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Integral solutions of the non homogeneous quintic equation with four unknowns $(x-y)(x^2+y^2) - (x^2-xy+y^2)z = 22zw^4$

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ABSTRACT:

This paper concerns with the problem of determining non-zero distinct integer solutions to the quintic equation with four unknowns given by $(x-y)(x^2+y^2)-(x^2-xy+y^2)z = 22zw^4$. Employing the method of factorization ten different choices of integer solutions to the above equation are obtained. Also a few interesting relations among the solutions are presented.

KEY WORDS: Quintic equation with four unknowns, integer solutions, Polygonal numbers, Pyramidal numbers, Jacobsthal number, Kynea number.

SUBJECT CLASSIFICATION: M.SC.,2000 subject classification:11D41

NOTATIONS:

$t_{m,n} = n[1+\frac{(n-1)(m-2)}{2}]$	Polygonal number of rank n with sides m
$P_n^r = \frac{1}{6}n(n+1)[(r-2)n+(5-r)]$	Pyramidal number of rank n with sides r
$Pt_n = \frac{n(n+1)(n+2)(n+3)}{24}$	Pentatope number of rank n
$SO_n = n(2n^2-1)$	Stella Octangula number of rank n
$\Pr_n = n(n+1)$	Pronic number of rank n
$gn_a = 2a-1$	Gnomonic number of rank a
$J_n = \frac{1}{3}[2^n-(-1)^n]$	Jacobsthal number of rank n
$j_n = 2^n+(-1)^n$	Jacobsthal Lucas number of rank n
$Ky_n = (2^n+1)^2-2$	Kynea number of rank n

INTRODUCTION:

Diophantine equations, homogeneous and non-homogeneous, have aroused the interest of numerous mathematicians since antiquity. The problem of finding all integer solutions of a Diophantine equation with three or more variables and degree at least three, in general, presents a good deal of difficulties. There is a vast general theory of homogeneous quadratic equations with three variables. Cubic equations in two variables fall into the theory of elliptic curves which is a very developed theory but still an important topic of current research. A lot is known about equations in two variables in higher degrees. For equations with more than three variables and degree at least three, very little is known. It is worth to note that undesirability appears in equations, even perhaps at degree four with fairly small coefficients. Also, one may refer [1-8] for a few Diophantine equations of degree five in which general form of integral solutions has been exhibited. The above results have motivated us to search for integral solutions in solving higher order Diophantine equations. Thus, in this communication, yet another non-homogeneous Diophantine equation of degree five with four unknowns represented by $(x-y)(x^2+y^2)-(x^2-xy+y^2)z = 22zw^4$ is analyzed for its non-zero integral solutions. A few interesting relations between the values of x, y, z, w and special numbers are presented.

METHOD OF ANALYSIS:

The quintic equation with four unknowns under consideration is

$$(x-y)(x^2+y^2)-(x^2-xy+y^2)z = 22zw^4 \quad (1)$$

Introducing the linear transformations

$$x = u+v,\ y = u-v,\ z = 2v \quad (2)$$

in (1), it is written as

$$u^2 - v^2 = 22w^4 \quad (3)$$

We solve (3) through different approaches and thus obtain different choices of integer solutions to (1) which are illustrated.

Approach 1:

Equation (3) is written as the system of double equations in 7 ways as follows:

System (1):

$u + v = w^4$ & $u - v = 22$.

System (2):

$u + v = 2w^4$ & $u - v = 11$.

System (3):

$u + v = w^3$ & $u - v = 22w$.

System (4)
$u + v = 11w^4$ & $u – v = 2$.

System (5)
$u + v = 11w^3$ & $u – v = 2w$.

System (6)
$u + v = 11w^2$ & $u – v = 2w^2$.

System (7)
$u + v = 11w$ & $u – v = 2w^3$.

Solving each of the above system of equations and employing (2), the corresponding integer solutions obtained are presented below.

Solutions for system (1):
$x = 16k^4$, $y = 22$, $z = 16k^4 – 22$, $w = 2k$.

Properties:
(i) Each of the following expressions represents a nasty number
(a) $6 (z+2w^2+23)$
(b) $6(y+z)$
(c) $3[y + z + w^3 - 32t_{3,k}^2 + 12P_k^4 - 12Pr_k]$
(ii) $z^3 + y^3 - x^3 = -3xyz$
(iii) y + z = a quartic number
(iv) (x + y + z) k = a quintic integer
(v) $24Pt_k + t_{6,k}^2 + (SO_k * gn_k) + 7Pr_k^2 - 7Pr_k - 12P_k^5 - z - w + 22$ = a perfect square
(vi) (x +y) z = difference between two perfect squares.

