 u' \ldots. \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q'_r}{\partial y'}$$
$$+\frac{Q_0}{\rho C_p}(T'-T'_\infty) + Du\frac{\partial^2 C'}{\partial y'^2} \ldots(2) \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K'_r\left(C'-C'_\infty\right) \ldots(3) \tag{3}$$

Equations (1), (2) and (3) refers Momentum equation, Energy Equation and Species Equation respectively.

Where u is the velocity of the fluid, β is Casson parameter, Q_0 is the heat source/sink parameter, D is the molecular diffusivity, k is thermal conductivity, C is mass concentration, t is time, υ is the kinematics viscosity, g is the gravitational constant, β *and* β^* are the thermal expansions of fluid and concentration, T is temperature of fluid , ρ is density, c_p is the specific heat capacity at constant pressure, y is distance, q_r is the radiative flux, β_0 is the magnetic field, kr_0 is the chemical reaction rate constant.

R.H.S. of equation (1), third term is thermal concentration effect, fourth term is magnetic effect, second term is thermal buoyancy effect.
R.H.S. of equation (2) second term is thermal radiation flux and third term is thermal radiation and fourth term is Dufour effect.
R.H.S. of equation (3), second term is chemical reaction and third term Dufour (Diffusion Thermo) effect.
Under the above assumptions the physical variables are functions of y and t.
The boundary conditions for the velocity, temperature and concentration fields are:

$$u' = U,\ \ T' = T' + \varepsilon(T' - T'_\infty)e^{i\omega t},$$
$$C' = C' + \varepsilon(C' - C'_\infty)e^{i\omega t}\ \ at\ \ y = 0$$
$$u' \to 0, T' \to 0, C' \to 0,$$
$$as\quad y \to \infty..... \tag{4}$$

Introducing the dimensionless quantities with thermal radiation flux gradient expressed andwe assume that the temperature differences within the flow are sufficiently small so that T'^4 can be expressed as a linear function of T' after using Taylor's series to expand T'^4 about the free stream temperature T'_∞ and neglecting higher-order terms. This results in the following approximation:

$$\frac{\partial q_r}{\partial y'} = -4a'\sigma'\left(T'^4_\infty - T'^4\right) and$$
$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty\ ;$$

$$\left.\begin{aligned}
&u = \frac{u'}{U}, y = \frac{Uy'}{v}, t = \frac{t'U^2}{v},\\
&\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Q = \frac{Q_0 \nu}{\rho C_p U^2}\\
&K = \frac{K' u_0^2}{v^2}, \Pr = \frac{\mu C_p}{k},\ Sc = \frac{v}{D},\\
&M = \frac{\sigma B_0^2 v}{\rho U^2}, Gr = \frac{v\beta g\left(T'_w - T'_\infty\right)}{U^3},\\
&Gm = \frac{v\beta^* g\left(C'_w - C'_\infty\right)}{U^3}, K_r = \frac{K'_0 v}{U_0^2},\\
&R = \frac{16a'v\sigma^* T'^3_\infty}{\rho C_p U^2}, \mu = v\rho, D_0 = \frac{Du}{v\left(T'_w - T'_\infty\right)}
\end{aligned}\right\}...(5) \tag{5}$$

The thermal radiation flux gradient may be expressed as follows

$$-\frac{\partial q_r}{\partial y'} = 4a\sigma^*(T'_\infty - T'^4) \tag{6}$$

Considering the temperature difference by assumption within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is attained by expanding in T'^4 taylor's series about T'_∞ and ignoring higher orders terms.

$$T'^4 = 4T'^3_\infty T' - 3T'^4_\infty \tag{7}$$

Substituting the dimensionless variables (5) into (1) to (3) and using equations (6) and (7), reduce to the following dimensionless form.
Substituing the dimentionless variables(5) into above all equations and reduces to the following.

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gc\phi-Mu \tag{8}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}-R\theta+Q\theta+D_0\frac{\partial^2 C}{\partial y^2} \tag{9}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-KrC \tag{10}$$

The corresponding boundary conditions of (4) in dimensionless form are

$$\begin{aligned} &u=1, \theta=1+\in e^{iwt}, \\ &C=1+\in e^{iwt} \quad at \quad y=0 \\ &u\to 0, \theta\to 0, C\to 0 \\ &\qquad as \quad y\to\infty \end{aligned} \tag{11}$$

Where Gr is thermal Grashof number, Pr is the prandtl number, kr is the chemical reaction parameter, R is the thermal radiation conduction number, M is Hartmann number, Gc is the mass Grashof number, Q is the heat source/sink parameter.

