Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 6 (2015), pp. 4691-4702

http://www.ripublication.com

Point and Interval Estimation Based on Progressive First-Failure Times Censored Samples Under Modified Weibull Distribution with application

Saieed F.Ateya[1, 2] and Mohamed M. Rizk[1,3]

[1] *Mathematics & Statistics Department,*
Faculty of Science, Taif University, Taif, Saudi Arabia.
[2] *Department of Mathematics, Faculty of Science, Assiut University, Egypt.*
[3] *Mathematics Department,*
Faculty of Science, Menoufia University, Shebin El-Kom, Egypt.

Abstract

In this paper, point and interval estimates for the parameters of the modified Weibull (MW) distribution are studied based on progressive first-failure censored data. The Bayes estimates are computed based on squared error (SE) and Linex loss functions and using Markov Chain Monte Carlo ($MCMC$) algorithm. Also, based on this censoring scheme, the interval estimation problem of the parameters of MW distribution have been studied. A Monte Carlo simulation study is carried out to compare the performances of the different methods by computing the mean squared errors ($MSE's$). Finally, point and interval estimation of all parameters are studied based on this real data set as illustrative example.

Keywords: Modified Weibull distribution; Progressive first-failure censoring; Markov Chain Monte Carlo method.

2010 Mathematics Subject Classification: 62F10; 62F15; 62N01; 62N02

1 Introduction

The Weibull distribution is one of the most popular and widely used models of failure time in life testing and reliability theory. The Weibull distribution has been shown to be useful for modeling and analysis of life time data in medical, biological and engineering sciences. Applications of the Weibull distribution in various fields are given in Zaharim et al. [20], Gotoh et al. [5], Shamilov et al. [13], Vicen-Bueno et al.

[18], Niola et al. [12] and Green et al. [6]. A great deal of research has been done on estimating the parameters of the Weibull distribution using both classical and Bayesian techniques, and a very good summary of this work can be found in Johnson et al. [9]. Hossain and Zimmer [7] have discussed some comparisons of estimation methods for Weibull parameters using complete and censored samples. Jaheen and Harbi[8] studied the Bayesian estimation of the exponentiated Weibull distribution using Markov chain Monte Carlo simulation. The modified Weibull distribution was proposed by Lai et al. [10] as a new lifetime distribution. They have shown the capability of the model for modeling a bathtub-shaped hazard-rate function. In addition, they characterized the model through the Weibull plot paper. Further, they have shown that the modified Weibull model compares well with other competing models to fit data that exhibit a bathtub-shaped hazard-rate function. Sultan [14] studied the record values from the modified Weibull distribution and studied its applications. Ateya and Alharthi[2,3] studied the estimation problem under a finite mixture of MW distribution using the traditional maximum likelihood method and using the EM algorithm. Vasile et al. [17] used the Bayes method to estimate the parameters of the modified Weibull distribution and Upadhyaya and Gupta [15] studied the Bayes analysis of the modified Weibull distribution using MCMC simulation. Ateya[1] study the estimation problem under a censored sample of generalized order statistics from MW distribution.

A random variable X is said to have a MW distribution with the parameters β, τ and λ if its probability density function (pdf) is given by

$$f(x \mid \beta, \tau, \lambda) = \tau(\beta + \lambda x)x^{\beta-1}\exp(\lambda x)\exp(-\tau x^{\beta} e^{\lambda x}), x \geq 0, (\tau > 0, \beta \geq 0, \lambda \geq 0). \quad (1.1)$$

The cumulative distribution function (cdf) of this distribution can be written as

$$F(x \mid \beta, \tau, \lambda) = 1 - \exp(-\tau x^{\beta} e^{\lambda x}). \quad (1.2)$$

2 A Progressive First-Failure Censoring Scheme

In this section, the first-failure censoring is combined with progressive censoring scheme as in Wu and Kus [19]. Suppose that n independent groups with k items within each group are put on life test. R_1 groups and the group in which the first failure is observed are randomly removed from the test as soon as the first failure $X^{R}_{1;m,n,k}$ has occurred, R_2 groups and the group in which the second failure is observed are randomly removed from the test as soon as the second failure $X^{R}_{2;m,n,k}$ has occurred, and finally when the mth failure $X^{R}_{m;m,n,k}$ is observed, the remaining groups R_m are removed from the test. Then $X^{R}_{1;m,n,k} < X^{R}_{2;m,n,k} < \ldots < X^{R}_{m;m,n,k}$ are called progressively first-failure censored order statistics with the progressive censored scheme $R = (R_1, R_2, ..., R_m)$. It is clear that $n = m + \sum_{i=1}^{m} R_i$. If the failure times of the $n \times k$ items originally in the test are from a continuous population with cdf $F(x)$ and pdf $f(x)$, the joint pdf for $X^R_{1;m,n,k}, X^R_{2;m,n,k}, ..., X^R_{m;m,n,k}$ is given by Wu and Kuş [19] as follows

