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Effects Of Free Convection On Steady Flow Through A Vertical Deformable Porous Layer With Constant Heat Source

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Abstract

The flow of a viscous fluid in a vertical deformable porous layer bounded by parallel plates is investigated. The vertical plates $y=0$ and $y=h$ are maintained at constant temperatures T_0 and T_w respectively. A heat source of strength Q_0 is introduced in the deformable layer. The expressions for the solid displacement, the fluid velocity and the temperature distribution are obtained. The effects of various physical parameters such as ϕ^f and η on the velocity and displacement are discussed in detail. It is found that the effect of heat source parameter is to decrease the solid displacement while it has increasing effect on the temperature. Further it is observed that the velocity decreases with the increasing drag in the deformable layer.

Keywords: natural convection; deformable porous layer; heat source.

Nomenclature

Q Volume flow rate
δ Viscous drag
x, y Cartesian coordinates
μ_a the apparent viscosity of the fluid in the porous material
K the drag coefficient

μ the Lame constant

μ_f the coefficient of viscosity

v the flow velocity

U the displacement

h width of the wall

θ Temperature

Gr Grashoff number

T_w Wall Temperature

T_0 Ambient Temperature

ρ Density

Q_0 Heat Source

C_p Specific heat at constant pressure

Nu Nusselt number

$\frac{\partial p}{\partial x}$ the pressure gradient

1. Introduction

The study of deformation in porous materials with coupled fluid movement was initiated by Terzaghi (1925). Later Biot (1941, 1955, 1956) developed a successful theory of soil consolidation and acoustic propagation. Atkin and Craine (1976), Bowen (1980) and Bedford and Drumheller (1983) made important contributions to the theory of mixtures. Applying this theory arterial wall permeability is discussed by Jayaraman (1983). Mow et al. (1984) developed a similar theory for the study of biological tissue mechanics. Appling this theory, Holmes and Mow (1990) analyzed rectilinear cartilages. Barry (1991) made a systematic study on the fluid flow over a deformable porous layer. Farina et al. (1997) discussed moulding processes using the theory of deformable porous media. Ambrosi (2002) studied infiltration phenomena through deformable porous media.

Klubertanz et al. (2003) are investigated multiphase flow in deformable porous media. Irfan Khan (2010) made numerical simulation and analysis of saturated deformable porous media. Sreenadh et al. (2014) studied the Couette flow over a deformable permeable bed. In view of several applications, it will be interesting to study fluid flow through and past deformable porous layers.

Ostrach (1952**)** has studied laminar natural convection flow and heat transfer of fluids with and without heat sources in the channels with constant wall temperatures. Beckerman et al. (1988) studied natural convection in vertical enclosures containing simultaneously fluid and porous layers. Tanda et al. (1988) examined natural convection in partially heated vertical channels. Kolar and Vafai (1988) studied numerically free convection transpiration over a vertical plate. Kim et al. (1989) analyzed natural convection about a vertical plate embedded in porous medium.

Vajravelu et al. (2011) studied the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Sanvicente et al. (2013) investigated the natural convection flow and heat transfer in an open channel. Motivated by the above studies, steady flow and heat transfer in a vertical deformable porous layer is investigated. The solid displacement, the fluid velocity and the temperature distribution are obtained. The results are discussed for various physical parameters.

2. Mathematical formulation

Consider a steady, fully developed flow through a vertical deformable porous layer bounded by rigid walls as shown in Fig. 1. Let the x-axis be taken along one of the plates. The plates are maintained at temperatures T_w and T_0 respectively and the $y-$axis is chosen perpendicular to $x-$axis. A pressure gradient $\frac{\partial p}{\partial x}$ is applied, producing an axially directed flow.