Method of Solution

Equations (8),(9) and (10) represents a set of partial differential equations that cannot be solved in closed form. However, it can be reduced to a set of ordinary differential equations in dimensionless form that can be solved analytically. This can be done by representing the velocity, temperature and concentration as

$$\begin{aligned} &u=u_0+\in e^{iwt}u_1+O(\in^2)+\ldots\ldots\ldots\ldots\ldots\ldots, \\ &\theta=\theta_0+\in e^{iwt}\theta_1+O(\in^2)+\ldots\ldots\ldots\ldots\ldots\ldots, \\ &C=C_0+\in e^{iwt}C_1+O(\in^2)+\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{12}$$

Where $u_0(y), u_1(y), \theta_0(y), \theta_1(y), C_0(y), and C_1(y)$ have to be determined.

$$\begin{aligned} &\left(1+\frac{1}{\beta}\right)u''_0-Mu_0=-Gr\theta_0-GcC_0 \\ &A_4u''_1-(M+iw)u_1=-Gr\theta_1-GcC_1 \\ &\frac{1}{\Pr}\theta''_0=(R-Q)\theta_0-D_0C''_0 \\ &\frac{1}{\Pr}\theta''_1=(R-Q+iw)\theta_1-\Pr D_0C''_1 \\ &C''_0-Sck_rC_0=0 \\ &C''_1-Sc(k_r+iw)C_1=0 \end{aligned} \tag{13}$$

All primes denote differentiation with respect to y.

The boundary conditions are

$$u_0 = 1, \theta_0 = 1, C_0 = 1 \quad at \quad y = 0,$$
$$u_1 = 0, \theta_1 = 1, C_1 = 1 \quad at \quad y = 0,$$
$$u_0 \to 0, \theta_0 \to 0, C_0 \to 0 \quad as \quad y \to \infty,$$
$$u_1 \to 0, \theta_1 \to 0, C_1 \to 0 \quad as \quad y \to \infty. \tag{14}$$

Solving the system (11) subject to the boundary conditions (12),
We obtain

$$u_0 = g_1 e^{-\sqrt{A_5}\,y} + \left(\frac{A_6}{A_1 - A_5}\right) e^{-\sqrt{A_1}\,y} +$$
$$\left(\frac{A_{10}}{Sckr - A_5}\right) e^{-\sqrt{Sckr}\,y}$$
$$u_1 = B_{15} e^{-\sqrt{A_8}\,y} + \left(\frac{g_2}{A_2 - A_8}\right) e^{-\sqrt{A_2}\,y}$$
$$+\left(\frac{g_3}{A_3 - A_8}\right) e^{-\sqrt{A_3}\,y} + \left(\frac{g_4}{A_3 - A_8}\right) e^{-\sqrt{A_3}\,y}$$
$$\theta_0 = B_{12} e^{-\sqrt{A_1}\,y} + B_{11} e^{-\sqrt{Sckr}\,y},$$
$$\theta_1 = B_{14} e^{-\sqrt{A_2}\,y} + B_{13} e^{-\sqrt{A_3}\,y},$$
$$C_0 = e^{-\sqrt{Sckr}\,y}, C_1 = e^{-\sqrt{A_3}\,y} \ldots.. \tag{15}$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer becomes

$$u = u_0 + \varepsilon\, e^{iwt} u_1,$$
$$\theta = \theta_0 + \varepsilon\, e^{iwt} \theta_1,$$
$$C = C_0 + \varepsilon\, e^{iwt} C_1 \tag{16}$$

The skin friction(C_f), Nusselt number(Nu) and Sherwood number(Sh) are obtained from equation(14) when differentiated aty=0.

$$C_f = \begin{pmatrix} -g_1\sqrt{A_5} + \dfrac{A_6}{A_1 - A_5}\sqrt{A_1} + \\ \dfrac{A_{10}}{Sckr - A_5}\sqrt{Sckr} \end{pmatrix} +$$
$$\varepsilon\, e^{iwt} \begin{pmatrix} -B_{15}\sqrt{A_8} - \dfrac{g_2}{A_2 - A_8}\sqrt{A_2} \\ -\dfrac{(g_3 + g_4)}{A_3 - A_8}\sqrt{A_3} \end{pmatrix},$$
$$Nu = \begin{pmatrix} (-B_{12}\sqrt{A_1} - Sckr.B_{11}) + \\ \varepsilon\, e^{iwt}(-B_{14}\sqrt{A_2} - B_{13}\sqrt{A_3} \end{pmatrix},$$
$$Sh = \sqrt{Sckr} + \varepsilon\, e^{iwt}\sqrt{A_3} \tag{17}$$

Using equation (16),(17) the results analysed and determined.

Results and Discussion

Casson MHD flow over a vertical plate with dufour parameter has been formulated and analysed analytically. Only three computations are performed for Variation of the velocity with thermal coefficient, Variation of the temperature with thermal radiation conduction, Variation of the concentration with chemical reaction parameter.