$$f_{1,2,...,m}(X^R_{1;m,n,k}, X^R_{2;m,n,k}, ..., X^R_{m;m,n,k}) = Ak^m \prod_{i=1}^{m} f(x^R_{i;m,n,k})[1-F(x^R_{i;m,n,k})]^{k(R_i+1)-1}, \quad (2.1)$$

$$0 < x^R_{1;m,n,k} < x^R_{2;m,n,k} < ... < x^R_{m;m,n,k} < \infty,$$

where

$$A = n(n-R_1-1)(n-R_1-R_2-2)... (n-R_1-R_2-...-R_{m-1}-m+1). \quad (2.2)$$

This censoring scheme has advantages in terms of reducing test time, in which more items are used but only m of $n \times k$ items are failures. Note that using the above notation, some censoring rules can be accommodated such as the first-failure censored order statistics when $R = (0,0,...,0)$, a progressive type-II censored order statistics when $k=1$, a usual type-II censored order statistics when $k=1$ and $R=(0,0,...,n-m)$, and a complete sample if $k=1$ and $R=(0,0,...,0)$, with $n=m$. Also, it should be noted that the progressive first-failure censored sample $X^R_{1;m,n,k}, X^R_{2;m,n,k}, ..., X^R_{m;m,n,k}$ with cdf $F(x)$, can be viewed as a progressive type-II censored sample from a population with cdf $1-(1-F(x))^k$.

3 Maximum Likelihood Estimation

Let $X_i = X^R_{i;m,n,k}$, $i = 1,2,...,m$, be the progressive first-failure censored order statistics from MW distribution with censored scheme $R = (R_1, R_2, ..., R_m)$ and its realization denoted by $x^R_{i;m,n,k}$, $i = 1,2,...,m$ which can be written for simplicity as $\underline{x} = (x_1, ..., x_m)$. The likelihood function of the parameters β, τ and λ given the vector of observations $\underline{x}$ can be obtained by substituting from (1.1) and (1.2) in (2.1) to be of the form

$$L(\beta,\tau,\lambda \mid \underline{x}) \propto \tau^m \prod_{i=1}^{m} [(\beta+\lambda x_i) x_i^{\beta-1} \exp(\lambda x_i) \exp[-k(R_i+1)\tau x_i^{\beta} e^{\lambda x_i}]], \quad \beta>0, \tau>0, \lambda>0. \quad (3.1)$$

By taking the natural logarithm for the likelihood function (3.1), differentiating with respect to all parameters and then setting to zero, three nonlinear equations will be obtained. By solving these nonlinear equations numerically, the maximum likelihood estimates $(MLE's)$ of all parameters have been obtained.

4 Bayesian Estimation

Suppose that the prior belief of the experimenter is measured by the trivariate prior suggested by Ateya[1] which of the form

$$\pi(\beta,\tau,\lambda) \propto \frac{1}{\Gamma(\beta)} \beta^{c_1+c_3-1} \tau^{\beta+c_3-1} \lambda^{\beta-1} \exp[-\beta(\tau+c_2)-\tau\lambda], \quad \beta>0, \tau>0, \lambda>0, (c_1>0, c_2>0, c_3>0), \tag{4.1}$$

where c_1, c_2 and c_3 are the prior parameters (also known as hyperparameters).

Therefore, the joint posterior pdf of the parameters β, τ and λ can be obtained from (4.1) and (3.1) in the form

$$\pi^*(\beta,\tau,\lambda \mid \underline{x}) = \frac{A}{\Gamma(\beta)} \beta^{c_1+c_3-1} \tau^{\beta+c_3+m-1} \lambda^{\beta-1} \exp[-\beta(\tau+c_2)-\tau\lambda] \prod_{i=1}^{m} [(\beta+\lambda x_i) x_i^{\beta-1} \exp(\lambda x_i) \exp[-k(R_i+1)\tau x_i^{\beta} e^{\lambda x_i}]], \tag{4.2}$$

where A is a normalizing constant.

