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# Analysis of DART II based Tsunami Warning System using Stochastic Petri Nets

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## ABSTRACT

This work focuses on utilization of Stochastic Petri Nets as a modeling, simulation and evaluation tool for analyzing DART II buoy based Tsunami warning system. In order to assess the readiness to deal with natural calamities like tsunami, effective modeling, simulation and evaluation based analysis is essential for effective disaster management. Petri Net a modeling and simulation tool with strong mathematical fundamentals allows evaluation of such systems and their dynamic behavior graphically. By mapping of Tsunami warning system scenarios into Stochastic Petri Nets such complex systems can be evaluated. This work is a novel attempt to build, simulate and analyze the responsiveness of Tsunami Warning System model based on America's National Oceanic and Atmospheric Administration's Deep-ocean Assessment and Reporting of Tsunamis (DART) II buoy architecture using Stochastic Petri Net. This work also attempts to evaluate the built model by doing structural analysis using Petri Net tools like incidence matrices, reachability trees, reachability graph and uses Erlang distribution to evaluate the responsiveness of the model.
**Keywords:** Stochastic Petri Net, Erlang Distribution, Tsunami warning system.

## 1. INTRODUCTION

The devastation and deaths caused by the December 2004 Tsunami has made development of new methods for monitoring essential. The Tsunami warning system (TWS) detects tsunamis and issues timely warnings to prevent loss of life and property. It consists of two equally important components: a network of sensors to detect tsunamis and a communications infrastructure to issue timely alarms to permit evacuation of coastal areas. Whenever there is an earthquake in the sea it puts the observation team in alert mode, the data about observed sea level height via DART II buoys are used to verify the existence of a tsunami.

Because of its importance, a lot of countries have spent a lot of resources in setting up Tsunami warning systems and deploying DART II buoys. Time is of importance in the case of an actual tsunami. Quick detection and response are of essence. The responsiveness and effectiveness of the tsunami warning systems has to be periodically assessed.

This work uses Petri Nets [1] as a modeling tool for the study of Tsunami Warning systems. Modeling and simulation helps in designing, fine tuning and checking the effectiveness of an integrated tsunami warning system. Detailed information on Petri Nets can be found in the works [1][2][3]. Modeling the interdependencies amongst the various components of the tsunami warning system is an essential part of successful modeling and evaluation. Other important considerations include representing cause effect relationships among them, conditional branching, hierarchical representation. These help understand, simulate and analyze the dynamics of the system. This work summarizes the steps required to model and simulate a tsunami warning system and demonstrate how Petri Net can be used effectively to analyze the responsiveness of such complex, dynamic environments.

## 2. MODELING OF THE TSUNAMI WARNING SYSTEM

A model is a set of components that work together to portray a system. A system can be modeled in various ways, either mathematically or via prototypes or scale models etc

We use mathematical modeling and simulation of the Tsunami warning system. It has been modeled in two parts and structural analysis is done using Petri Net analysis techniques. In this case we can model the system via the definition as follows

A system is a 5 tuple S = (T,R,PRE,TS,MT) satisfying the following requirements.

(1) T is a finite set of tasks.

(2) R is a finite set of resources.

(3) $PRE \subseteq T \times T$ is a partial order, the precedence relation.

(4) TS is the time set.

(5) $MT \in (T \times \mathcal{P}(R)) \nrightarrow TS$ defines for each task t :

The resource sets capable of manipulating task t and

The manipulating time required to manipulate t by a specific resource set.

The definition specifies the data required to formulate a general system. Resources and tasks are defined using the mapping technique [6].

