Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 6 (2015), pp. 4789-4799

http://www.ripublication.com/gjpam.htm

On the generalized partially degenerate Genocchi polynomials

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Abstract

In this paper, we consider the partially degenerate Genocchi polynomials and investigate some properties and identities of these polynomials. Futhermore, we define the generalized partially degenerate Genocchi polynomials and give some interesting identities of these polynomials.

AMS subject classification: 28E10, 28E20, 03E72, 26E50 11B68.
Keywords: Euler polynomials, Genocchi polynomials, partially degenerate Genocchi polynomials, p-adic invariant integrals.

1. Introduction

Let p be a fixed odd prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$, and $\mathbb{C}_p$ will denote the ring of p-adic integers, the field of p-adic rational numbers, and the completion of algebraic closure of $\mathbb{Q}_p$, respectively. The p-adic norm $|\cdot|_p$ is normalized as $|p|_p = p^{-1} = \frac{1}{p}$. Let $UD(\mathbb{Z}_p)$ be the space of $\mathbb{C}_p$-valued uniformly differentiable functions on $\mathbb{Z}_p$.

For $f \in UD(\mathbb{Z}_p)$, the p-adic invariant integral on $\mathbb{Z}_p$ is defined as

$$\begin{aligned} I_0(f) &= \int_{\mathbb{Z}_p} f(x)d\mu_0(x) \\ &= \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)\mu_0(x+p^N\mathbb{Z}_p) \\ &= \lim_{N\to\infty} \frac{1}{p^N} \sum_{x=0}^{p^N-1} f(x) \quad \text{(see [2,3,7,8,14,23,27]).} \end{aligned} \tag{1}$$

For $f \in UD(\mathbb{Z}_p)$, the fermionic p-adic integral on $\mathbb{Z}$ is defined by Kim to be

$$\begin{aligned} I_{-1}(f) &= \int_{\mathbb{Z}_p} f(x)d\mu_{-1}(x) = \lim_{N\to\infty} \\ &\times \sum_{x=0}^{p^N-1} f(x)(-1)^x \quad \text{(see [1,4-6,9-19,21,22, 24-27]).} \end{aligned} \tag{2}$$

From (1) and (2), we have

$$I_0(f_n) - I_0(f) = \sum_{l=0}^{n-1} f'(l), \quad (n \in \mathbb{N}). \tag{3}$$

and

$$I_{-1}(f_n) - (-1)^n I_{-1}(f) = 2\sum_{a=0}^{n-1}(-1)^{n-1-a} f(a), \quad (n \in \mathbb{N}). \tag{4}$$

As is well-known, the Bernoulli polynomials are defined by the generating function to be

$$\frac{t}{e^t-1}e^{xt} = \sum_{n=0}^{\infty} B_n(x)\frac{t^n}{n!} \quad \text{(see [3,27]).} \tag{5}$$

and the Euler polynomials are defined by the generating function to be

$$\frac{2}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!} \quad \text{(see [4,7,10, 12, 14, 15-20, 22, 24, 26]).} \tag{6}$$

When $x = 0$, $B_n = B_n(0)$ and $E_n = E_n(0)$ are called the Bernoulli numbers and the Euler numbers, respectively. Recall that the Genocchi polynomials are defined by the generating function to be

$$\frac{2t}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} G_n(x)\frac{t^n}{n!} \quad \text{(see [2,3,5,8,9,11,13-19, 25])}. \tag{7}$$

Recently, Qi-Dolgy-Kim-Ryoo introduced the partially degenerate Bernoulli polynomials which are given by the degenerating function to be

$$\frac{\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^t-1}e^{xt} = \sum_{n=0}^{\infty} B_{n,\lambda}(x)\frac{t^n}{n!} \quad \text{(see [23])}. \tag{8}$$

When $x = 0$, $B_{n,\lambda} = B_{n,\lambda}(0)$ are called partially degenerate Bernoulli numbers. In this paper, we consider the partially degenerate Genocchi polynomials and investigate some properties and identities of these polynomials. Furthermore, we define the generalized partially degenerate Genocchi polynomials and give some interesting identities and properties of these polynomials which are derived from the generating functions and p-adic integral equations.

