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Theoretical Foundations of Using Econometric Methods of Time Series Forecasting

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Abstract

the human, ever since his emergence on the Earth, has always wanted to know what the future would bring, what events could happen. People wanted to know this not out of idle curiosity, but to be better prepared for these events. That's the way forecasting appeared.Currently, there are different kinds of forecasts. Forecasts can be divided into short-term, middle-term and long-term. They can also be individual,local, regional, etc. But whatever be the forecast, it isbased on a forecasting model, i.e. the tool which is usedfor forecasting. The present paper is devoted to the analysis of the main models used for time series forecasting. The paper deals with the following types of forecasting models: regression and autoregression models,exponential smoothing models,neural network models,Markov chain models, models based on classification and regression trees, models based on the genetic algorithm,support vector and transfer function models, fuzzy logic models, singular spectrum analysis models,local approximation models, models based on fractal time series, models based on wavelet transformation, models based onFourier transformation. Along with studying the structure or algorithm of each model, the paper also attempts to identify their strengths and weaknesses.

Keywords: Forecasting Models, Time Series,Moving Average Models, Exponential Smoothing, Neural Networks, Hybrid Models.

Introduction

In the context of the modern economy, one of important tasks for any economic system is forecasting its development. The extent of the accuracy of determining the line of development can be essential for the existence of this system.The issues of forecasting are discussed by many researchers, both in Russia and abroad. Among the domestic scientists who dealt with this problem there areN.N. Kondrat'ev, A.G. Aganbegyan, A.I. Anchishkin, Yu.V. Yakovets, E.E. Slutskiy, A.A. Dynkin, V.V. Ivanter, V.V. Kuleshov, etc. The studies by foreign scientists are represented by the works by C. Juglar, J. Schumpeter, J. Tinbergen, J. Kitchin, etc.

There are different approaches and methods of forecasting. For example, qualitative and quantitative forecasting methods can be identified. The first ones have become widespread, as far as getting a subjective opinion of an expert is concerned. The second group is used when it is necessary to get an independent assessment or whenit can bebelieved that past activities could have an impact on the present and evenfuture situation. Among the quantitative methods most widely spread are various time series models. There are a large number of models used for forecasting on the basis of time series, starting from the "simplest" regression models and exponential smoothing models and ending with "complex",combined models. **The aim of this paper** is the analysis of the majority of existing models, the identification of the strengths and weaknesses of each of them.

Research Methodology

In the course of this study, the authors relied on the classificationoftime series forecasting models proposed by I.I. Chuchueva and took thefollowing models [1]:

- Regression models (linear and non-linear regression);
- Autoregression models (an AR-model and models built on its basis);
- Exponential smoothing models;
- Neural network models;
- Markov chain models;
- Models based on classification and regression trees;

- Model based on the genetic algorithm;
- Support vector models;
- Transfer function models;
- Fuzzy logic models;
- Singular spectrum analysis models;
- Local approximation models;
- Fractal time series models;
- Wavelet transformation models;
- Fourier transformation models.

The authors studied each model, identified its strengths and weaknesses, andconsidered the possible options for their application.

Research Results

At first, regression models were studied. This category includes the following models:

- Regression models (linear and non-linear regression)
- autoregression models (an AR-model and models built on its basis);

Regression models are the models of the type:

$$Y(t) = f(X(t), X_2(t) \dots X_n(t)) + \varepsilon_t, \text{ where}$$

$X_1(t)$, $X_2(t) \dots X_n(t)$ areexogenous variables,$Y(t)$ is an endogenous variable, ε_t is a random component.