Using the MCMC method, the Bayes estimate of any function $\eta(\beta,\tau,\lambda)$ under SE and Linex loss functions are given, respectively, by

$$\hat{\eta}_{BS} = \frac{1}{N-M} \sum_{i=M+1}^{N} \eta(\beta_i, \tau_i, \lambda_i), \tag{4.3}$$

and

$$\hat{\eta}_{BL} = -\frac{1}{a} \ln[\frac{1}{N-M} \sum_{i=M+1}^{N} \exp(-a\eta(\beta_i, \tau_i, \lambda_i))], \tag{4.4}$$

where β_i, τ_i and λ_i are generated from the posterior pdf, M is the burn-in period (that is, a number of iterations before the stationary distribution is achieved) and a is a constant.

For more details about MCMC methods, see, for example, Upadhyaya et al.[16] and Upadhyaya and Gupta[15]. The Gibbs is an algorithm for simulating from the full conditional posterior distributions while the Metropolis-Hatings algorithm generate sampling from an (essentially) arbitrary proposal distribution (i.e., a Markov transition kernel).

5 Interval Estimation

In this section, we will study the approximate confidence interval (ACI), credibility interval (CI) and highest posterior density interval (HPD) for the two parameters β, τ and λ.

5.1 Approximate confidence interval

Let $X_{1;m,n,k}^{R} < X_{2;m,n,k}^{R} < \ldots < X_{m;m,n,k}^{R}$ denote a progressive first-failure censored sample from MW distribution with parameters β, τ and λ. In this section, the approximate confidence intervals for the parameters of MW distribution are obtained based on progressive first-failure censored using the Fisher information matrix $I(\beta,\tau,\lambda)$ which can be estimated by $I(\hat{\beta},\hat{\tau},\hat{\lambda})$ in the form

$$I(\hat{\beta},\hat{\tau},\hat{\lambda}) = \begin{bmatrix} -\frac{\partial^2 \ell}{\partial \beta^2} & -\frac{\partial^2 \ell}{\partial \beta \partial \tau} & -\frac{\partial^2 \ell}{\partial \beta \partial \lambda} \\ -\frac{\partial^2 \ell}{\partial \beta \partial \tau} & -\frac{\partial^2 \ell}{\partial \tau^2} & -\frac{\partial^2 \ell}{\partial \tau \partial \lambda} \\ -\frac{\partial^2 \ell}{\partial \beta \partial \lambda} & -\frac{\partial^2 \ell}{\partial \tau \partial \lambda} & -\frac{\partial^2 \ell}{\partial \lambda^2} \end{bmatrix}_{(\hat{\beta},\hat{\tau},\hat{\lambda})}, \tag{5.1}$$

where ℓ is the log likelihood of the parameters β,τ and λ.

The Approximate confidence intervals for β,τ and λ can be obtained, respectively, by

$$\hat{\beta} \mp z_{\frac{\alpha}{2}}\sqrt{\nu_{11}} \quad \hat{\tau} \mp z_{\frac{\alpha}{2}}\sqrt{\nu_{22}} \quad \text{and} \quad \hat{\lambda} \mp z_{\frac{\alpha}{2}}\sqrt{\nu_{33}}, \tag{5.2}$$

where ν_{11}, ν_{22} and ν_{33} are the elements on the main diagonal of the covariance matrix $I^{-1}(\hat{\beta},\hat{\tau},\hat{\lambda})$ and $z_{\frac{\alpha}{2}}$ is the standard normal variate.

5.2 Credibility interval

For a specified value of α, $(1-\alpha)\times 100\%$ CI (L_β, U_β) for β, $(1-\alpha)\times 100\%$ CI (L_τ, U_τ) for τ and $(1-\alpha)\times 100\%$ CI (L_λ, U_λ) for λ have been defined, respectively by

$$\begin{aligned} &\int_{L_\beta}^{\infty} \pi_1^*(\beta \mid \underline{x})d\beta = 1-\frac{\alpha}{2}, \quad \int_{U_\beta}^{\infty} \pi_1^*(\beta \mid \underline{x})d\beta = \frac{\alpha}{2}, \\ &\int_{L_\tau}^{\infty} \pi_2^*(\tau \mid \underline{x})d\tau = 1-\frac{\alpha}{2}, \quad \int_{U_\tau}^{\infty} \pi_2^*(\tau \mid \underline{x})d\tau = \frac{\alpha}{2}, \\ &\int_{L_\lambda}^{\infty} \pi_3^*(\lambda \mid \underline{x})d\lambda = 1-\frac{\alpha}{2}, \quad \int_{U_\lambda}^{\infty} \pi_3^*(\lambda \mid \underline{x})d\lambda = \frac{\alpha}{2}, \end{aligned} \tag{5.3}$$