2. Partially degenerate Genocchi polynomials

In this section, we assume that $\lambda, t \in \mathbb{C}_p$ with $|\lambda t|_p < p^{-\frac{1}{p-1}}$. The partially degenerate Genocchi polynomials are defined by the generating function to be

$$\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^t+1}e^{xt} = \sum_{n=0}^{\infty} G_{n,\lambda}(x)\frac{t^n}{n!}. \tag{9}$$

When $x = 0$, $G_{n,\lambda} = G_{n,\lambda}(0)$ are called the degenerate Genocchi numbers.

Now, we observe that

$$\log(1+x) = \sum_{n=1}^{\infty} -\frac{(-1)^n}{n}x^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}x^{n+1}, \tag{10}$$

and hence

$$\frac{\log(1+\lambda t)}{\lambda t} = \sum_{m=0}^{\infty} \frac{(-1)^m}{m+1}(\lambda t)^m. \tag{11}$$

By (7)and (11), we have

$$\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^t+1}e^{xt} \quad = \quad \frac{\log(1+\lambda t)}{\lambda t}\frac{2t}{e^t+1}e^{xt}$$

$$= \left(\sum_{m=0}^{\infty}\frac{(-1)^m}{m+1}(\lambda t)^m\right)\left(\sum_{l=0}^{\infty}G_l(x)\frac{t^l}{l!}\right)$$
$$= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{m}\frac{(-1)^m}{m+1}m!G_{n-m}(x)\right)\frac{t^n}{n!}. \tag{12}$$

From (9) and (12), we obtain the following theorem.

Theorem 2.1. Let $n \in \mathbb{N} \cup \{0\}$. Then, we have

$$G_{n,\lambda}(x) = \sum_{m=0}^{n}(n)_m m!(-\lambda)^m G_{n-m}(x). \tag{13}$$

It is well known that

$$t\int_{\mathbb{Z}_p} e^{(x+y)t}d\mu_{-1}(y) = \frac{2t}{e^t+1}e^{xt} \tag{14}$$

and

$$\int_{\mathbb{Z}_p}(1+\lambda t)^x d\mu_0(x) = \frac{\log(1+\lambda t)}{\lambda t}. \tag{15}$$

Recall that the Daehee numbers are given by the generating function to be

$$\frac{\log(1+t)}{t} = \sum_{n=0}^{\infty}D_n\frac{t^n}{n!}. \tag{16}$$

By (15) and (16), we have

$$\begin{aligned}
\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^t+1}e^{xt} &= \frac{\log(1+\lambda t)}{\lambda t}\frac{2t}{e^t+1}e^{xt} \\
&= \left(\sum_{m=0}^{\infty}D_m\frac{(\lambda z)^m}{m!}\right)\left(\sum_{n=0}^{\infty}G_n(x)\frac{t^n}{n!}\right) \\
&= \sum_{n=1}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{m}\lambda^m D_m G_{n-m}(x)\right)\frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n+1}\binom{n+1}{m}\lambda^m D_m G_{n-m+1}(x)\right)\frac{t^{n+1}}{(n+1)!} \\
&= t\sum_{n=0}^{\infty}\left(\sum_{m=0}^{n+1}\binom{n+1}{m}\lambda^m D_m\frac{G_{n-m+1}(x)}{n+1}\right)\frac{t^n}{n!}.
\end{aligned} \tag{17}$$

and

$$\frac{1}{t}\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^t+1}e^{xt} = \frac{1}{t}\sum_{n=0}^{\infty}G_{n,\lambda}(x)\frac{t^n}{n!}$$

$$
\begin{aligned}
&= \frac{1}{t}\sum_{n=1}^{\infty} G_{n,\lambda}(x)\frac{t^n}{n!}\\
&= \frac{1}{t}\sum_{n=0}^{\infty} G_{n+1,\lambda}(x)\frac{t^{n+1}}{(n+1)!}\\
&= \sum_{n=0}^{\infty} \frac{G_{n+1,\lambda}(x)}{n+1}\frac{t^n}{n!}. \qquad (18)
\end{aligned}
$$

From (17) and (18), we obtain the following theorem.