This type of models currently doesn't have a wide application. In the authors' opinion, this is due to the following reasons. First, as noted by I.A. Chuchueva, in order to determine the future value of the variable Y, it is necessary to know the future values of the variables$X_1(t)$, $X_2(t) \dots X_n(t)$[1]. For this end, the values of these variables should be knownin advance (which is virtually impossible), or it is necessary to carry out forecasting. Secondly, the choice of the type of dependency is subjective. And there is no certainty that the researcher chose the right type of dependency. Thirdly, there can be a functional dependency or a strong correlation betweenexogenous variables. The presence of multicollinearity (which is the case here) can make a model unsuitable for forecasting.
Another model type, which the authors included in this group, is the autoregression model. This type of models includes those, in which the subsequent values of Y (t) depend on the previous values Y (t-p) [2]. The simplest model of this type (AR-model) has the following form [3]:

$$Y(t) = a_1 Y(t-1) + a_2 Y(t-2) \dots + a_n Y(t-p) + C + \varepsilon,$$

Where a_1, a_2, a_n are coefficients, $Y(t-1)$, $Y(t-2)$, $Y(t-p)$are "previous" values of $Y(t)$, C is a constant, ε is the process of "white noise" or a random component.

In practice, however, the variations of this model have become widespread, namely: ARMA (p, q), ARIMA (p, q, d), ARFIMA (p,q,d), ARIMAX (p, q, d), SAR (p), SARIMA (p, q, d), SARIMAX (p,q,d), ARCH (q), GARCH (p,q), EGARCH (p,q), QGARCH (p,q), FIGARCH (p,d,q), VAR (p), ARDLM (p, I), etc. We will not discusseach of the above models in detail. An inquisitive reader will find a detailed description of each of them inworks like [4-5]. Let us dwell only on the strengths and weaknesses of the models of this class. In the authors' opinion, the main advantages are the simplicity and transparency ofmodel building. All models of this class are characterized by the uniformity of the process of model building. They have found wide application in many areas. The main disadvantages of these models are the following. Firstly, theprocedure of model identification is complicated. Secondly, models of this class are characterized by low adaptedness.

The following class of models is the exponential smoothing models. In econometric literature the followingfour of them are most popular[6]: exponential smoothing model (Brown's model), Holt's model, Holt-Winters model, and Theil-Wage model. The best known of them isthe Brown's model. In general terms, it has the following form [7]:

$$\hat{Y}(t+1) = \alpha Y(t) + (1-\alpha)\widehat{Y(t)},$$

Where α is the smoothing coefficient, takingvalues from 0 to 1; $\widehat{Y(t)}$ is a previous predicted value; $Y(t)$ is an actual value.

The main advantage of using this model is its simplicity. The main disadvantage is that this model can be used only for a short-term forecasting horizon.

For a partial solution of the first problem, it is possible to use the Holt's model. This model takes into account the trend, but does not take into account the seasonal component. Another disadvantage of this model, as well as of theBrown's model, is a small forecasting horizon. The seasonalcomponent and the trend are taken into account by the third exponential smoothing model – the Holt-Winters model. There are two varieties of this model: a model with additive seasonality and the model with multiplicativeseasonality [8]. The main disadvantage of this model is the need to find optimal parameters experimentally [8]. The Theil-Wage modelalso takes into account trend and seasonality. Its distinctive feature is the ability to take into account the additive seasonality and linear growth. However, as noted by the V.M. Vartanyan, Yu.A. Romanenkov, V.Yu.Kashcheev, "in the absence of the information on the exact value of the seasonality period and the small length of the series the Theil-Wage forecasting modelhas a low accuracy and becomes practically inapplicable" [9].

The following group of models includes models based on neural networks. As noted by I.V. Nikolaeva, "the role of a neural network consists in predicting the future reaction of a system based on its previous behavior" [10]. A neural network is based on a neuron, consisting ofthree main types of elements: synapses, an adder and a nonlinear converter (Figure 1).

