Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 1 (2016), pp. 147–155

http://www.ripublication.com/gjpam.htm

Inverse Secure Domination in Graphs

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Abstract

In this paper, we show that every integers k, m, and n with $1 \leq k \leq m < n$ is realizable as inverse domination number, inverse secure domination number, and order of G respectively. Further, we give the characterization of the inverse secure dominating set with inverse secure domination number of one and two and give some important results.

AMS subject classification: 05C69.
Keywords: Domination, inverse dominating set, secure dominating set, inverse secure dominating set.

1. Introduction

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [8]. However, it was not until 1977, following an article [2] by Ernie Cockayne and Stephen Hedetniemi, that domination in graphs became an area of study by many researchers. One type of domination parameter is the secure domination in graphs. This

[1]This research is partially funded by the Office of Research University of San Carlos and Cebu Normal University, Philippines.

was studied and introduced by E.J. Cockayne et al. [3, 4, 1]. Recently, Enriquez and Canoy, introduced a new domination parameter, the concept of secure convex domination in graphs [5]. The inverse domination in graph was first found in the paper of Kulli [9] and further read in [6, 10]. Moreover, for the general concepts, the reader may refer to [7]. In this study, we introduce a new domination parameter, the inverse secure domination in graphs, and give some important results.

Let $G = (V(G), E(G))$ be a connected simple graph and $v \in V(G)$. The neighborhood of v is the set $N_G(v) = N(v) = \{u \in V(G) : uv \in E(G)\}$. If $S \subseteq V(G)$, then the *open neighborhood* of S is the set $N_G(S) = N(S) = \bigcup_{v \in S} N_G(v)$. The *closed neighborhood* of S is $N_G[S] = N[S] = S \cup N(S)$. A subset S of $V(G)$ is a *dominating set* of G if for every $v \in (V(G)\backslash S)$, there exists $x \in S$ such that $xv \in E(G)$, i.e., $N[S] = V(G)$. The *domination number* $\gamma(G)$ of G is the smallest cardinality of a dominating set of G.

A dominating set S in G is called a *secure dominating set* in G if for every $u \in V(G) \setminus S$, there exists $v \in S \cap N_G(u)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set. The minimum cardinality of secure dominating set is called the *secure domination number* of G and is denoted by $\gamma_s(G)$. A secure dominating set of cardinality $\gamma_s(G)$ is called γ_s-*set* of G. Let D be a minimum dominating set in G. The dominating set $S \subseteq V(G) \setminus D$ is called an *inverse dominating set* with respect to D. The minimum cardinality of inverse dominating set is called an *inverse domination number* of G and is denoted by $\gamma^{-1}(G)$. An inverse dominating set of cardinality $\gamma^{-1}(G)$ is called γ^{-1}-*set* of G. Motivated by the definition of inverse domination in graph, we define a new domination parameter. Let C be a minimum secure dominating set in G. The secure dominating set $S \subseteq V(G) \setminus C$ is called an *inverse secure dominating set* with respect to C. The minimum cardinality of inverse secure dominating set is called an *inverse secure domination number* of G and is denoted by $\gamma_s^{-1}(G)$. An inverse secure dominating set of cardinality $\gamma_s^{-1}(G)$ is called γ_s^{-1}-*set* of G.

2. Results

One of the classical result in the domination theory which was introduced by Ore in 1962 state the following theorem:

Theorem 2.1. [8] Let G be a graph with no isolated vertex. If $S \subseteq V(G)$ is a γ-*set*, then $V(G) \setminus S$ is also a dominating set in G.

This motivate a new domination parameter, the inverse secure domination in graphs. Theorem 2.1 guarantees the existence of γ_s^{-1}-*set* in some graph G. Since the inverse secure dominating set of any graph G of order n cannot be $V(G)$, it follows that $\gamma_s^{-1}(G) \neq n$ and hence $\gamma_s^{-1}(G) < n$.

Since $\gamma_s^{-1}(G)$ does not always exists in a connected nontrivial graph G, we denote by $\mathcal{G}_s^{-1}$ be a family of all graphs with inverse secure dominating set. Thus, for the purpose of this study, it is assumed that all connected nontrivial graphs considered belong to the family $\mathcal{G}_s^{-1}$. From the definitions, the following result is immediate.