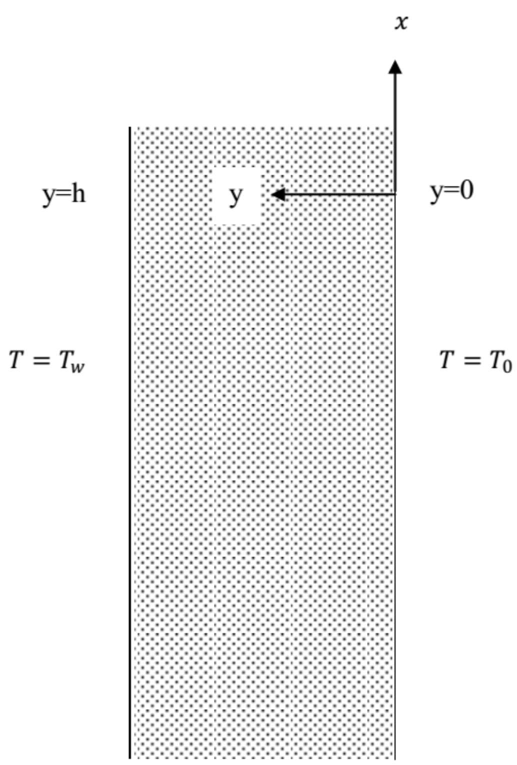

Fig. 1 Physical Model

The basic equations of the problem are

$$\mu \frac{\partial^2 u}{\partial y^2} - (1-\varphi^f)\frac{\partial p}{\partial x} + Kv = 0 \tag{1}$$

$$\frac{1}{\eta}\left(\frac{\mu_f}{\rho}\right)\frac{\partial^2 v}{\partial y^2}-\frac{\varphi^f}{\rho}\frac{\partial p}{\partial x}-\frac{Kv}{\rho}+g\beta(T-T_0)=0 \quad (2)$$

$$K\frac{\partial^2 T}{\partial y^2}+Q_0=0 \quad (3)$$

where μ_a is the apparent viscosity of the fluid in the porous material, K is the drag coefficient, μ is the Lame constant, μ_f is the coefficient of viscosity, v is the flow velocity, Q_0 is the constant heat addition/absorption, u is the solid displacement, $\frac{\partial p}{\partial x}$ is the pressure gradient.

It is convenient to introduce the following non-dimensional quantities:

$$y^* = \frac{y}{h},\ v^* = \frac{v}{U},\ \theta^* = \frac{T-T_0}{T_w-T_0},\ p^* = \frac{hp}{\mu_f U},\ x^* = \frac{x}{h}, u^* = \frac{u\mu}{h^2 G_0},$$

$$\delta = \frac{Kh}{\mu_f}, Gr = \frac{g\beta(T_w-T_0)\rho h^2}{\mu_f U}, \eta = \frac{\mu_f}{2\mu_a}, \beta = Q_0\left(\frac{h^2}{K(T_w-T_0)}\right), \frac{dp}{dx} = G$$

In view of the above dimensionless quantities, the equations (1)-(3) take the following form. The asterisks $(*)$ are neglected hereafter.

$$\frac{d^2u}{dy^2}=(1-\varphi^f)G-\delta v \quad (4)$$

$$\frac{d^2v}{dy^2}-\eta\delta v=(\varphi^f\eta)G-\eta Gr\theta \quad (5)$$

$$\frac{d^2\theta}{dy^2}+\beta=0 \quad (6)$$

The parameter δ is a measure of the viscous drag of the outside fluid relative to drag in the porous medium. The parameter η is the ratio of the bulk fluid viscosity to the apparent fluid viscosity in the porous layer.

The boundary conditions are

$u=0$ at $y=o$

$u=1$ at $y=1$ (7)

$v=0$ at $y=0$

$v=1$ at $y=1$ (8)

$\theta=0$ at $y=0$

$\theta=1$ at $y=1$ (9)