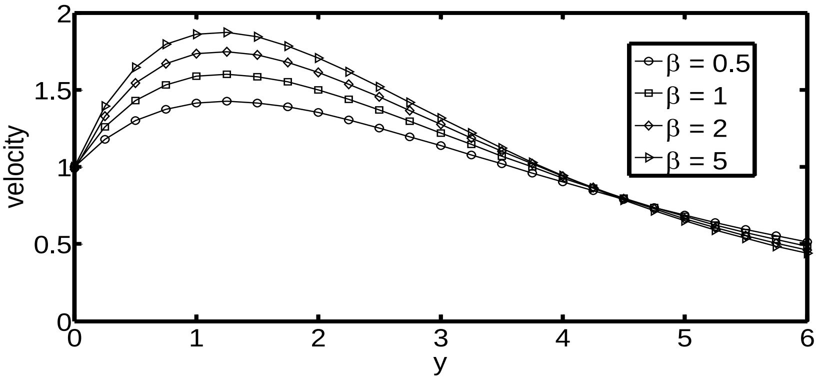

Figure 1: Variation of the velocity with thermal coefficient

The dimensional governing equations are solved by two term perturbation technique in this article with Pr = 2, G_r = 2.0, G_C = 2.0, ε = 0.01, M = 0.5, t =1.0,Sc=2.0, kr = 0.5,R =0.2,Q=0.1,ω= 1.0,Du=0.03. All graphs therefore correspond to these unless specifically indicated on the appropriate graph.The influence of Casson parameter in velocity is shown in Fig.1. It is recognised that velocity decreases far way the plate and increases near the plate while increase in beta. Fig.2. tells the influence of thermal radiation conduction on the temperature. It is cleared that temperature is decrease when R is increase. The influence of kr on the concentration is illustrated in Fig.3. The concentration is decreases as the chemical reaction parameter increases.

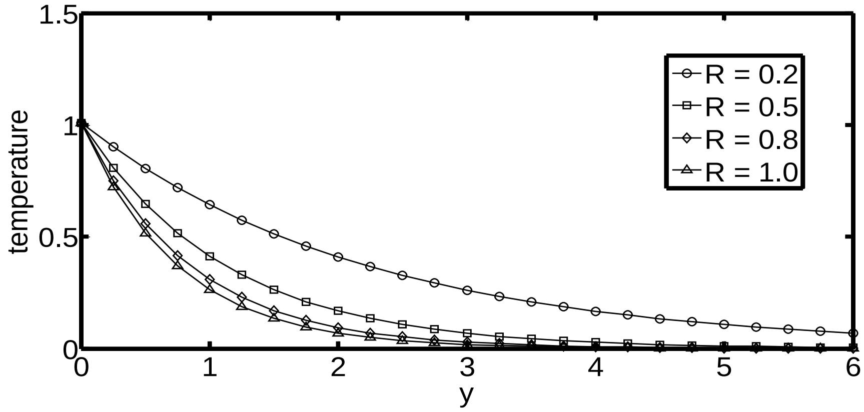

Figure 2: Variation of the temperature with thermal radiation conduction

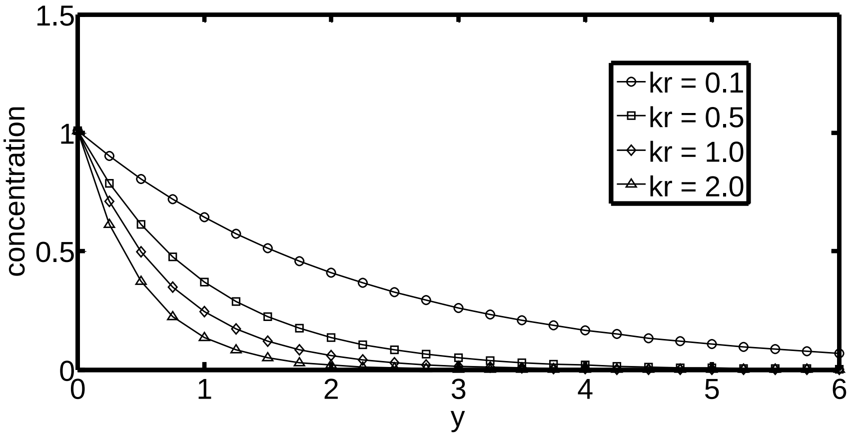

Figure 3: Variation of the concentration with chemical reaction parameter.

Table 1: Variation of the the Skin-friction coefficient, Nusselt number and Sherwood number .

β	M	R	C_f	Nu	Sh
3	0.5	0.6	1.1741	0.8945	1.0001
4	0.5	0.6	1.3136	0.8945	1.0001
2	1	0.6	0.6970	0.8945	1.0001
2	2	0.6	0.1149	0.8945	1.0001
2	0.5	1	0.9919	1.265	1.0001
2	0.5	2	0.8069	1.8976	1.0001

Table 1 tells the effects of Casson parameter, Hartmann number and thermal radiation conduction with Pr = 2, G_r = 2.0, G_C = 2.0, ε = 0.01, M = 0.5, t =1.0,Sc=2.0, kr = 0.5,R =0.2,Q=0.1,ω= 1.0, Du=0.03.

Conclusion

In this article it is examined Chemical reaction, Radiation and Dufour effects on Casson MHD fluid flow over a vertical plate with heat source/sink. This Problem is solved numerically using the perturbation technique for the velocity, the temperature and the concentration species. The skin friction, Nusselt number and Sherwood number are also obtained and are shown in tabular form. The skin-friction is increases with the effect of casson parameter, at the plate. As perturbation parameter increases, the Skin-friction coefficient, Nusselt number and Sherwood number are decreases at the plate and hence some will not discuss any further due to brevity.

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