Remark 2.2. Let G be a connected graph of order $n \geq 4$. Then

(i) $1 \leq \gamma_s^{-1}(G) < n$.

(ii) $\gamma(G) \leq \gamma^{-1}(G) \leq \gamma_s^{-1}(G)$; and

The next result says that the value of the parameter γ_s^{-1} ranges over all positive integers.

Theorem 2.3. Given positive integers k and n such that $n \geq 4$ and $1 \leq k < n$, there exists a connected nontrivial graph G with $|V(G)| = n$ and $\gamma_s^{-1}(G) = k$.

Proof. Consider the following cases:

Case 1. Suppose $k = 1$.
Let $G = K_n$. Then, clearly, $|V(G)| = n$ and $\gamma_s^{-1}(G) = 1$.

Case 2. Suppose $k = 2$.
Let $H_1 = K_r$ and $r = n - 1$. Let $a, b \in V(H_1)$ and consider the graph G obtained from H_1 by adding a vertex $v \notin V(H_1)$ and a pendant edge vb (see Figure 1).

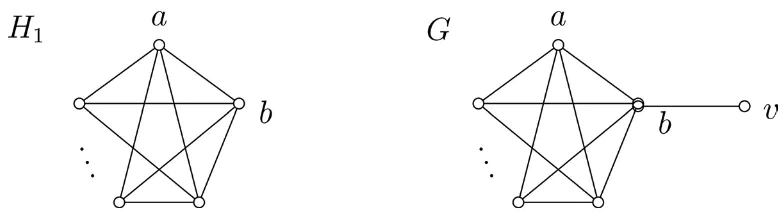

Figure 1: A graph G with $\gamma_s^{-1}(G) = 2$

Since $C = \{a, b\}$ is a γ_s-*set* in G. The set $S = \{v, x\}$ is a γ_s^{-1}-*set* of G for any $x \in V(H_1) \setminus \{a, b\}$. Hence, $|V(G)| = r + 1 = n$ and $\gamma_s^{-1}(G) = 2 = k$.

Case 3. Suppose $3 \leq k < n$.
Let $H_2 = K_r$ $(r \geq 3)$ and $P_m = [a_1, a_2, \ldots, a_m]$ $(m \geq 4)$. Consider the graph G obtained from H_2 by adding the edges va_1, va_2, …, and va_m (see Figure 2).

Subcase 1. Suppose that $k = 3$.
Let $m = 4$. Then the set $C = \{u, v, a_1\}$ is a γ_s-*set* of G and $S = \{a_2, a_3, x\}$ for all $x \in V(H_2) \setminus \{u, v\}$ is a γ_s^{-1}-*set* of G. Thus, $\gamma_s^{-1}(G) = 3 = k$.

Subcase 2. Suppose that $3 < k < n$.
Let $n = r + m$, $r \geq 3$. If $2k = m + 2$ and $m = 6s$ for some $s \in \mathbb{N}$, then set $C = \left\{a_{3j+1} : j = 1, 2, \ldots, \dfrac{m-3}{3}\right\} \cup \{a_m, u, v\}$ is a γ_s-*set* of G and the set $S =$

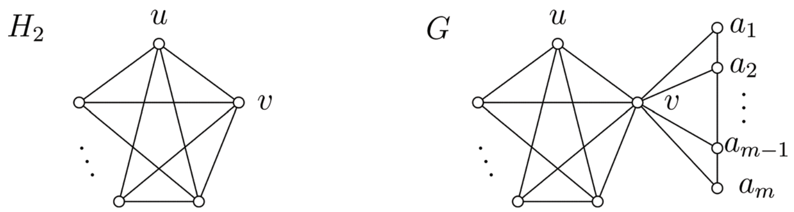

Figure 2: A graph G with $\gamma_s^{-1}(G) = k$

$\left\{a_{3j} : j = 1, 2, \ldots, \frac{m-3}{3}\right\} \cup \left\{a_{6j+2} : j = 1, 2, \ldots, \frac{m-6}{6}\right\} \cup \{a_1, a_{m-1}, x\}$ for all $x \in V(H_2)\setminus\{u, v\}$ is a γ_s^{-1}-*set* of G. Thus, $\gamma_s^{-1}(G) = \frac{m-3}{3} + \frac{m-6}{6} + 3 = \frac{m+2}{2} = k$.