3. Solution of the problem

The governing equations (4) to (6) are coupled differential equations that can be solved by using the boundary conditions (7) and (9). The solid displacement, fluid velocity and temperature in the free flow region and porous regions are obtained as

$$u=\frac{b_1 y^2}{2}-\frac{\delta}{\alpha^2}\left(Ae^{\alpha}+Be^{\alpha} \;-\left\{\frac{a_0 y^4+2a_1 y^3+6a_2 y^2}{12}\right\}\right)+Cy+D \tag{10}$$

$$v=Ae^{\alpha y}+Be^{-\alpha y}-\frac{a_0 y^2+a_1 y+a_2}{\alpha^2} \tag{11}$$

$$\theta=-\beta\frac{y^2}{2}+\left(1+\frac{\beta}{2}\right)y \tag{12}$$

Where

$$\left.\begin{aligned} & b_1=(1-\phi^f)\frac{dp}{dx}, a_0=\frac{\eta Gr\beta}{2}, a_1=-\eta Gr\left(1+\frac{\beta}{2}\right), a_2=\phi^f\eta\frac{dp}{dx}, \\ & A=\frac{1}{\alpha^2}\left(\frac{a_2e^{-\alpha}-(\alpha^2+a_0+a_1+a_2)}{e^{-\alpha}-e^{\alpha}}\right), B=\frac{1}{\alpha^2}\left(\frac{a_2e^{\alpha}-(\alpha^2+a_0+a_1+a_2)}{e^{\alpha}-e^{-\alpha}}\right), \alpha=\sqrt{\delta\eta}, \\ & C=1-\frac{b_1}{2}+\frac{\delta}{\alpha^2}\left(Ae^{\alpha}+Be^{-\alpha} \;-\left\{\frac{a_0+2a_1+6a_2}{12}\right\}\right)-\delta\left(\frac{A+B}{\alpha^2}\right), D=\delta\left(\frac{A+B}{\alpha^2}\right) \end{aligned}\right\} \tag{13}$$

4. Nusselt Number

The Nusselt numbers at the two walls are given by

$$Nu_{1,2}=-\left(\frac{d\theta}{dy}\right)_{y=0,1} \tag{14}$$

5. Volume flow rate

Another interesting phenomenon in this study is to understand the effect of Gr and φ^f on the volume flow rate. It is given by

$$Q=\int_0^1 v\,dy. \tag{15}$$

$$=A\frac{e^{\alpha}}{\alpha}-B\frac{e^{-\alpha}}{\alpha}-\frac{a_0}{36}-\frac{a_1}{24}-a_2-\frac{A}{\alpha}+\frac{B}{\alpha}. \tag{16}$$

where the constants A, B etc. are listed in (13).

6. Results and discussions

In this paper, natural convection effects on the steady flow through a vertical deformable porous layer are investigated and the results are discussed for various physical parameters ϕ^f, Gr, δ and η.

The variation of solid displacement u with y is calculated from equation (10) for different values of β, δ and η and is shown in Figures 1, 2 and 3 for fixed $\phi_f=0.6$, $Gr=5$ and $G=-1$. It is seen that the solid displacement increases with decreasing heat source parameter β or drag δ or the viscous ratio parameter η.

The variation of fluid velocity v with y is calculated from equation (11) for different values of Gr, η and is shown in Figures 4 and 5 for fixed $\beta = 4, G = -1, \varphi^f = 0.6$ and $\delta = 1.0$. It is observed that the velocity increases with the increasing Grashoff number Gr or viscosity ratio parameter η.

The variation of fluid velocity v with y is calculated for different values of φ^f and is shown in Figure 6 for fixed $\beta = 4, G = -1, Gr = 0, \eta = 0.5$ and $\delta = 1.0$. It is observed that the velocity increases with the increasing φ^f.

The variation of fluid velocity v with y is calculated for different values of β and is shown in Figure 7 for fixed $\phi_f = 0.6, \eta = 0.5, G = -1, \delta = 1$ and $Gr = 5$. It is observed that the fluid velocity increases with the increasing β. So the addition of heat to systems increases the fluid velocity whereas the heat absorption ($\beta = -4$) give rise to opposite behavior of fluid velocity.

The variation of fluid velocity v with y is calculated for different values of δ and is shown in Figure 8 for fixed $\phi_f = 0.6, \eta = 0.5, G = -1, \beta = 4$ and $Gr = 5$. It is observed that the fluid velocity decreases with the increasing of drag δ.

The variation of temperature θ with y is calculated from equation (12) for different values of heat source/sink parameter β and is shown in Figure 9. It is observed that the temperature increases with increasing β, that is the presence of heat source increases the temperature in the porous layer.