## 3. TIMED PETRI NET MODEL OF TSUNAMI WARNING SYSTEM

The example taken for the case here is tsunami warning system which consists of two physical components, a tsunameter on the ocean floor and a surface buoy with satellite telecommunications capability [10]. Tsunami Warning system (DART II) have bi-directional communication links and are able to send and receive data from the tsunami warning centre. For easy understanding of the concepts explained in above sections, we have done the modelling in two parts. In first part we have done the model of one way communication (i.e.,) model of the tsunami warning system sending the measured values, calculated data and sending warning if necessary. In the second part we have modelled the other way communication (i.e.,) model of warning centre requesting data from the tsunameter via satellite[7]. The Algorithm for the first model is defined and the Petri Net model is presented.

## 4. ALGORITHM OF A TSUNAMI WARNING SYSTEM

(1) Tsunameter measures water pressure on the ocean floor.
(2) Computer reads pressure readings and runs tsunami detection algorithm.
(3) If amplitude exceeds the threshold the tsunameter goes to event mode. Wait for one more value. If two consecutive measures exceeds threshold value gives tsunami warning. Transmits the measurements and other in formations to an anchored buoy on the surface.
(4) If amplitudes do not exceed the threshold tsunameter goes to standard mode and transmits the measurement to the buoy.
(5) Buoy sends/transmits the measures received to the satellite.
(6) Satellite transmits the measure to the early warning station on the earth.
(7) Early warning stations sends alarm if the tsunameter is in event mode or just saves the data.