For simplicity, the integer solutions obtained from system(2) to system(7), each satisfying (1), are presented below in the following table.

System No.	x	y	z	w
2	$32k^4$	11	$32k^4$-11	2k
3	$8k^3$	44k	$8k^3$-44k	2k
4	$176k^4$	2	$176k^{4-}2$	2k
5	$88k^3$	4k	$88k^3$-4k	2k
6	44k	$8k^2$	44k-$8k^2$	2k
7	22k	$16k^3$	22k-$16k^3$	2k

Approach 2:
Note that (3) is satisfied by

$$u = 22p^2 + q^2, v = 22p^2 – q^2, w^2 = 2pq \quad (4)$$

Replacing p by 2^{2k-1}q and using (2), the corresponding integer solution to (1) are given by

$x = 11.2^{4k}.q^2$
$y = 2q^2$
$z = 11.2^{4k}.q^{2-}2q^2$
$w = 2^k q$

Properties:

1. Each of the following expressions represents a nasty number

(i) $66(y+z)$

(ii) $\left(\frac{22w^4}{y} - z\right)$

(iii) 3 (x – z)

2. $\frac{2w^2}{y} + 1 = j_{2k}$

3. $\frac{2w^2}{y} - 1 = 3J_{2k}$

4. $xy = 11w^4$

5. $x^3 - 3xyz = y^3 + z^3$

6. $2x + 44w^2 - 11y = 11y.Ky_{2k}$

Approach 3:

Assume $u = a^2 + 22b^2$ (5)

Using (5) in (3) and applying the method of factorization, define

$v + i\sqrt{22}\, w^2 = (a + i\sqrt{22}\, b)^2$

Equating the real and imaginary parts, we've

$v = a^2 - 22b^2,$
$w^2 = 2ab$ (6)

Replacing 'a' by 2^{2k-1}b in (5) & (6) and applying (2), we have

$u = (2^{4k-2} + 22)\, b^2$; $v = (2^{4k-2} - 22)\, b^2$; $w = 2^k b$

and hence the integer solutions of (1) are obtained as

$x = 2^{4k-1}b^2$
$y = 44b^2$
$z = (2^{4k-1} - 44)\, b^2$
$w = 2^k b$

Properties:

1. Each of the following expressions represents a nasty number

(i) 12(z+y)

(ii) 12x

(iii) 66y

2. $\frac{2w^2}{b^2} = 2.2^{2k}$
3. 88z + 87y + $88w^2$ = y ky_{2k}
4. $\frac{22(2x+w^2)}{y} = t_{3,2^{2k}}$
5. $\frac{22*44(2x+w^2)w^2}{y^2} = P^5_{2^{2k}}$
6. $\frac{44(2x+w^2)(22w^2+y)}{y^2} = 3P^3_{2^{2k}}$

Approach 4:

Equation (3) is written as

$$v^2 + 22w^4 = u^2 = u^2 * 1 \quad (7)$$

write 1 as

$$1 = \frac{(5n^2-21n+25+i2n\sqrt{22})(5n^2-21n+25-i2n\sqrt{22})}{(2n^2+11)^2} \quad (8)$$

Substituting (5), (8) in (7) and define

$$v + i\sqrt{22}w^2 = (a + i\sqrt{22}b)^2 \frac{(5n^2-21n+25+i2n\sqrt{22})}{(2n^2+11)} \quad (9)$$

For simplicity and clear understanding let us take n =1.
On equating the real and imaginary parts in (9), we get

we get $$v = \frac{9}{13}(a^2 - 22b^2) - \frac{88}{13}ab \quad (10)$$

$$w^2 = \frac{18ab}{13} + \frac{2}{13}(a^2 - 22b^2) \quad (11)$$

The R.H.S of (11) is a perfect square when

(i) $a = 4P^2 + 143Q^2$

(ii) $a = -22P^2 - 26Q^2$

Considering choice (i), the corresponding integer solutions to (1) are given by
x = $48334Q^4$, y = $208P^4$, z =$-208P^4+48334Q^4$, w = 26PQ

For choice (ii) , the corresponding integer solutions to (1) are obtained as
x = $1144P^4$, y = $8788Q^4$, z = $1144P^4-8788Q^4$, w = 26PQ

CONCLUSION :

In this paper, we've presented infinitely many non-zero distinct integer solutions to the quintic equation with four unknowns given by $(x-y)(x^2+y^2)-(x^2-xy+y^2)z = 22zw^4$. As the quintic equations are rich in variety, one may attempt to find integer solutions for quintic equations with four or more variables along with suitable properties.

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