From Table 1, it is found that the skin friction at the wall $y = 0$ increases with increasing Gr or ϕ^f or η. Opposite behavior is observed on the other plate $y = 1$. Further Volume flow rate Q increases with Gr or ϕ^f or η.

From Table 2, it is noticed that the Nusselt numbers both the walls increases with addition of heat to the system.

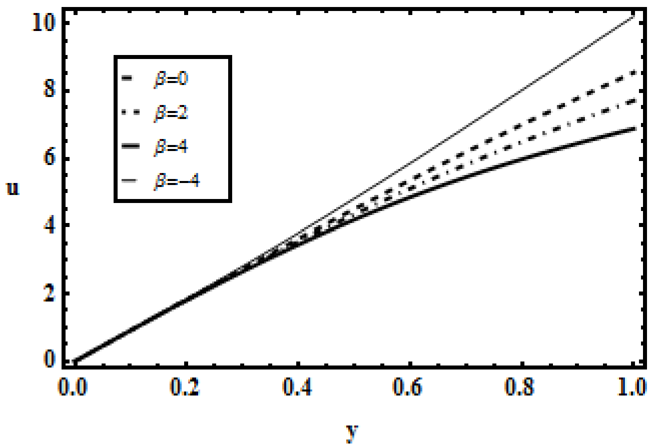

Figure 1. Displacement profiles for different values of β.

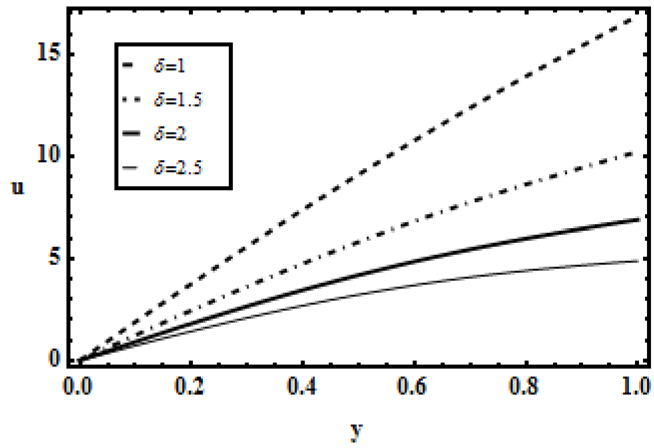

Figure 2. Displacement profiles for different values of δ .

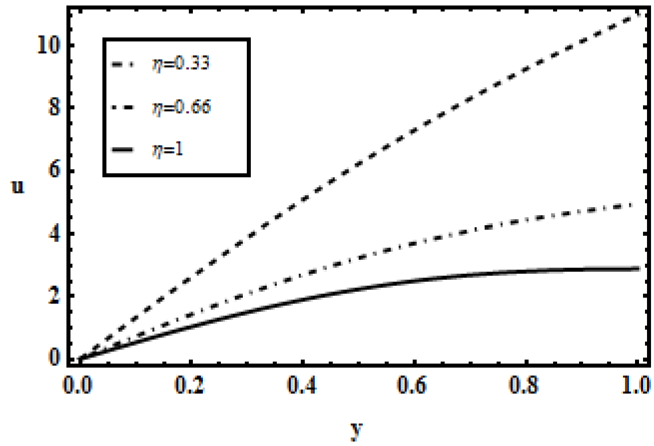

Figure 3. Displacement profiles for different values of η .

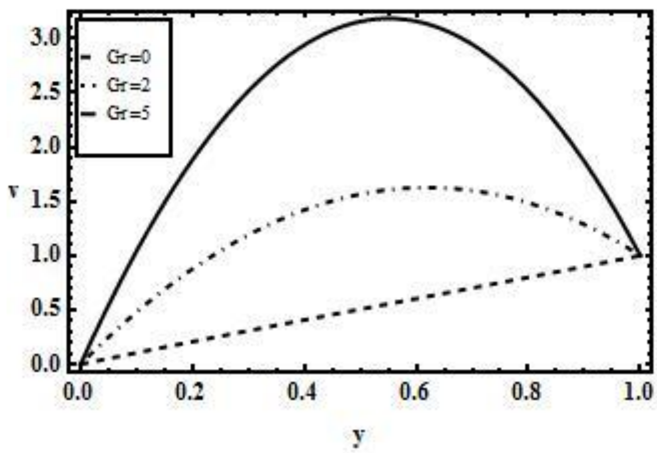

Figure 4. Velocity profiles for different values of Gr .