If $2k = m + 3$ and $m = 6s + 1$ for some $s \in \mathbb{N}$, then the set $C = \left\{a_{3j+1} : j = 1, 2, \ldots, \frac{m-1}{3}\right\} \cup \{u, v\}$ is a γ_s-*set* in G and the set $S = \left\{a_{3j-1} : j = 1, 2, \ldots, \frac{m-1}{3}\right\} \cup \left\{a_{6j-3} : j = 1, 2, \ldots, \frac{m-1}{6}\right\} \cup \{a_{m-1}, x\}$ for all $x \in V(H_2) \setminus \{u, v\}$ is a γ_s^{-1}-*set* of G. Thus, $\gamma_s^{-1}(G) = \frac{m-1}{3} + \frac{m-1}{6} + 2 = \frac{m+3}{2} = k$.

If $2k = m + 3$ and $m = 6s + 3$ for some $s \in \mathbb{N}$, then the set $C = \left\{a_{3j+1} : j = 1, 2, \ldots, \frac{m-3}{3}\right\} \cup \{a_m, u, v\}$ is a γ_s-*set* in G and the set $S = \left\{a_{3j-1} : j = 1, 2, \ldots, \frac{m}{3}\right\} \cup \left\{a_{6j} : j = 1, 2, \ldots, \frac{m-3}{6}\right\} \cup \{a_3, x\}$ for all $x \in V(H_2)\setminus \{u, v\}$ is a γ_s^{-1}-*set* of G. Thus, $\gamma_s^{-1}(G) = \frac{m}{3} + \frac{m-3}{6} + 2 = \frac{m+3}{2} = k$.
Morover, $|V(G)| = r + m = n$. This proves the assertion. ■

Theorem 2.4. Given positive integers k, m and n such that $n \geq 4$ and $1 \leq k \leq m < n$, there exists a connected nontrivial graph G with $\gamma^{-1}(G) = k$, $\gamma_s^{-1}(G) = m$, and $|V(G)| = n$.

Proof. Consider the following cases:

Case 1. Suppose $1 = k = m < n$.
Let $G = K_n$. Clearly, $\gamma^{-1}(G) = 1$, $\gamma_s^{-1}(G) = 1$, and $|V(G)| = n$.

Case 2. Suppose $1 = k < m < n$.
Let $G = K_2 + P_r$, $V(K_2) = \{u, v\}$, and $P_r = \{v_1, v_2, \ldots, v_r\}$ where $r \geq 3$ and $n = r + 2$. Then the set $A = \{u\}$ is the γ-*set*, and the set $B = \{v\}$ is the γ^{-1}-*set* of G. Thus, $\gamma^{-1}(G) = 1 = k$. Now, the set $C = \{u, v\}$ is a γ_s-*set* of G. Consider the following subcases:

Subcase 1. Suppose that r is even.
Let $r = 2m$. Then the set $S = \left\{v_{2i} : i = 1, 2, \ldots, \frac{r}{2}\right\}$ is a γ_s^{-1}-*set* of G. Thus, $\gamma_s^{-1}(G) = \frac{r}{2} = m$.

Subcase 2. Suppose that r is odd.
Let $r = 2m + 1$. The set $S = \left\{a_{2i} : i = 1, 2, \ldots, \frac{r-1}{2}\right\}$ is a γ_s^{-1}-*set* of G. Thus, $\gamma_s^{-1}(G) = \frac{r-1}{2} = m$.

Moreover, $|V(G)| = r + 2 = n$.

Case 3. Suppose $1 < k = m < n$.
Let $G = P_m \circ K_2$, where $P_m = [v_1, v_2, \ldots, v_m]$, $V(K_2) = \{a, b\}$, $m \geq 2$, and $n = 3m$. Since the set $D = V(P_m)$ is a γ-*set* of G, it follows that the γ^{-1}-*set* of G is

$$A = \bigcup_{i=1}^{m} (V(H^{v_i}) \setminus \{a\}), \text{ where } H = K_2.$$

Thus,

$$\gamma^{-1}(G) = \sum_{i=1}^{m} |V(H^{v_i}) \setminus \{a\}| = \sum_{i=1}^{m} |b| = m \cdot 1 = m = k.$$

Now, if the γ_s-*set* of G is $C = D$, then the γ_s^{-1}-*set* of G is $S = A$. Hence, $\gamma_s^{-1}(G) = m$. Moreover, $|V(G)| = m + 2m = n$.