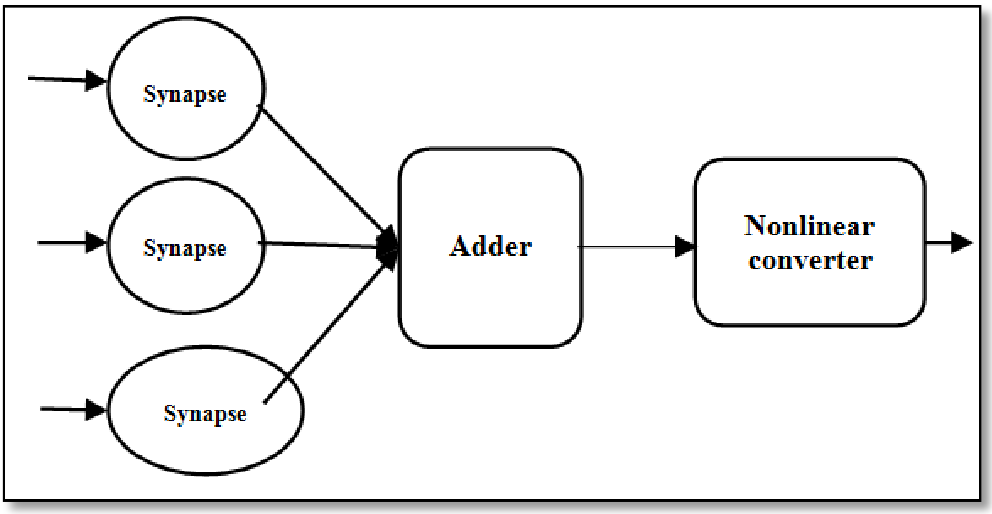

Figure 1: Structure of a neuron

Compiled by the authors

The set of synapses receives incoming signals. The adder adds up signals, and the nonlinear converter limits the amplitude of the output signal. One of the disadvantages of using neural networks for time series forecasting, in the authors' opinion, is the correct choice of the architecture of a neural network. Depending on the task at hand, neural networks can be of several types. For example, it is possible to identifysingle-layer feedforward networks,multi-layer feedforward networks, recurrent networks, etc. [11]. Another problem is the need to train a neural network. There are several ways of neuralnetwork training. The following ways of training can be identified [12]:

- training with a teacher;
- training without a teacher;
- training algorithm with fixed weights.

Each of these methods has both positive and negative aspects.

In general, the advantages of using neural networks intime series forecasting include the following. Firstly, it is possible to buildnonlinear models. Secondly, neural networks exhibit highadaptability. Thirdly, they show the consistency of analysis.
Along withneural networks, time series forecastingcan also be implemented on the basis ofMarkov chains. Markov chain is "a Markov process with a finite or countable set of states" [13]. In the process of forecasting it is assumed thatthe subsequent state of the object of study doesn't depend on its previous states, but is determined by its current state. The main advantages of using this model are the following. Firstly, it is the relative ease of building a Markov chain. Secondly, it is theconsistency of analysis. Thenegative aspects include the following. Firstly, this approach cannot be

used if it is necessary to simulate the processwith a long memory. Secondly, it is the narrowapplicability of the model. Thirdly, it is the nondeterminism of neural networks themselves.

In practice models based on classification andregression trees are also used. This algorithm was introduced byLeo Breiman, Jerome Friedman, Charles Stone, and Richard Olshen. This type of model usesbinary splitting. This means that every node in a tree has only two descendants. The main advantages of this type of models are the possibility of wide application, intelligibility and the ease of calculation. The forecasted valuedepends onprevious values and some independent variables. There are many varieties of this model. Here are some of them: C4.5, CHAID, CN2, Newld, etc.

A more extensivemodel of time series forecasting is the modelbased on the genetic algorithm. The genetic algorithm came to be widely used in the early 1990s. Nowadays, it is used for forecasting fuzzy time series. It is appropriate to recall that a fuzzy time series is "an ordered sequence of observations over some phenomenon, if the values taken bysome variable at a certain moment of time are expressed using a fuzzy label" [14].In case of using the genetic algorithm, all original values make up genotypes with subsequent implementation of procedures of mating and mutation. This is done for the purpose of creating phenotypes. If the result proved unsuccessful, the procedure is repeated again. The main advantage of this algorithm is, in the authors' opinion,its adaptability. Among the weaknesses the following should be mentioned: the algorithm has limited possibilities ofapplication, its building is time-consuming, and there isno guarantee of finding an optimal solution.