where $\pi_1^*(\beta \mid \underline{x})$, $\pi_2^*(\tau \mid \underline{x})$ and $\pi_3^*(\lambda \mid \underline{x})$ are the marginal density functions of β, τ and λ, respectively. In many cases it will be very difficult to obtain the marginal pdf from the posterior pdf. So, we will use Gibbs sampler and Metropolis Hastings algorithms to generate $(\beta_1,\tau_1,\lambda_1),(\beta_2,\tau_2,\lambda_2),\ldots,(\beta_N,\tau_N,\lambda_N)$ from $\pi^*(\beta,\tau,\lambda \mid \underline{x})$.

Using these generated values of β, τ and λ, we have

$$\pi_1^*(\beta \mid \underset{-}{x}) = \frac{1}{N}\sum_{i=1}^{N}\pi^*(\beta, \tau_i, \lambda_i \mid \underset{-}{x}),$$
$$\pi_2^*(\tau \mid \underset{-}{x}) = \frac{1}{N}\sum_{i=1}^{N}\pi^*(\tau, \beta_i, \lambda_i \mid \underset{-}{x}), \tag{5.4}$$
$$\pi_3^*(\lambda \mid \underset{-}{x}) = \frac{1}{N}\sum_{i=1}^{N}\pi^*(\lambda, \beta_i, \tau_i \mid \underset{-}{x}).$$

Substituting from (5.4) in (5.3), simple formulas have been obtained to compute the credibility intervals for β, τ and λ in the following form

$$\frac{1}{N}\sum_{i=1}^{N}\int_{L_\beta}^{\infty}\pi^*(\beta, \tau_i, \lambda_i \mid \underset{-}{x})d\beta = 1-\frac{\alpha}{2}, \quad \frac{1}{N}\sum_{i=1}^{N}\int_{U_\beta}^{\infty}\pi^*(\beta, \tau_i, \lambda_i \mid \underset{-}{x})d\beta = \frac{\alpha}{2}$$
$$\frac{1}{N}\sum_{i=1}^{N}\int_{L_\tau}^{\infty}\pi^*(\tau, \beta_i, \lambda_i \mid \underset{-}{x})d\tau = 1-\frac{\alpha}{2}, \quad \frac{1}{N}\sum_{i=1}^{N}\int_{U_\tau}^{\infty}\pi^*(\tau, \beta_i, \lambda_i \mid \underset{-}{x})d\tau = \frac{\alpha}{2}, \tag{5.5}$$
$$\frac{1}{N}\sum_{i=1}^{N}\int_{L_\lambda}^{\infty}\pi^*(\lambda, \beta_i, \tau_i \mid \underset{-}{x})d\lambda = 1-\frac{\alpha}{2}, \quad \frac{1}{N}\sum_{i=1}^{N}\int_{U_\lambda}^{\infty}\pi^*(\lambda, \beta_i, \tau_i \mid \underset{-}{x})d\lambda = \frac{\alpha}{2}.$$

5.3 Highest posterior density interval

A $(1-\alpha)\times 100\%$ HPD interval for β is obtained by solving the following two nonlinear equations

$$\frac{1}{N}\sum_{i=1}^{N}\int_{L_\beta}^{U_\beta}\pi^*(\beta, \tau_i, \lambda_i \mid \underset{-}{x})d\beta = 1-\alpha, \quad \sum_{i=1}^{N}\pi^*(L_\beta, \tau_i, \lambda_i \mid \underset{-}{x}) = \sum_{i=1}^{N}\pi^*(U_\beta, \tau_i, \lambda_i \mid \underset{-}{x})). \tag{5.6}$$

Similarly, the $(1-\alpha)\times 100\%$ HPD interval for τ are obtained by solving the following two nonlinear equations

$$\frac{1}{N}\sum_{i=1}^{N}\int_{L_\tau}^{U_\tau}\pi^*(\tau, \beta_i, \lambda_i \mid \underset{-}{x})d\tau = 1-\alpha, \quad \sum_{i=1}^{N}\pi^*(L_\tau, \beta_i, \lambda_i \mid \underset{-}{x}) = \sum_{i=1}^{N}\pi^*(U_\tau, \beta_i, \lambda_i \mid \underset{-}{x})). \tag{5.7}$$