Theorem 2.2. Let $n \in \mathbb{N} \cup \{0\}$. Then, we have

$$
G_{m,\lambda}(x) = \sum_{m=0}^{n+1} \binom{n+1}{m} \lambda^m D_{n+1} G_{n+1-m}(x). \qquad (19)
$$

By (14) and (15), we have

$$
\begin{aligned}
\sum_{n=0}^{\infty} G_{n,\lambda}(x)\frac{t^n}{n!} &= t\frac{\log(1+\lambda t)}{\lambda t}\frac{2}{e^t+1}e^{xt}\\
&= t\int_{\mathbb{Z}_p}(1+\lambda t)^z d\mu_0(z)\int_{\mathbb{Z}_p} e^{(x+y)t} d\mu_{-1}(y)\\
&= t\int_{\mathbb{Z}_p}\int_{\mathbb{Z}_p}(1+\lambda t)^{x_1} e^{(x_2+y)t} d\mu_0(x_1)d\mu_{-1}(x_2)\\
&= t\int_{\mathbb{Z}_p}\int_{\mathbb{Z}_p}\left(\sum_{m=0}^{\infty}(x_1)_m\frac{(\lambda t)^m}{m!}\right)\left(\sum_{l=0}^{\infty}(x_2+x)^l\frac{t^l}{l!}\right) d\mu_0(x_1)d\mu_{-1}(x_2)\\
&= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\binom{n}{m}\int_{\mathbb{Z}_p}\int_{\mathbb{Z}_p}(x_1)_m(x_2+x)^{n-m}d\mu_0(x_1)d\mu_{-1}(x_2)\frac{t^{n-1}}{n!}\\
&= \sum_{n=1}^{\infty}\sum_{m=0}^{n-1} n\binom{n-1}{m} D_m E_{n-m-1}(x)\frac{t^n}{n!}. \qquad (20)
\end{aligned}
$$

From (20), we obtain the following theorem.

Theorem 2.3. Let $n \in \mathbb{N} \cup \{0\}$. Then, we have

$$
G_{n,\lambda}(x) = \sum_{m=0}^{n-1} n\binom{n-1}{m} D_m E_{n-m-1}(x). \qquad (21)
$$

3. The generalized partially degenerate Genocchi-polynomials

Let $d \in \mathbb{N}$ with $d \equiv 1(mod 2)$, let χ be a Dirichlet character with conductor d.

The generalized partially degenerate Genocchi polynomials attached to χ which are given by the generating function to be

$$\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^{dt}+1}\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{(a+x)t}=\sum_{n=0}^{\infty}G_{n,\chi,\lambda}(x)\frac{t^n}{n!}. \tag{22}$$

Note that $\lim_{\lambda\to 0}G_{n,\chi,\lambda}(x)=G_{n,\chi}(x)\quad(n\in\mathbb{N}\cup\{0\})$. When $x=0$, $G_{n,\chi,\lambda}=G_{n,\chi,\lambda}(0)$ are called the generalized partially degenerate Genocchi numbers attached to χ. Recall that the generalized Genocchi polynomials are defined by the generating function to be

$$\frac{2t}{e^{dt}+1}\sum_{a=0}^{d-1}\chi(a)e^{(a+x)t}=\sum_{n=0}^{\infty}G_{n,\chi}(x)\frac{t^n}{n!}. \tag{23}$$

When $x=0$, $G_{n,\lambda}=G_{n,\lambda}(0)$ are called the generalized Genocchi numbers. From (22), we have

$$\begin{aligned}
&\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^{dt}+1}\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{(a+x)t}\\
&=\left(\frac{\log(1+\lambda t)}{\lambda t}\right)\left(\frac{2t}{e^{dt}+1}\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{(a+x)t}\right)\\
&=\left(\sum_{l=0}^{\infty}D_l\frac{\lambda^l t^l}{l!}\right)\left(\sum_{m=0}^{\infty}G_{n,\chi}(x)\frac{t^m}{m!}\right)\\
&=\sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}\lambda^l D_l G_{n-l,\chi}(x)\binom{n}{l}\right)\frac{t^n}{n!}.
\end{aligned} \tag{24}$$

Thus from (22) and (24), we obtain the following theorem.