## 5. PETRI NET MODEL OF A TSUNAMI WARNING SYSTEM.

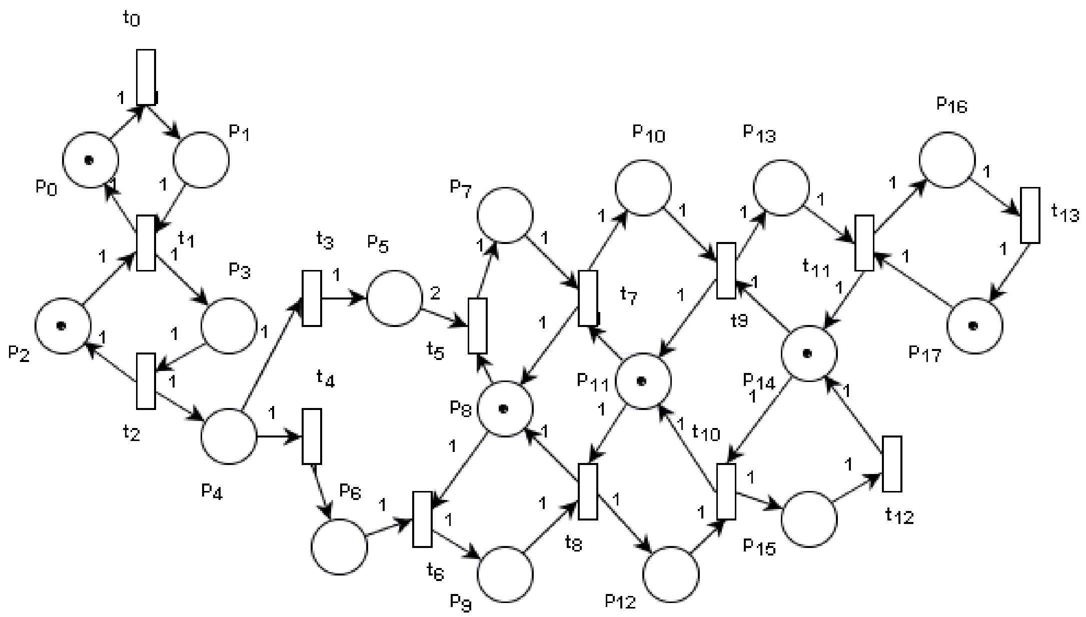


**FIGURE 1. Petri Net model of Tsunami warning system.**

**RESOURCES:** {Tsunameter Sensor, Buoy, Satellite, Early Warning Station, Alert Mechanism}
**TASKS:** {Measuring, transmitting, Receiving, Monitoring the pressure, Provoking alert interface, raising alert}.
We now define the transitions and places in the Petri Net model as follows:
**Places** - denotes stages
$p_0$,$p_2$,$p_8$,$p_{11}$,$p_{14}$,$p_{17}$-denotes the necessary resources
$p_0$- Tsunameter available
$p_1$ - Tsunameter ready with measured pressure value
$p_2$- computer ready to analyze (resource)
$p_3$- computer received the pressure
$p_4$- pressure compared value
$p_5$- tsunameter turned to event mode(tsunami detected)
$p_6$- tsunameter in the standard mode
$p_7$ – buoy received the value from event mode
$p_8$- buoy ready(resource)
$p_9$- buoy received the value from standard mode
$p_{10}$- satellite received the values from the event mode
$p_{11}$- satellite ready(resource)
$p_{12}$ - satellite received the value from the standard mode
$p_{13}$ - warning station received the values from the event mode
$p_{14}$ - warning station availability (resource)
$p_{15}$- warning station received the values from the standard mode
$p_{16}$- tsunami warning alarm raised stage
$p_{17}$- alarm mechanism availability(resource)

**Transitions**
$t_0$- start reading water pressure
$t_1$- send it to computer
$t_2$- run tsunami detection algorithm
$t_3$-go to event mode( if pressure difference abnormal)
$t_4$- go to standard mode(if pressure difference normal)
$t_5$- send it to buoy after two consecutive monitoring of pressure
$t_6$- send it to buoy from standard mode(pressure difference is normal condition)
$t_7$- buoy sends the values to satellite from event mode
$t_8$- buoy sends the value to satellite from standard mode
$t_9$-satellite sends the value to warning station from event mode
$t_{10}$-satellite sends it to warning station from standard mode
$t_{11}$-raise alarm
$t_{12}$-store data
$t_{13}$-store abnormal data

Firing transitions $t_3$ and $t_4$ give two different cases. The first case uses $t_3$ when the pressure difference is abnormal and enters into the event mode. The second case uses $t_4$ when the pressure difference is normal and enters into the standard mode [16]. The time delay of the transitions $t_3$ and $t_4$ are negligible.

Standard mode reports once every six hours. So time delay for the transition $t_6$ is 6 hours. Event mode transmits the value to buoy immediately less than a three minute delay. The time delay of $t_5$ is 3 minutes. Event mode transmission of buoy to satellite is immediately transmitted due to their importance and urgency. $t_7$ -approximately 30 seconds.

Standard mode transmission of buoy to satellite occurs once every six hours. $t_8$ time delay is 6 hours, the time delay for the other transitions are assumed as 30 secs.

## 6. STRUCTURAL ANALYSIS

**The structural analysis of the constructed Tsunami Warning System model using invariant techniques is as follows.**

Place and transition invariants are powerful tools for studying structural properties of Petri Nets. The structural behaviour of the net can be assessed using the algebraic analysis of the incidence matrix [invariant analysis][9]

The incidence matrix is defined as A= $[a_{ij}]$ where

$a_{ij} = a^{+}_{ij} - a^{-}_{ij}$

$a^{+}_{ij}$ = w(i,j) is the weight of the arc from $t_i$ to $p_j$ and

$a^{-}_{ij}$ = w(j,i) is the weight of the arc from $p_j$ to $t_i$.

The incidence matrix of the net is given below;

$$\begin{pmatrix}
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{pmatrix}$$
a

**Figure 2. Incidence matrix**

The order of the places in the matrix is
$P=\{p_0, p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}\}$ (columns)
and the order of the transitions is
$T = \{t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13}\}$ (rows)
A P- invariant is a vector that satisfies the equation $Ax = 0$
A T- invariant is a vector that satisfies the equation $A^T y=0$
The following invariants are obtained from the incidence matrix A.