Case 4. Suppose $1 < k < m < n$.
Let $G = P_n = [v_1, v_2, \ldots, v_n]$. Consider n is even. Let $n = 2m$. If $n = 6s - 2$ for some $s \in \mathbb{N}$ and $3k = n + 2$, then the set $A = \left\{v_{3i-2} : i = 1, 2, \ldots, \frac{n+2}{3}\right\}$ is a γ^{-1}-*set* of G. Thus, $\gamma^{-1}(G) = \frac{n+2}{3} = k$. If $n = 6s$ for some $s \in \mathbb{N}$ and $3k = n + 3$, then the set $A = \left\{v_{3i-2} : i = 1, 2, \ldots, \frac{n}{3}\right\} \cup \{v_n\}$ is a γ^{-1}-*set* of G. Thus, $\gamma^{-1}(G) = \frac{n}{3} + 1 = k$. If $n = 6s + 2$ for some $s \in \mathbb{N}$ and $3k = n + 1$, then the set $A = \left\{v_{3i-2} : i = 1, 2, \ldots, \frac{n+1}{3}\right\}$ is a γ^{-1}-*set* of G. Thus, $\gamma^{-1}(G) = \frac{n+1}{3} = k$. Moreover, the sets $C = \left\{v_{2i} : i = 1, 2, \ldots, \frac{n}{2}\right\}$ and $S = \left\{v_{2i-1} : i = 1, 2, \ldots, \frac{n}{2}\right\}$ are γ_s-*set* and γ_s^{-1}-*set* of G respectively. Thus, $\gamma_s^{-1}(G) = \frac{n}{2} = m$.

Consider n is odd. Let $n = 2m - 1$. If $n = 6s - 1$ for some $s \in \mathbb{N}$ and $3k = n + 1$, then the set $A = \left\{v_{3i-2} : i = 1, 2, \ldots, \frac{n+1}{3}\right\}$ is a γ^{-1}-*set* of G. Thus, $\gamma^{-1}(G) = \frac{n+1}{3} = k$. If $n = 6s + 1$ for some $s \in \mathbb{N}$ and $3k = n + 2$, then the set $A =$

$\left\{v_{3i-2} : i = 1, 2, \ldots, \dfrac{n+2}{3}\right\}$ is a γ^{-1}-*set* of G. Thus, $\gamma^{-1}(G) = \dfrac{n+2}{3} = k$. If $n = 6s+3$ for some $s \in \mathbb{N}$ and $3k = n+3$, then the set $A = \left\{v_{3i-2} : i = 1, 2, \ldots, \dfrac{n}{3}\right\} \cup \{v_n\}$ is a γ^{-1}-*set* of G. Thus, $\gamma^{-1}(G) = \dfrac{n}{3} + 1 = k$. Moreover, the sets $C = \left\{v_{2i} : i = 1, 2, \ldots, \dfrac{n-1}{2}\right\}$ and $S = \left\{v_{2i-1} : i = 1, 2, \ldots, \dfrac{n+1}{2}\right\}$ are γ_s-*set* and γ_s^{-1}-*set* of G respectively. Thus, $\gamma_s^{-1}(G) = \dfrac{n+1}{2} = m$.

Moreover, $|V(G)| = n$. This proves the assertion. ■

Corollary 2.5. The difference $\gamma_s^{-1} - \gamma^{-1}$ can be made arbitrarily large.

Since $\gamma_s(G)$ is the order of the minimum secure dominating set of G, it follows that $\gamma_s(G) \leq \gamma_s^{-1}(G)$. The following remark holds.

Remark 2.6. Let G ba a connected nontrivial graph of order $n \geq 4$. Then $\gamma_s(G) \leq \gamma_s^{-1}(G)$.

Theorem 2.7. [1] Let G ba a graph of order $n \geq 1$. Then $\gamma_s(G) = 1$ if and only if $G = K_n$.

Theorem 2.8. Let G ba a connected nontrivial graph of order n. Then $\gamma_s^{-1}(G) = 1$ if and only if $G = K_n$.