Theorem 3.1. Let $n\in\mathbb{N}\cup\{0\}$. Then, we have

$$G_{n,\chi,\lambda}(x)=\sum_{l=0}^{n}\binom{n}{l}\lambda^l D_l G_{n-l,\chi}(x). \tag{25}$$

Now, we observe that

$$\begin{aligned}
&2\log(1+\lambda)^{\frac{1}{\lambda}}\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{(a+x)t}\\
&=\frac{1}{d}\sum_{a=0}^{d-1}(-1)^a\chi(a)\frac{2\log(1+\lambda t)^{\frac{d}{\lambda}}}{e^{dt}+1}e^{(\frac{a+x}{d})dt}
\end{aligned}$$

$$= \frac{1}{d}\sum_{a=0}^{d-1}\sum_{n=0}^{\infty}(-1)^a\chi(a)G_{n,\frac{\lambda}{d}}(\frac{a+x}{d})\frac{(dt)^n}{n!}$$
$$= \sum_{n=0}^{\infty}\left(d^{n-1}\sum_{a=0}^{d-1}\chi(a)G_{n,\frac{\lambda}{d}}(\frac{a+x}{d})\right)\frac{t^n}{n!}. \tag{26}$$

From (22) and (26), we obtain the following theorem.

Theorem 3.2. Let $n \in \mathbb{N} \cup \{0\}$. Then, we have

$$G_{n,\chi,\lambda}(x) = d^{n-1}\sum_{a=0}^{d-1}(-1)^a\chi(a)G_{n,\frac{\lambda}{d}}(\frac{a+x}{d}). \tag{27}$$

From (3) and (4), we can derive

$$\begin{aligned} & t\int_X\int_X(1+\lambda t)^z\chi(y)e^{(x+y)t}d\mu_0(z)d\mu_{-1}(y) \\ = & \left(\frac{log(1+\lambda t)}{\lambda t}\right)\left(\frac{2t\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{at}}{e^{dt}+1}e^{xt}\right) \\ = & \frac{\log(1+\lambda t)^{\frac{1}{\lambda}}\sum_{a=0}^{d-1}(-1)^a e^{(a+x)t}}{e^{dt}+1} \\ = & \sum_{n=0}^{\infty}G_{n,\chi,\lambda}(x)\frac{t^n}{n!}. \end{aligned} \tag{28}$$

On the other hand,

$$\begin{aligned} & \int_X\int_X(1+\lambda t)^z\chi(y)e^{(x+y)t}d\mu_0(z)d\mu_{-1}(y) \\ = & \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}\binom{n}{l}\lambda^l\int_X(z)_l d\mu_0(z)\int_X\chi(y)(x+y)^{n-l}d\mu_{-1}(y)\right)\frac{t^n}{n!}. \end{aligned} \tag{29}$$

Thus, by (28) and (29), we obtain the following theorem.

Theorem 3.3. Let $n \in \mathbb{N} \cup \{0\}$. Then, we have

$$\sum_{l=0}^{n}\binom{n}{l}\lambda^l\int_X(z)_l d\mu_0(z)\int_X\chi(y)(x+y)^{n-l}d\mu_{-1}(y) = G_{n,\chi,\lambda}(x). \tag{30}$$

where $(z)_l = z(z-1)\cdots(z-l+1)$.

By (22), we can derive

$$\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^{dt}+1}\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{(a+x)t}$$

$$\begin{aligned}
&= \left(\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^{dt}+1}\sum_{a=0}^{d-1}(-1)^a\chi(a)e^{at}\right)e^{xt}\\
&= \left(\sum_{l=0}^{\infty}G_{l,\chi,\lambda}\frac{t^l}{l!}\right)\left(\sum_{m=0}^{\infty}x^m\frac{t^m}{m!}\right)\\
&= \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}\binom{n}{l}G_{l,\chi,\lambda}x^{n-l}\right)\frac{t^n}{n!}. \qquad (31)
\end{aligned}$$

From (22) and (31), we obtain the following theorem.