Finally, the $(1-\alpha)\times 100\%$ HPD interval for λ are obtained by solving the following two nonlinear equations the $(1-\alpha)\times 100\%$ HPD interval for τ are obtained by solving the following two nonlinear equations

$$\frac{1}{N}\sum_{i=1}^{N}\int_{L_\lambda}^{U_\lambda}\pi^*(\lambda, \beta_i, \tau_i \mid \underset{-}{x})d\lambda = 1-\alpha, \quad \sum_{i=1}^{N}\pi^*(L_\lambda, \beta_i, \tau_i \mid \underset{-}{x}) = \sum_{i=1}^{N}\pi^*(U_\lambda, \beta_i, \tau_i \mid \underset{-}{x})). \tag{5.8}$$

6 Numerical Computations

In the following, the maximum likelihood and Bayes estimates are compared based on a Monte Carlo simulation study.

1. For a given vector of prior parameters (c_1, c_2, c_3) the vector of population parameters (β, τ, λ) have been generated from the joint prior (4.1).

2. Making use of the generated population parameters, a progressive first-failure censored samples from the MW distribution with pdf (1.1) have been generated. To generate progressive first failure samples, we used the algorithm proposed by Balakrishnan and Aggarwala[4], with the fact that, the progressive first-failure censored sample $X^R_{1;m,n,k}, X^R_{2;m,n,k}, ..., X^R_{m;m,n,k}$ with cdf $F(x)$, can be viewed as progressive type-II censored sample from a population with distribution function $1-(1-F(x))^k$. We assume that the number of items put on a life test is equal to $n \times k$, where n denotes the number of groups and k the number of items in each group. Using a progressive first-failure censoring scheme, only m observations are obtained from the test.
3. The ML estimates of β, τ and λ are computed as shown in section 3 using the software $Mathematica8$ for solving the resulting nonlinear equations.
4. The Bayes estimates for the parameter $\eta \equiv (\beta, \tau, \lambda)$ under SE and $Linex$ loss functions using $MCMC$ method are given, respectively, by using the formulas (4.3) and (4.4).
5. The above steps (2-4) are repeated 1000 times.
6. If $\hat{\theta}_j$ is an estimate of θ, based on sample $j, j = 1,2,...,1000$, then the average estimate over the 1000 samples is given by $\bar{\hat{\theta}} = \frac{1}{1000}\Sigma_{j=1}^{1000}\hat{\theta}_j$.
7. The $(MSE's)$ of $\hat{\theta}$ over the 1000 samples is given by $MSE(\hat{\theta}) = \frac{1}{1000}\Sigma_{j=1}^{1000}(\hat{\theta}_j - \theta)^2$.
8. From 7 and 8 we computed the average estimates and the $MSE's$ for all parameters.
9. The ACI, CI, HPD, lengths and finally the coverage probabilities $(CP's)$ for all parameters.

The computations are shown in Tables 1 and 2.

Table 1:- $MSE's$ of The Estimates Under SEL and $LINEX$ Loss Function ($a=-3, 0.0001, 3$), $(\beta=2.5, \tau=2.0, \lambda=2.7)$, $(c_1=1.4, c_2=0.8, c_3=2.5)$ Based on Progressive First-Failure Censored Data.

(n,m,k)	Method			$MSE(\hat{\beta})$	$MSE(\hat{\tau})$	$MSE(\hat{\lambda})$
$R=(1,2,1,1,3,1,1,2,1,2)$						
(25,10,1)	ML			0.3017	1.2013	1.0152
	B	SEL		0.2438	1.0031	0.8161
		LINEX	$a=-3.0$	0.2819	1.1163	0.9032
			$a=0.0001$	0.2438	1.0031	0.8161
			$a=3.0$	0.1671	0.5721	0.4301
$R=(1,1,0,0,0,1,0,1,0,0,3,0,1,0,2)$						
(25,15,3)	ML			0.2541	0.8603	0.9068
	B	SEL		0.1901	0.6691	0.8230
		LINEX	$a=-3.0$	0.2068	0.7105	0.8614
			$a=0.0001$	0.1901	0.6691	0.8230
			$a=3.0$	0.1005	0.4708	0.3552
$R=(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)$						
(25,25,5)	ML			0.1006	0.5104	0.7105
	B	SEL		0.0916	0.2217	0.4319
		LINEX	$a=-3.0$	0.0951	0.4416	0.5115
			$a=0.0001$	0.0916	0.2217	0.4319
			$a=3.0$	0.0103	0.1017	0.1506

Table 2:- $CI's$ of the parameters β, τ and λ.