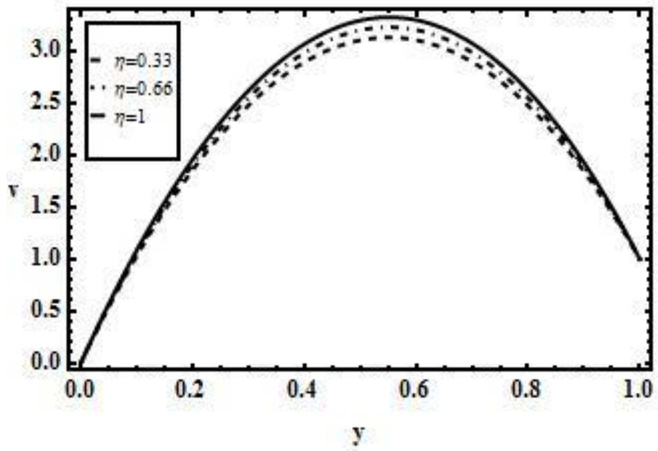

Figure 5. Velocity profiles for different values of η.

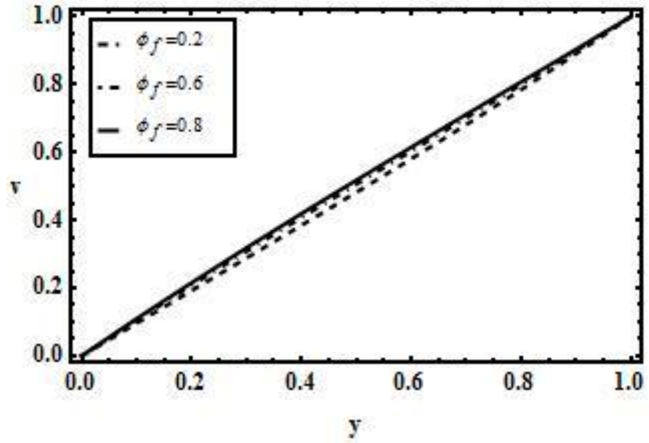

Figure 6. Velocity profiles for different values of φ^f.

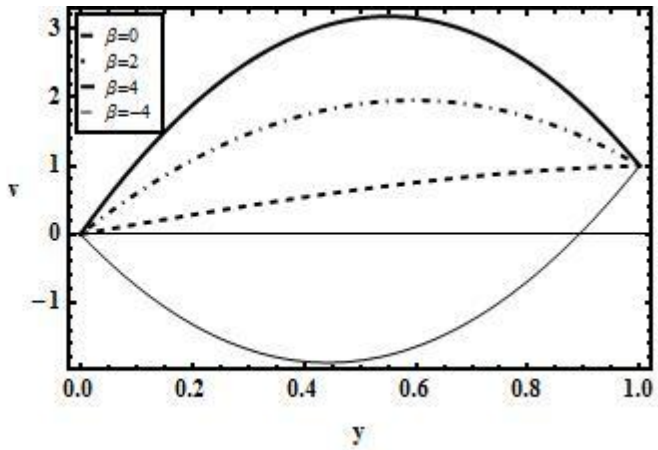

Figure 7. Velocity profiles for different values of β.