Theorem 3.4. Let $n \in \mathbb{N} \cup \{0\}$. Then, we have

$$G_{n,\chi,\lambda}(x) = \sum_{l=0}^{n}\binom{n}{l}G_{l,\chi,\lambda}x^{n-l}. \qquad (32)$$

Note that

$$\begin{aligned}
\frac{d}{dx}G_{n,\chi,\lambda}(x) &= \sum_{l=0}^{n}\binom{n}{l}(n-l)G_{l,\chi,\lambda}x^{n-l}\\
&= \sum_{l=0}^{n-1}n\binom{n-1}{l}G_{l,\chi,\lambda}x^{n-l-1}\\
&= nG_{n-1,\chi,\lambda}(x). \qquad (33)
\end{aligned}$$

It is not difficult to show that

$$\begin{aligned}
\frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^t+1}e^{xt} &= \frac{2\log(1+\lambda t)^{\frac{1}{\lambda}}}{e^{dt}+1}\sum_{a=0}^{d-1}e^{(a+x)t}\\
&= \sum_{n=0}^{\infty}\left(d^{n-1}\sum_{a=0}^{d-1}G_{n,\frac{\lambda}{d}}(\frac{a+x}{d})\right)\frac{t^n}{n!}. \qquad (34)
\end{aligned}$$

From (23) and (34), we get

$$G_{n,\lambda}(x) = d^{n-1}\sum_{a=0}^{d-1}G_{n,\frac{\lambda}{d}}(\frac{a+x}{d}) \quad \text{for } n \in \mathbb{N}\cup\{0\}. \qquad (35)$$

From (35), we can derive the following theorem.

Theorem 3.5. Let $k \in \mathbb{N} \cup \{0\}$ and let $\mu_{k,G,\lambda}$ be defined by

$$\mu_{k,G,\lambda}(a+dp^N\mathbb{Z}_p) = (dp^N)^{k-1}G_{n,\frac{\lambda}{dp^N}}\left(\frac{a}{dp^N}\right). \qquad (36)$$

Then $\mu_{k,G,\lambda}$ extends to a $\mathbb{C}_p$-valued distribution on the compact open set $U \subset X$.

Proof. By (35), we get

$$\begin{aligned}
&\sum_{i=0}^{p-1} \mu_{k,G,\lambda}(a + idp^N + dp^{N+1}\mathbb{Z}_p) \\
&= (dp^{N+1})^{k-1} \sum_{i=0}^{p-1} G_{k,\frac{\lambda}{dp^{N+1}}}\left(\frac{a+idp^N}{dp^{N+1}}\right) \\
&= (dp^N)^{k-1} p^{k-1} \sum_{i=0}^{p-1} G_{k,\frac{\lambda}{\frac{dp^N}{p}}}\left(\frac{\frac{a}{dp^N}+i}{p}\right) \\
&= (dp^N)^{k-1} G_{k,\frac{\lambda}{dp^N}}\left(\frac{a}{dp^N}\right) \\
&= \mu_{k,G,\lambda}(a + dp^N\mathbb{Z}_p).
\end{aligned} \tag{37}$$

■

From (37), we note that $\mu_{k,G,\lambda}$ extends to a $\mathbb{C}_p$-valued distribution on the compact set $U \subset X$.

References

[1] S. Araci, M. Açikgöz, *A note on the Frobenius-Euler numbers and polynomials associated with Bernstein polynomials*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **22(3)** (2012) 399–406.

[2] S. Araci, M. Açikgöz, H. Jolany, J. J. Seo, *A unified generating function of the q-Genocchi polynomials with their interpolation functions*, *Proc. Jangjeon Math. Soc.*, **15(2)** (2012) 227–233.

[3] A. Bayad. T. Kim, *Identities for the Bernoulli the Euler and the Genocchi numbers and polynomials*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **20(2)** (2010) 247–253.

[4] A. Bayad. J. Chikhi, *Apostol-Euler polynomials and asymptotics for negative binomial reciprocals*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **24(1)** (2014) 33–37.

[5] I. N. Cangul, V. Kurt, H. Ozden, Y. Simsek, *On the higher-order w-q-Genocchi numbers*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **19(1)** (2009) 39–57.

[6] C. O. Chen, L. Lin, *An inequality for the generalized-Euler-constant function*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **17(1)** (2008) 105–107.