$R=(1,2,1,1,3,1,1,2,1,2)$				
(n,m,k)	Method	(L_β, U_β)	(L_τ, U_τ)	(L_λ, U_λ)
		Length	Length	Length
		CP	CP	CP
(25,10,1)	ACI	(0.2178,5.0164)	(0.6822,2.7594)	(1.5993,4.3504)
		4.7986	2.0771	2.7511
		95.35	96.21	95.71.4
	CI	(1.3464,5.1343)	(1.3409,2.7701)	(1.8609,4.0924)
		3.7879	1.4292	2.2315
		98.0	95.4	96.9
	HPD	(1.7901,3.9010)	(1.0923,2.2021)	(1.5541,3.4347)
		2.1109	1.1098	1.8806
		95.76	95.45	95.98.01

R = (1,1,0,0,0,1,0,1,0,0,3,0,1,0,2)				
(25,10,3)		(0.7337,3.9824)	(0.5387,1.894)	(1.1332,3.2824)
	ACI	3.2387	1.3553	2.1482
		95.2	96.3	96.2
		(1.6038,3.5673)	(0.8091,2.1103)	(1.6583,3.4965)
	CI	1.9635	1.3012	1.8382
		99.8	97.8	95.4
		(1.7709,3.5822)	(1.0733,2.1747)	(1.6198,3.4019)
	HPD	1.8113	1.1014	1.7821
		96.01	96.44	97.85
R = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)				
(25,10,5)	ACI	(1.3309,4.2275)	(0.6914,1.9411)	(1.9415,3.9737)
		2.8966	1.2497	2.0322
		97.1	96.1	98.6
	CI	(2.9566,3.9545)	(1.0393,2.0473)	(2.1147,3.1856)
		0.9979	1.0080	1.0709
		96.76	96.6	95.4
	HPD	(2.6520,3.5264)	(0.9187,1.8955)	(2.8319,3.4446)
		0.8744	0.9768	0.8127
		95.87	95.63	96.32

7 Data Analysis And Application

In this section, we consider a real life data set and illustrate the methods proposed in the previous sections. The real data set is from Nicholas and Padgett[11]. The data concerning tensile strength of 100 observations of carbon fibers, they are: 3.7, 3.11, 4.42, 3.28, 3.75, 2.96, 3.39, 3.31, 3.15, 2.81, 1.41, 2.76, 3.19, 1.59, 2.17, 3.51, 1.84, 1.61, 1.57, 1.89, 2.74, 3.27, 2.41, 3.09, 2.43, 2.53, 2.81, 3.31, 2.35, 2.77, 2.68, 4.91, 1.57, 2.00, 1.17, 2.17, 0.39, 2.79, 1.08, 2.88, 2.73, 2.87, 3.19, 1.87, 2.95, 2.67, 4.20, 2.85, 2.55, 2.17, 2.97, 3.68, 0.81, 1.22, 5.08, 1.69, 3.68, 4.70, 2.03, 2.82, 2.50, 1.47, 3.22, 3.15, 2.97, 2.93, 3.33, 2.56, 2.59, 2.83, 1.36, 1.84, 5.56, 1.12, 2.48, 1.25, 2.48, 2.03, 1.61, 2.05, 3.60, 3.11, 1.69, 4.90, 3.39, 3.22, 2.55, 3.56, 2.38, 1.92, 0.98, 1.59, 1.73, 1.71, 1.18, 4.38, 0.85, 1.80, 2.12, 3.65.

These real data are analyzed using $\text{Weibull}(\alpha,\beta)$ distribution and using $\text{MW}(\beta,\tau,\lambda)$ by Ateya[1] and he found that the MW model fits these data better than the Weibull model.

To illustrate the use of the estimation methods proposed in this paper, firstly we order the data as follows 0.39, 0.81, 0.85, 0.98, 1.08, 1.12, 1.17, 1.18, 1.22, 1.25, 1.36, 1.41, 1.47, 1.57, 1.57, 1.59, 1.59, 1.61, 1.61, 1.69, 1.69, 1.71, 1.73, 1.80, 1.84, 1.84, 1.87, 1.89, 1.92, 2.00, 2.03, 2.03, 2.05, 2.12, 2.17, 2.17, 2.17, 2.35, 2.38, 2.41, 2.43, 2.48, 2.48, 2.50, 2.53, 2.55, 2.55, 2.56, 2.59, 2.67, 2.68, 2.73, 2.74, 2.76, 2.77, 2.79, 2.81, 2.81, 2.82, 2.83, 2.85, 2.87, 2.88, 2.93, 2.95, 2.96, 2.97, 2.97, 3.09, 3.11, 3.11, 3.15,