The followingmodel used to forecast the levels of a time series is the support vector model. The essence of this model isfinding "similar" samples in the original time series.The advantage of this model is the possibility of taking into account the current state of the studied system. However, this method has its drawbacks. The main disadvantage is, in the authors' opinion, the random selection of the area, which will be used for testing. Secondly, there is no calibration of the probability of getting into a certain class. Thirdly, the support vector method is suitable for solving problems with two classes. And fourthly, the parameters of the support vector method are very difficult to interpret.

There are also models based on transfer functions. This model is used in the process of forecasting taking into account external factors. The model has the following form [1]:

$$Q(t) = v(B)X + \eta(t),$$

Where B is a shift operator; $\eta(t)$characterizes external disturbance; v is the coefficient which describesthe dynamic change of v(B);

The main advantage of this model is, in the authors' view,the possibility of taking into accountexternal disturbance. The disadvantage of this model is theneed to find thev factor experimentally.

Another method is forecasting time seriesusing fuzzy logic. It is used in cases, whenusing expert knowledge is concerned [15]. This approach is based on the fact that there is a linguistic interpretation of the values of a time series. Both values and time points can be expressed in fuzzy linguistic assessments. The main advantage of this method is the appearance of a number of not only quantitative, but also qualitative ways of processing information. Its main disadvantage is the greatinfluence of an expert who makes an assessment.

To forecast the behavior of irregular time series, singular spectrum analysis is used. The use of this method makes it possible to [16]:

- distinguish the components of a time series;
- find the periodicities of a time series which are not known beforehand;
- smooth the input data on the basis of selected components;
- identify a component with a known period in the best possible way;
- etc.

This approach is based on building multiple delay vectors. This makes it possible to move from a scalar time series to a multidimensional view:

$$\mathrm{X}_{\mathrm{p}*(N-p+1)} = \left(\begin{bmatrix} X_{p+1} \\ \mathrm{X_p} \\ \cdots \\ \mathrm{x_2} \\ \mathrm{x_1} \end{bmatrix} \begin{bmatrix} \mathrm{X_{p+2}} \\ \mathrm{X_{p+1}} \\ \cdots \\ \mathrm{x_2} \\ \mathrm{x_1} \end{bmatrix} \cdots \begin{bmatrix} \mathrm{X}_N \\ \cdots \\ X_{N-p+2} \\ X_{N-p+1} \\ X_{N-p} \end{bmatrix} \right)$$

A specific feature of this method is the fact that the processing of the matrix is implemented using the method of principal components. On the basis of singular spectrum analysis the following methods of forecasting time series were developed: “Caterpillar” and the autoregression method, used separately for each individual component.

The main advantages of this method, in the authors’ opinion, are the following. Firstly, there is no requirement of thestationarity of a time series. Secondly, there is no need to know the trend model.The main disadvantages are the following. Firstly, the grouping of the components of the singular decompositionof the time series trajectory matrix is non-automatic. Secondly, the absence of a model does not make it possible to test the hypotheses onthe presence of acertain component in a time series. Thirdly, the use of this method makes it possible to obtain a result, which is just slightly more accurate as compared withparametric methods,in the analysis of a time series with a known model.

For the purpose of forecasting the dynamics of a time serieslocal methods are used, in particular, the method of local approximation. It may be used in case, if the researcher has to deal with a complex function. The main idea of the method of local approximation is breaking a complex function into severallocal areas with the purpose of building an approximating model and finding its parameters for each area. The main advantages of this method are the following. Firstly, this method is well-founded in terms ofchoosing the order of autoregression. It makes it possible to linearize nonlinear functions using linear piecewise functions [17].