Proof. Suppose that $\gamma_s^{-1}(G) = 1$. Let $S = \{v\}$ be a γ_s^{-1}-*set* of G. Then $S \subseteq V(G) \setminus C$ where C is a γ_s-*set* in G. In view of Remark 2.6, $\gamma_s(G) \leq \gamma_s^{-1}(G) = 1$. This implies $\gamma_s(G) = 1$. Thus, $G = K_n$ by Theorem 2.7.

For the converse, if $G = K_n$, then $\gamma_s^{-1}(G) = 1$ is immediate. ■

The following result is a quick consequence of Theorem 2.8.

Corollary 2.9. Let G and H be two graphs. Then $\gamma_s^{-1}(G + H) = 1$ if and only if G and H are complete graphs.

We need the following results for our next characterization of the inverse secure domination number of a graph G.

Theorem 2.10. [1] Let G be a connected graph of order $n \geq 3$. Then $\gamma_s(G) = 2$ if and only if G is non-complete and there exists distinct vertices x and y that dominate G and satisfy one of the following conditions:

(i) $N(x) \setminus \{y\} = N(y) \setminus \{x\} = V(G) \setminus \{x, y\}$.

(ii) $\langle N(x) \setminus N[y] \rangle$ and $\langle N(y) \setminus N[x] \rangle$ are complete and for each $u \in N(x) \cap N(y)$ either $\langle (N(x) \setminus N[y]) \cup \{u\} \rangle$ or $\langle (N(y) \setminus N[x]) \cup \{u\} \rangle$ is complete.

(iii) $N(x) \setminus \{y\} = V(G) \setminus \{x, y\}$, $N(x) \setminus N[y] \neq \varnothing$ and $\langle N(x) \setminus N[y] \rangle$ is complete.

Lemma 2.11. Let G be a connected non-complete graph of order $n \geq 4$. If $\gamma_s^{-1}(G) = 2$, then $\gamma_s(G) = 2$.

Proof. Suppose that $\gamma_s^{-1}(G) = 2$. Since $\gamma_s(G) \leq \gamma_s^{-1}(G) = 2$ by Remark 2.6, it follows that either $\gamma_s(G) = 1$ or $\gamma_s(G) = 2$. If $\gamma_s(G) = 1$, then $G = K_n$ by Theorem 2.7. This is contrary to our assumption that G is non-complete. Therfore $\gamma_s(G) = 2$. ■

The next result characterize the inverse secure domination number equal to 2 of a graph G.

Theorem 2.12. Let G be a connected non-complete graph of order $n \geq 4$. Then $\gamma_s^{-1}(G) = 2$ if and only if

(i) there exist distinct vertices x and y such that $\{x\}$ and $\{y\}$ are dominating sets of G and satisfy one of the following conditions:

a) $N(x) \setminus \{y\} = N(y) \setminus \{x\} = V(G) \setminus \{x, y\} = V(H)$ and $\gamma_s(H) = 2$.

b) $\langle N(x) \setminus N[y] \rangle$ and $\langle N(y) \setminus N[x] \rangle$ are complete and for each $u \in N(x) \cap N(y)$ either $\langle (N(x) \setminus N[y]) \cup \{u\} \rangle$ or $\langle (N(y) \setminus N[x]) \cup \{u\} \rangle$ is complete; or

(ii) there exist a vertex x such that $\{x\}$ is a dominating set in G and satisfy the following, $N(x) \setminus \{y\} = V(G) \setminus \{x, y\}$, $N(x) \setminus N[y] \neq \varnothing$ and $\langle N(x) \setminus N[y] \rangle$ is complete.