**P-INVARIANTS**
$x_1= [1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]^T$
$x_2= [0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0]^T$
$x_3= [0,0,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0]^T$
$x_4= [0,0,0,0,0,0,0,0,0,0,1,1,1,0,0,0,0,0]^T$
$x_5= [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1]^T$
$x_6= [0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,0,0]^T$

**T-INVARIANTS**
$y_1= [2,2,2,2,0,1,0,1,0,1,0,1,0,1]^T$
$y_2= [1,1,1,0,1,0,1,0,1,0,1,0,1,0]^T$

A net is said to be covered by P- invariants if and only if, for each place p in the net there exist a positive P-invariant x such that $x(p) > 0$. The net is not covered by P-invariants. We do not know if it is bounded. A net is covered by T- invariants if and only if for each transition t in the net, there exist a positive T- invariant y such that $y(t)>0$.The net is covered by T-invariant. It might be bounded and live.
A Petri Net is structurally bounded if it is covered by P-invariants and the initial marking $M_0$ is finite. Since it is not covered by P-invariants the net Figure 1 is not structurally bounded. The reachability graph will be too big to discuss.

**PETRI NET MODEL OF THE SECOND PART.**
**ALGORITHM FOR THE MODEL**
1. Warning centre sends request of data to satellite
2. Satellite sends request to buoy
3. Buoy sends request to tsunameter
4. Tsunameter process the data
5. Sends it to buoy
6. Buoy sends the data to satellite
7. Satellite sends the data to warning centre

**PETRI NET MODEL OF TSUNAMI WARNING CENTRE REQUESTING DATA FROM TSUNAMETER**

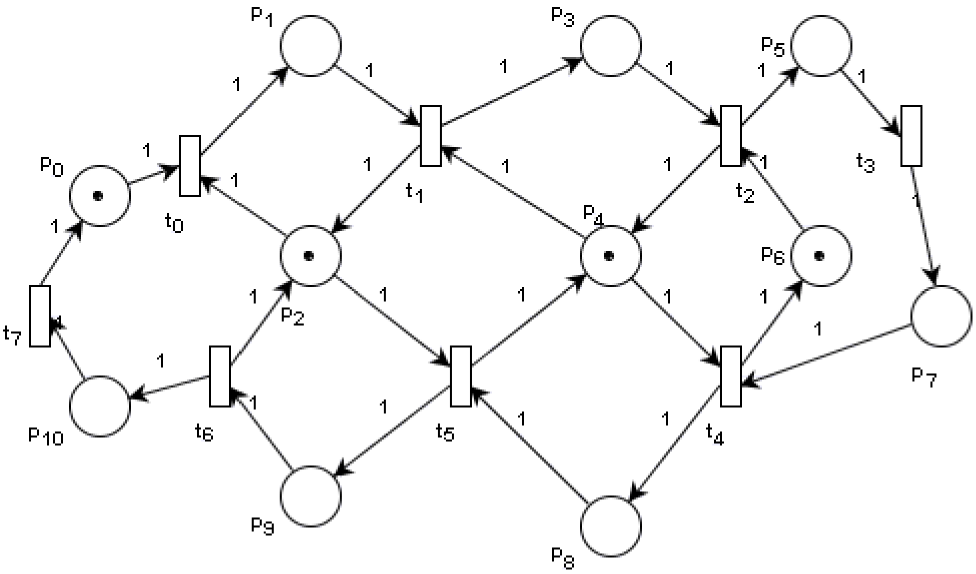


**FIGURE 3. PETRI NET MODEL OF TSUNAMI WARNING CENTRE REQUESTING DATA FROM TSUNAMETER**

**Resources:**{ satellite, buoy, tsunameter, warning station}
**Tasks:**{Sending request to satellite, sending request to buoy, sending request to tsunameter, processing, sending details, accepting, returning to next cycle.}