The followingmethod used in forecastingtime series is the model of fractal time series. This model is based on the calculation of the so-called fractal dimension of a time series. Fractal dimension “isunderstood to be a number that quantitatively describes how an object fills the space” [18]. There are two methodsof the calculation of the fractal dimension of a time series. The first way is to calculate the Hurst exponent. The main drawback of this method is the fact that it is necessary to have at one’s disposal large amounts of data. The second method is the calculation of thedimension of minimum covering. It is the calculation of this indicator which is the most appropriate for forecasting time series values. The main advantage of using this method is the fact that it does not impose a limitation on the distribution law of a time series. However, to obtainmore accurate predicted values, the series should contain not less than 32 observations [18].

For the purpose of forecastingdiscrete time series the algorithm of redundant discrete wavelet transformation can be used [19]. The term “wavelet” was introduced in the article byGrossman and Morlet [20]. Wavelets are specific functions having the form of brief oscillations. For the implementation of redundant discrete wavelet transformationthe “a trous” algorithmcan be used. According to this algorithm, the decomposition of a time series takes place. The main advantage of this algorithm is the ability to approximate any function, consisting of different non-stationary components [21].

The last modelwhich will be considered in this paper is the model based onFourier transformation. The essence of this method is representing a signal as an infinite sum of sinusoids. It is used to determine the seasonality of a time series. A Fourier series is used as a model of periodically changing levels. In general terms it can be presented as follows [16]:

$$\widehat{y_t} = a_0 + \sum(a_k \cos kt + b_k \sin kt),$$

Where k determines the number of a harmonic in a Fourier series.

Equation parameterscan be calculated using theleast square method.

Currently combined or hybrid models of forecasting time series are becoming increasingly popular. This is due to several reasons. According to A.A. Vasil'ev, it is already impossible to describe the dynamics of a time series using only one model, it is necessary to use a whole complex of models. According to A.A. Vasil'ev, all hybrid models can be divided into two big groups. The first group is a group of selective models. A selective model is a combined model, according to which “at each step offorecasting one best model is selected by the specified criteria” of a core set of models. A hybrid model “is a model,according to which the forecast is formed as a weighted sum of the forecasts of the models of a basic set” [22]. I.A. Chuchueva identifies the following types of combined models [1]:

- neural network models plus fuzzy logic;
- neural network model plus ARIMA;
- neural network model plus regression model;
- neural network model plus genetic algorithm plusfuzzy logic.

Discussion

Thus, there are a large number of models that can be used for forecastingtime series. In this paper five types of models were considered. As it can be seen, each of them has both advantages and disadvantages. For example, the advantage of using regression, autoregression and exponentialmodels is the ease ofbuilding. It is because of this fact that they have become widespreadin various fields of human knowledge. These models are used to analyze the behavior of the prices forvarious assets in the financial markets and to forecast the population of animals. The disadvantage of these models is, as already noted, a short-termforecasting horizon and the fact that a researcher should possess certain skills for a right choice of themodel structure.Besides, regression models are not suitable for forecasting because ofpossiblefunctional dependency between variables.The above-mentioned problems could be solved by using Markov chain models, models based on classification and regression trees, models based on the genetic algorithm, support vector models, transfer function models,fuzzy logic models, singular spectrum analysis models, local approximation models, models based on fractal time series,models based on wavelet transformation, models based onFourier transformation. However, these models are not perfect. Each of them has both advantages and disadvantages.

Conclusion

As it can be seen, the development of the methodologies for forecasting time series does not stand still. Fromregression and exponential models the science stepped towards the use of fuzzy logic and the genetic algorithm. New methods and models emerge. Combined or hybrid models are used more and more actively. Unfortunately, within the framework of this paper they were only listed and not discussed in more detail. The authors will deal with this problemin the following papers, dedicated to time series forecasting models.