Proof. Suppose that $\gamma_s^{-1}(G) = 2$. Then $\gamma_s(G) = 2$ by Lemma 2.11. This implies that there exist distinct vertices x and y such that $\{x\}$ and $\{y\}$ are dominating set of G and satisfy Theorem 2.10 (i), (ii) or (iii). Suppose first that Theorem 2.10(i) holds. Let $C = V(K_2) = \{x, y\}$ be a γ_s-*set* of G and let $H = \langle V(G) \setminus C \rangle$. Since $\gamma_s^{-1}(G) = 2$, let $S = \{u, v\}$ be a γ_s^{-1}-*set* of G. Then $S \cap C = \varnothing$. Thus $S \subseteq (V(G) \setminus C) = V(H)$, that is, $\gamma_s^{-1}(H) \leq |S| = 2$. If $\gamma_s^{-1}(H) = 1$, then H is a complete graph by Theorem 2.8. Thus $G = K_2 + H$ is a complete graph contrary to our assumption that G is non-complete. Thus, $\gamma_s^{-1}(H) = 2$. This proves (ia). If Theorem 2.10 (ii) holds, then (ib) holds directly. Since $\gamma_s^{-1}(G) = 2$, let $S = \{x, y\}$ be a γ_s^{-1}-*set* of G. Then $\{x\}$ or $\{y\}$ is a dominating set in G. Since $\gamma_s(G) = 2$, $N(x) \setminus \{y\} = V(G) \setminus \{x, y\}$, $N(x) \setminus N[y] \neq \varnothing$ and $\langle N(x) \setminus N[y] \rangle$ is complete, by Theorem 2.10. This proves (ii).

For the converse, suppose that (i) or (ii) holds. Then $\gamma_s(G) = 2$ by Theorem 2.10. If (i) holds, then there exist distinct vertices x and y such that $\{x\}$ and $\{y\}$ are dominating sets of G and satisfy condition (ia) or (ib). Suppose first that (ia) holds. Let $C = V(K_2) = \{x, y\}$. Then C and $(C \setminus \{x\}) \cup \{u\}$ for all $u \in V(G) \setminus C$ are dominating sets. This implies that C be a γ_s-*set* of G. Since $n \geq 4$, let $a, b \in (V(G) \setminus C) = V(H)$. Since $\gamma_s(H) = 2$, let $S = \{a, b\}$ be a γ_s-*set* of H. Now, $S \cap C = \varnothing$ implies that $S \subseteq (V(G) \setminus C)$ where C is a γ_s-*set* of G. Thus, S is a γ_s^{-1}-*set* of G and hence $\gamma_s^{-1}(G) = 2$.

Suppose that (ib) holds. Since $\gamma_s(G) = 2$, let $C = \{x, y\}$ be a γ_s-*set* in G. Let $a \in N(x) \setminus N[y]$ and $b \in N(y) \setminus N[x]$. Then $S = \{a, b\}$ and $(S \setminus \{a\}) \cup \{z\}$ are dominating sets in G for all $z \in N(x) \setminus N[y]$ or S and $(S \setminus \{b\}) \cup \{w\}$ are dominating sets in G for all $w \in N(y) \setminus N[x]$. Thus, S is a secure dominating set in G. Moreover, $a \in N(x) \setminus N[y]$ and $b \in N(y) \setminus N[x]$ implies that $S \cap C = \varnothing$. Thus, $S \subseteq (V(G) \setminus C)$, where C is a γ_s-*set* in G and hence $\gamma_s^{-1}(G) = 2$.

Finally, suppose that (ii) holds. Since $N(x) \setminus N[y] \neq \varnothing$ and $n \geq 4$, let $a, b \in N(x) \setminus N[y]$ and since $\langle N(x) \setminus N[y] \rangle$ is complete, $ab \in E(G)$. Let $C = \{a, y\}$. Then, $V(G) \setminus \{x, y\} = N(x) \setminus \{y\} = N[C] \setminus \{x, y\}$ implies that $N[C] = V(G)$, that is C is a dominating set of G. Similarly, $C \setminus \{a\} \cup \{b\} = \{y, b\}$ is a dominating sets of G. Thus, C is a secure dominating set of G and hence C is γ_s-*set* since $\gamma_s(G) = 2$ in view Theorem 2.10. Now, $\{x\}$ is a dominating set of G implies that $S = \{x, b\}$ and $S \setminus \{b\} \cup \{a\}$ are dominating sets of G. Thus, S is a secure dominating set of G. Since $S \cap C = \varnothing$, it follows that $S \subseteq (V(G) \setminus C)$ where C is a γ_s-*set* of G. Hence, $\gamma_s^{-1}(G) = 2$. ■

The following result is a direct consequence of Theorem 2.12.

Corollary 2.13. Let G be a connected non-complete graph of order $n \geq 4$. Then $\gamma_s^{-1}(G) = 2$ if $G = \langle \{x, y\} + (K_r \cup K_{n-r-2}) \rangle$.

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