**Places:** defines stages
$p_0$- warning station ready with request
$p_1$- satellite received the request
$p_2$- satellite (resource)
$p_3$- buoy received the request
$p_4$- buoy (resource)
$p_5$- tsunameter received the request
$p_6$- tsunameter(resource)
$p_7$- tsunameter ready with the processed data to send
$p_8$ -buoy received the data
$p_9$- satellite received the data
$p_{10}$-warning station received the requested data

**Transitions:**
$t_0$- warning station sending the request to satellite
$t_1$- satellite sending the request to buoy
$t_2$-buoy sending request to tsunameter

$t_3$- tsunameter processing the requested data
$t_4$- tsunameter sending the requested data to buoy
$t_5$- buoy sending the data to satellite
$t_6$- satellite sending the data to warning station
$t_7$- warning station returns with the data
We assume that the time delay for each transition 5ms.

**Analysis using Reachability Tree and Reachability Graph**
The reachability tree and reachability graph of the Petri Net model (FIGURE 3) is as follows:

$M_0$[1,0,1,0,1,0,1,0,0,0,0]
↓$t_0$
$M_1$[0,1,0,0,1,0,1,0,0,0,0]
↓$t_1$
$M_2$[0,0,1,1,0,0,1,0,0,0,0]
↓$t_2$
$M_3$[0,0,1,0,1,1,0,0,0,0,0]
↓$t_3$
$M_4$[0,0,1,0,1,0,0,1,0,0,0]
↓$t_4$
$M_5$[0,0,1,0,0,0,1,0,1,0,0]
↓$t_5$
$M_6$[0,0,0,0,1,0,1,0,0,1,0]
↓$t_6$
$M_7$[0,0,1,0,1,0,1,0,0,0,1]
↓$t_7$
$M_0$[1,0,1,0,1,0,1,0,0,0,0]
Old

**FIGURE 4. REACHABILITY TREE (TSUNAMI)**

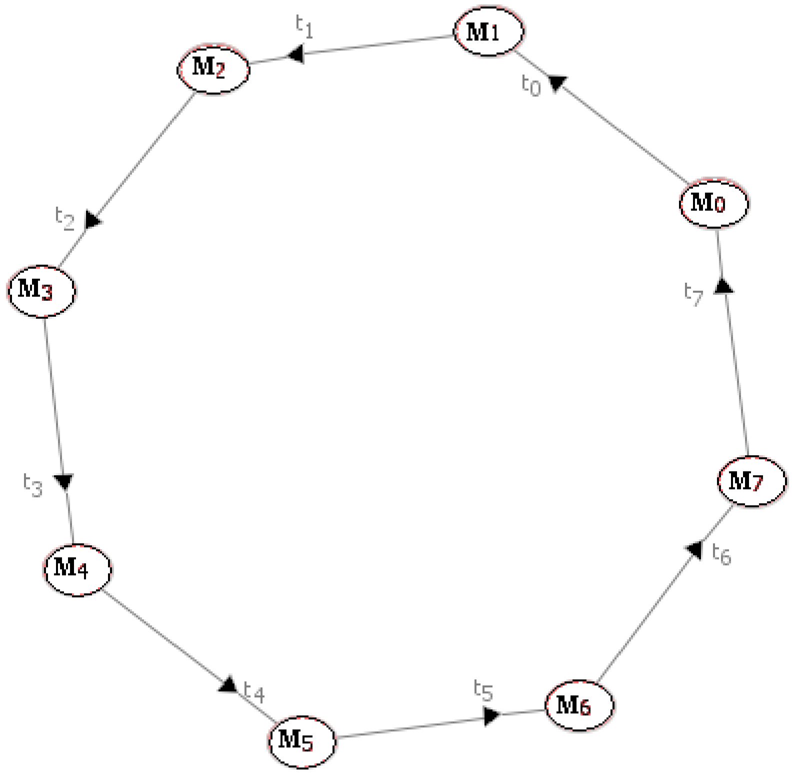


**FIGURE 5. REACHABILITY GRAPH (TSUNAMI)**

**Analysis of reachability graph**

An assessment of boundedness and liveness properties based on the initial marking of the net can be performed by analysing the reachability tree of the net using $M_0$ as the root node. A reachability tree is a graph representation of the markings of a net. Each node in the tree represents a marking and the edges represent a transition firing. The resulting tree is small enough to be analysed by inspection and it can be concluded that

a) The reachability set $R(M_0)$ is finite

b) The number of tokens of every place in all marking is bounded ( 1-bounded)

c) There are no dead transitions (all transitions can fire)

As a result it can be concluded that the net is bounded.