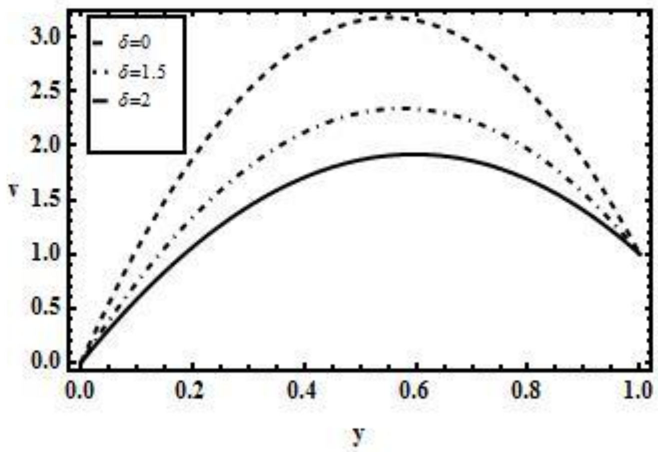

Figure 8. Velocity profiles for different values of δ .

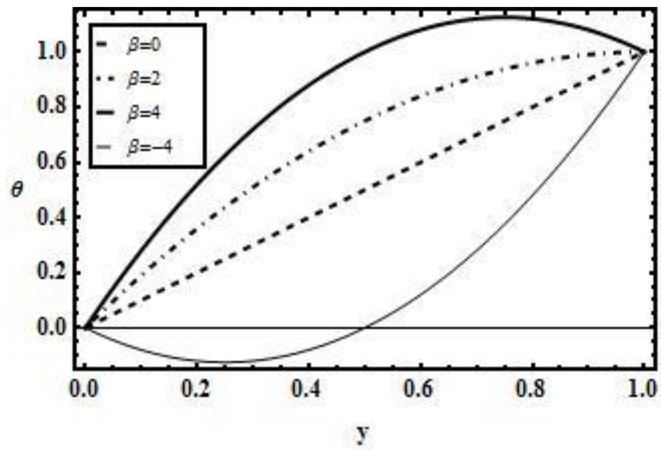

Figure 9. Tempurature profiles for different values of β

Table1: Effect of Gr, ϕ^f and η on skin friction τ_0, τ_1 and Volume fraction Q or fixed values of $\beta = 0, \delta = 2$ and $P = -1$.

Gr	ϕ^f	η	τ_0	τ_1	Q
2	0. 2	0. 333	1. 0315	0. 968432	0. 50520
	0. 4		1. 0631	0. 936865	0. 51407
	0. 8		1. 1262	0. 873732	0. 52081
4	0. 2	0. 666	1. 2518	0. 530942	0. 55878
	0. 4		1. 3119	0. 470866	0. 56857
	0. 8		1. 4320	0. 350798	0. 58817
8	0. 2	1	1. 8936	-0. 86178	0. 722309
	0. 4		1. 9797	-0. 94789	0. 73620
	0. 8		2. 1519	-1. 12008	0. 76399

Table2: Effect of Gr and β on skin friction τ_0, τ_1, Volume fraction Q and Nusselt numbers or fixed values of $\delta = 2, \varphi^f = 0.5, \eta = 0.5$ and $P = -1$.

Gr	β	τ_0	τ_1	Q	Nu_0	Nu_1
2	-2	0. 11552	1. 88447	0. 35227	0	-2
	-1	0. 61552	1. 38447	0. 43508	0. 5	-1. 5
	0	1. 11553	0. 88447	0. 51894	1	-1
	1	1. 61553	0. 38443	0. 60227	1. 5	-0. 5
	2	2. 11553	-0. 11552	0. 68560	2	0
4	-2	-0. 73538	2. 57143	0. 22349	0	-2
	-1	0. 26461	1. 57143	0. 39015	0. 5	-1. 5
	0	1. 26461	0. 571436	0. 55682	1	-1
	1	2. 26461	-0. 42856	0. 72349	1. 5	-0. 5
	2	3. 26461	-1. 42856	0. 89015	2	0
8	-2	-2. 43723	3. 94537	-0. 03407	0	-2
	-1	-0. 43722	1. 94537	0. 29925	0. 5	-1. 5
	0	1. 56277	-0. 05462	0. 63259	1	-1
	1	3. 56277	-2. 05462	0. 96592	1. 5	-0. 5
	2	5. 56277	-3. 05462	1. 29926	2	0

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