3.15, 3.19, 3.19, 3.22, 3.22, 3.27, 3.28, 3.31, 3.31, 3.33, 3.39, 3.39, 3.51, 3.56, 3.60, 3.65, 3.68, 3.68, 3.70, 3.75, 4.20, 4.38, 4.42, 4.70, 4.90, 4.91, 5.08, 5.56.
Secondly, we assume that the carbon fibers are randomly grouped into 25 groups with $k = 4$ carbon fibers within each group. The tensile strength of carbon fibers of the groups are: 0.39, 0.81, 0.85, 0.98, 1.08, 1.12, 1.17, 1.18, 1.22, 1.25, 1.36, 1.41, 1.47, 1.57, 1.57, 1.59, 1.59, 1.61, 1.61, 1.69, 1.69, 1.71, 1.73, 1.80, 1.84, 1.84, 1.87, 1.89, 1.92, 2.00, 2.03, 2.03, 2.05, 2.12, 2.17, 2.17, 2.17, 2.35, 2.38, 2.41, 2.43, 2.48, 2.48, 2.50, 2.53, 2.55, 2.55, 2.56, 2.59, 2.67, 2.68, 2.73, 2.74, 2.76, 2.77, 2.79, 2.81, 2.81, 2.82, 2.83, 2.85, 2.87, 2.88, 2.93, 2.95, 2.96, 2.97, 2.97, 3.09, 3.11, 3.11, 3.15, 3.15, 3.19, 3.19, 3.22, 3.22, 3.27, 3.28, 3.31, 3.31, 3.33, 3.39, 3.39, 3.51, 3.56, 3.60, 3.65, 3.68, 3.68, 3.70, 3.75, 4.20, 4.38, 4.42, 4.70, 4.90, 4.91, 5.08, 5.56. Suppose that the pre-determined progressively first-failure censoring plan is applied using progressive censoring plan is applied using progressive censoring scheme $R = (2,2,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)$. The following progressively first-failure censored data of size $(m = 20)$ out of 25 groups of carbon fibers were observed: 0.39, 1.47, 1.84, 1.92, 2.05, 2.43, 2.53, 2.59, 2.74, 2.81, 2.85, 2.95, 3.09, 3.15, 3.22, 3.31, 3.51, 3.68, 4.20, 4.90. For this example 5 groups are censored and 20 first failure are observed. The estimates of the parameters β, τ and λ are obtained in Table3. Moreover, the result of 95% ACI , CI and HPD for β, τ and λ are given in Table 4.

Table 3:- Estimates of the parameters β, τ and λ using ML and Bayes methods (under SEL and LINEX loss functions) $(a = 0,1,2)$ based on progressive first-failure censored scheme from real data

(n, m, k)	Method			$\hat{\beta}$	$\hat{\tau}$	$\hat{\lambda}$
(25,20,4)	ML			2.5901	0.0659	1.4413
	B	SEL		2.4901	0.0641	1.3413
			$a = 0.00001$	2.4901	0.0641	1.3413
		LINEX	$a = 1.0$	2.6012	0.0649	1.4719
			$a = 2.0$	2.5983	0.0652	1.4715

Table 4:- Confidence intervals for the parameters β, τ and λ based on progressive first-failure censored scheme from real data.

(n, m, k)	Method	(L_β, U_β)	(L_τ, U_τ)	(L_λ, U_λ)
(25,20,4)		Length	Length	Length
	ACI	(1.8856, 3.2946)	(0.0032, 0.1287)	(0.9154, 2.0158)
		1.4090	0.1256	1.1004
	CI	(1.9618,3.0720)	(0.0115,0.1130)	(0.9781, 1.9452)
		1.1102	0.1015	0.9671

	HPD	(2.0191, 3.1112)	(0.0317,0.1279)	(1.0051, 1.8224)
		1.0921	0.0962	0.8173

8 Concluding Remarks

In this paper, the estimation problem (point and interval) is studied based on progressive first failure censoring scheme of MW distribution. Also, a real data set is introduced as illustrative example. A simulation study is carried out to examine and compare the performance of the proposed methods for different sample sizes and different censoring schemes. From the results which are summarized in tables 1 and 2, we observe the following.

1 The $MSE's$ of the $BE's$ based on SEL function and $LINEX$ loss function are less than that obtained for the $MLE's$ which means that the $BE's$ are better than the $MLE's$.