By looking at the reachability graph FIGURE 5 of the net FIGURE 3 with initial marking $M_0$ it can be seen that there is always an active transition regardless of the state of the net and all the transitions of T are included in the graph. From this it can be concluded that the Petri Net model of the tsunami warning centre requesting data is bounded and live. For models with a large number of places and transitions, the reachability graph construction is not practical without the use of computer tools to automate the reachability graph generation and the assessment of properties.

**Structural analysis.**
The incidence matrix of the Petri Net given in Figure 3 is as follows.

$$\begin{pmatrix} -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

**FIGURE 6. Incidence matrix of net (Tsunami)**

The order of the places in the matrix is
P $=\{p_0,p_1,p_2,p_3,p_4,p_5,p_6,p_7,p_8,p_9,p_{10}\}$ (columns) and the order of the transitions is T = $\{t_0,t_1,t_2,t_3,t_4,t_5,t_6,t_7\}$ (rows)
The following invariants are obtained from the incidence matrix
P-invariants
$x_1= [1,1,0,1,0,1,0,1,1,1,1]^T$
$x_2= [0,1,1,0,0,0,0,0,0,1,0]^T$
$x_3= [0,0,0,1,1,0,0,0,1,0,0]^T$
$x_4= [0,0,0,0,0,1,1,1,0,0,0]^T$
T-invariants
$y_1= [1,1,1,1,1,1,1,1]^T$

The net is covered by P- invariants and T-invariants. Therefore the net is bounded and live.
A Petri Net is structurally bounded if it is covered by P-invariants and the initial marking $M_0$ is finite. Since it is covered by P-invariants the net FIGURE 3, it is structurally bounded. From the reachability tree it can be concluded the net is reversible.

**Mean response time of the tsunami warning system (request of data)**
Mean response time is the average time taken for the Tsunami warning system to complete one cycle of data fetch from the time of request for data.
Time is critical in a Tsunami warning system. Metrics that evaluate the effectiveness of the system are important. Mean response time acts as an important metric to evaluate the performance of the data fetch cycle from the tsunameter. It indicates the average time difference between the request being sent and the response received.
To find mean response time of the system we have taken the second part of the

system, (ie) The model of warning centre requesting data from the tsunameter. We have constructed a Generalized Stochastic Petri Net (GSPN)[8] model of the system.
SPNs have been extended to a class of GSPN. GSPN comprise two types of transitions.Timed transitions which are associated with random, exponentially distributed firing delays. Immediate transitions which fire in zero time with priority over timed transitions [4][5].

**Definition: Erlang distribution**
A random variable X has an Erlang-k distribution with mean k/λ if X is the sum of k independent random variable $x_1,x_2,x_3,\ldots x_k$ having a common exponential distribution with mean 1/λ.The common notation is $E_k(\lambda)$ or briefly E.
The probability density function of the Erlang distribution is

$$f(x;k,\lambda)=\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} \quad \text{for } x,\lambda \geq 0$$

The parameter k is called the shape parameter and the parameter λ is called the rate parameter.
Because of the factorial function in the denominator, the Erlang distribution is only defined when the parameter *k* is a positive integer. In fact, this distribution is sometimes called the **Erlang-k distribution** (e.g., an Erlang-2 distribution is an Erlang distribution with *k=2*).
The cumulative distribution function is