[7] D. Ding, J. Yang, *Some identities related to the Apostol-Euler and Apostol-Bernoulli polynomials*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **20(1)** (2010) 7–21.

[8] S. Gaboury, R. Tremblay, B. J. Fugère, *Some explicit formulas for certain new*

classes of Bernoulli, Euler and Genocchi polynomials, *Proc. Jangjeon Math. Soc.*, **17(1)** (2014) 115–123.

[9] Jang, Lee-Chae, *A study on the distribution of twisted q-Genocchi polynomials*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **18(2)** (2009) 181–189.

[10] J. H. Jeong, J. H. Jin, J. W. Park, S. H. Rim, *On the twisted week q-Euler numbers and polynomials with weight 0*, *Proc. Jangjeon Math. Soc.*, **16(2)** (2013) 157–163.

[11] J. Y. Kang, C. S. Ryoo, *Interpolation function of the multiple twisted q-Genocchi polynomials with weak weight α*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **23(3)** (2013) 491–500.

[12] D. S. Kim, *Symmetry identities for generalized twisted Euler polynomials twisted by unramified roots of unity*, *Proc. Jangjeon Math. Soc.*, **15(3)** (2012) 303–316.

[13] T. Kim, S. H. Rim, D. V. Dolgy, S. H. Lee, *Some identities of Genocchi polynomials arising from Genecchi basis*, *J. Inequal. Appl.*, **2013** (2013:43), 6 pages.

[14] T. Kim, *Some identities for the Bernoulli the Euler and the Genocchi numbers and polynomials*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **20(1)** (2010) 23–28.

[15] T. Kim, *New approach to q-Euler, Genocchi numbers and their interpolation functions*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **18(2)** (2009) 105–112.

[16] T. Kim, *On the multiple q-Genocchi and Euler numbers*, *Russ. J. Math. Phys.*, **15(4)** (2008) 481–486.

[17] T. Kim, *Note on q-Genocchi numbers and polynomials*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **17(1)** (2008) 9–15.

[18] T. Kim, *On the q-extension of Euler and Genocchi numbers*, *J. Math, Anal. Appl.*, **326(2)** (2007) 1458–1465.

[19] T. Kim, L. C. Jang, H. K. Pak, *A note on q-Euler and Genocchi numbers*, *Proc. Japan Acad. Ser. A Math Sci.*, **77(8)** (2001) 139–141.

[20] T. Kim, *Some identities on the q-Euler polynomials of higher order and q-Stiring number by the fermionic p-adic integra $\mathbb{Z}_p$*, *Russ. J. Math. Phys.*, **16(4)** (2009) 484–491.

[21] T. Kim, *Symmetry of power sum polynomials and multivariate fermionic p-adic invariant integral on $\mathbb{Z}_p$*, *Russ. J. Math. phys.*, **16(1)** (2009) 93–96.

[22] Q. M. Luo, *q-analogues of some rusult for the Apostol-Euler polynomials*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **20(1)** (2010) 103–113.

[23] F. Qi, D.V. Dolgy, T. Kim, C.S. Ryoo, *On the partially degenerate Bernoulli polynomials of the first kind*, *Global J. of Pure and Applied Math.*, **11(4)** (2015) 425–438.

[24] S. H. Rim, J. Joung, J. H. Jin, S. J. Lee, *A note on the weighted Carlitz's type q-Euler numbers and q-Bernstein polynomials*, *Proc. Jangjeon Math. Soc.*, **15(2)** (2012) 195–201.

[25] C. S. Ryoo, *A numerical computation on the structure of the roots of (h, q)-extension of Genocchi polynomials associated with p-adic q-inegral on $\mathbb{Z}_p$*, *Proc. Jangjeon Math. Soc.*, **16(3)** (2013) 425–438.

[26] E. Şen, *Theorems on Apostol-Euler polynomials of highr order arising from Euler basis*, *Adv. Stud. Contemp. Math. (Kyngshang)*, **23(2)**(2013) 337–345.

[27] Z. Zhang, H. Yang, *Some closed formulas for generalized Bernoulli-Euler numbers and polynomials*, *Proc. Jangjeon Math. Soc.*, **11(2)**(2008) 191–198.