2 The $MSE's$ of the $BE's$ based on $LINEX$ loss function decrease by increasing a.

3 The $MSE's$ of the $BE's$ based on $LINEX$ loss function are the same as that obtained based on SEL function when $a \to 0$.

4 In all cases, the $CP's$ of all $CI's$ of all methods close to the desired level of 0.95.

5 The length of the $ACI >$ that computed for $CI >$ that computed for the HPD interval.

References

[1] S.F. Ateya, Estimation Under Modified Weibull Distribution Based on Right Censored Generalized Order Statistics, J. Applied Statistics, 40(2013), 2720-2734.

[2] S.F. Ateya and A.S. Alharthi, Maximum likelihood estimation under a finite mixture of modified Weibull distributions based on censored data with application. JASS. 0(2014), 231-239.

[3] S.F. Ateya and A.S. Alharthi, Estimation under a finite mixture of modified Weibull distributions based on censored data via EM algorithm with application. JSTA. 13(2014), 196-204.

[4] N. Balakrishnan and R. Aggarwala, *Progressive Censoring-Theory, Methods, and Applications*. Birkhäuser, Boston(2000).

[5] T. Gotoh, M. Fukuhara, and K. I. Kikuchi, Mathematical model for change in size distribution of baculovirus-infected Sf-9 insect cells, The 3rd WSEAS International Conference on Cellular and Moleculaz Biology, Biophysics and Bioengineering, 2007, 25-29.

[6] E.J. Green, F.A. Jr. Roesh, A.F.M. Smith and W. E. Strawderman, Bayes estimation for the three parameter Weibull distribution with tree diameters data, Biometrics, 50(1994), 254-269.

[7] A.M. Hossain and W.J. Zimmer, Comparison of estimation methods for Weibull parameters: complete and censored samples, J. Statist. Comput. Simulation, 73(2003), 145-153.

[8] Z.F. Jaheen and M.M. Al Harbi, Bayesian Estimation for the Exponentiated Weibull Model via Markov Chain Monte Carlo Simulation, Commun. Statist.-Simul. Comput., 40(2011), 532-543.

[9] N.L. Johnson, S. Kotz and N. Balakrishnan, Continuous Univariate Dis tributions,Wiley, NewYork, (1994).

[10] C.D. Lai, M. Xie and D.N. Murthy, A modified Weibull distribution, IEEE Trans. Reliab., 52(2003), 33-37.

[11] M.D. Nicholas and W. J. Padgett, A bootstrap control chart for Weibull percentiles, Quality and Reliability Engineering International, 22(2006), 141-151.

[12] V. Niola, R. Oliviero and G. Quaremba, The application of wavelet transform for estimating the shape parameter of a Weibull pdf, Proceedings 5th Wseas International Conference on Signal Processing, (2005), 126-130.

[13] A. Shamilov, I. Usta and Y.M. Kantar, The Distribution of Minimiz- ing Maximum Entropy: Alternative to Weibull distribution for Wind Speed,WSEAS Transactions on Mathematics, 5(2006), 695-700.

[14] K.S. Sultan, Record values from the modified Weibull distribution and applications, International Mathematical Forum, 2(2007), 2045-2054.

[15] S.K. Upadhyaya, and A. Gupta, A Bayes analysis of modified Weibull distribution via Markov chain Monte Carlo simulation. J. Statist. Comput. Simul. 80(2010), 241-254.

[16] S.K. Upadhyaya, N. Vasishta and A.F.M. Smith, Bayes inference in life testing and reliability via Markov chain Monte Carlo simulation. *Sankhya A*, 63(2001):15-40.

[17] P. Vasile, P. Eugenia and C. Alina, Bayes estimators of modified Weibull distribution parameters using Lindley's approximation, WSEAS Transactions on Mathematics, 9(2010), 539-549.

[18] R. Vicen-Bueno, M. Rosa-Zurera, L. Cuadra- Rodriguez and D. De La Mata-Moya, Models of radar clutter from the Weibull distribution, Proceedings of the 5th WSEAS International Conference on Signal Processing, Robotics and Automation, (2006), 376-380.

[19] S.J. Wu and C. Kus, On estimation based on progressive first-failure- censored sampling. Comput. Statist. Data Anal., 53(2009), 3659-3670.

[20] A. Zaharim, S.K. Najid, A.M. Razali and K. Sopian, The Suitability of Statistical Distribution in Fitting Wind Speed Data, WSEAS Transactions on Mathematics, 7(2008), 718-727.