References

[1] Chuchueva, I.A., 2012. A Forecasting Model Based on a Maximum Likelihood Sample. Date Views 16.11.2015 www.mbureau.ru/sites/default/files/pdf/Chuchueva-Dissertation.pdf.
[2] Mkhitaryan, V.S., 2008. Econometrics: Textbook. Moscow: Prospect.
[3] Fedorova, E.A., 2009. Statistical Modelling of the Assessment of the Change of Stock Market Efficiency and Its Practical Application. Audit and Financial Analysis, 6: 1.
[4] Duka, O.S., 2007. Analysis of the Return of Assets Using the Models ARIMA – (E)GARH и AFRIMA –FIGARCH. Date Views 17.11.2015 digitalphysics.ru/pdf/Analiz_dohodnosti_i_volatilnosti_finansovyh_aktivov_-_Olga_Duka.pdf.
[5] Kantorovich, G.G., 2002. Time Series Analysis. Date Views 17.11.2015 www.hse.ru/data/2010/12/31/1208182146/06_04_06.pdf.

[6] Svetun′kov, I.S. and S.G. Svetun′kov, 2014. Methods of Socio-Economic Forecasting: Vol. 2. Methods and Models. Moscow: Urait.
[7] Semenenko, M.G. and I.V. Knyazeva, 2012. Brown’s Model. Date Views 17.11.2015 mas.exponenta.ru/literature/server%20model.pdf.
[8] Semenenko, M.G., 2008. Analysis of Time Series of Financial Indicators in the Holt-Winters Model. Date Views 17.11.2015 mas.exponenta.ru /literature/Semenenko.pdf.
[9] Vartanyan, V.M., Yu.A. Romanenkov and V.Yu. Kashcheeva, 2011. Assessment of the Frequency Parameters of the Theil-Wage Model in Short-Term Forecasting Tasks. East European Journal of Enterprise Technologies, 1(5): 49.
[10] Nikolaeva, I.V, 2012. Application of Artificial Neural Networks for Prediction of
Dynamic Economic Indicators. Date Views 17.11. 2015 journal.kfrgteu.ru /files/1/2012_8_22.pdf.
[11] Karasev, D.S., 2012. Artificial Intelligence Today. Date Views 17.11.2015 na-journal.ru/stati/83-iskusstvennyj-intellekt-segodnja?format=pdf.
[12] Lapyrova, R., 2012. Neural Networks. Moscow: LAP.
[13] Zorin, A.V., V.A. Zorin, E.V. Proydakova, M.A. Fedotkin et al., 2013. Introduction to General Markov Chains: Study Guide. Nizhny Novgorod: Nizhny Novgorod State University.
[14] Yunusov, T.R., 2007. Mathematical Modeling of Terminal Calculated Networks Based on Fuzzy Time Series. Date Views 17.11. 2015 www.ulstu.ru/main?cmd=file&object=1144.
[15] Afanas'eva, T.A., 2013. Modeling of Fuzzy Time Series Trends. Ulyanovsk: Ulyanovsk State Technical University.
[16] Loskutov, A.Yu., 2014. Time Series Analysis. Date Views 17.11.2015 chaos.phys.msu.ru/loskutov/PDF/Lectures_time_series_analysis.pdf.
[17] Istomin, I.A., O.L. Kotlyarov and A.Yu. Loskutov, 2013. Applications of a Local Approximation Technique for Forecasting of Economic Indicators. Date Views 17.11.2015 ics.org.ru/doc?dir=r&pdf=403.
[18] Beloliptsev, I.I. and S.A. Farkhieva, 2014. Time Series Forecasting on the Basis of the Index of Fractality. World of Science, 3: 1.
[19] Anushina, E.S., 2009. Time Series Forecasting on the Basis of Intelligent Computational Technologies. Intelligent Systems, 1: 84.
[20] Yakovlev, A.A., 2003. Introduction to Wavelet Transformation: Tutorial. Novosibirsk: NSTU.
[21] Seraya, O.V., 2011. Forecasting the Wavelet Approximation of a Time Series. Mathematics and Cybernetics – Fundamental and Applied Aspects, 4: 49.
[22] Vasil'ev, A.A., 2014. Genesis of Hybrid Forecasting Models Based on Combining Forecasts. Bulletin of the Tver State University: Economics and Management, 23.