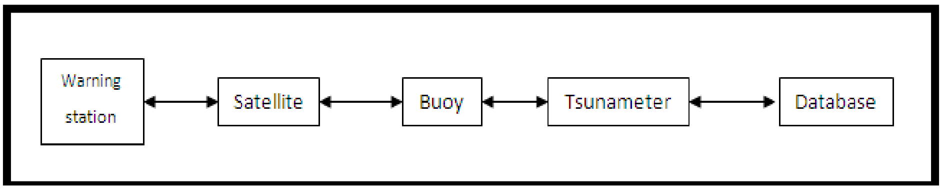


**FIGURE 7. Flow Diagram of warning centre requesting data from tsunameter**

The GSPN Model of warning centre requesting data from tsunameter is as follows

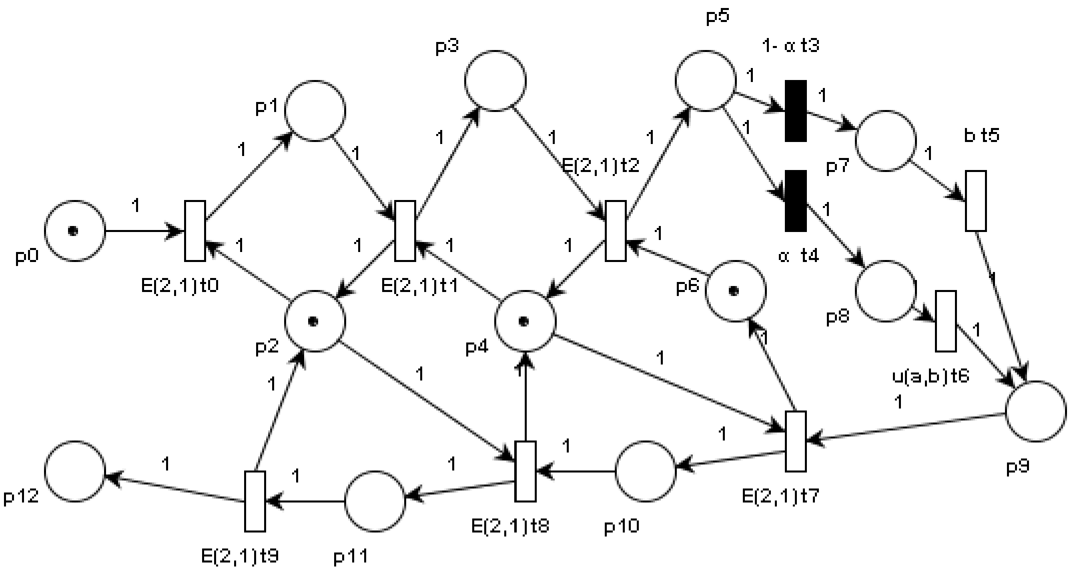


**FIGURE 8. The GSPN model of warning centre requesting data from Tsunameter**

$$F(x;k,\lambda) = 1 - \sum_{n=0}^{k-1} \frac{1}{n!} e^{-\lambda x} (\lambda x)^n$$

The mean, variance and squared co-efficient of variation are equal to
$E(X) = k/\lambda$ $Var(X) = k/\lambda^2$ $SCV = 1/k$

Flow Diagram of the Model of warning centre requesting data from tsunameter is as follows

Let us consider the message transmissions between warning station to satellite($t_0$), satellite to buoy ($t_1$),buoy to tsunameter($t_2$), tsunameter to buoy($t_7$), buoy to satellite($t_8$), satellite to warning station($t_9$). These message transmission times are expressed as k stage Erlang distribution with parameter λ for each stage. Random variable expressed by Erlang distribution with parameter λ is denoted by E(k, λ) or $E_k(\lambda)$.Exponential distribution is a special case of Erlang distribution and is denoted by $E_1(\lambda)$.In our approximation method it will be assumed that k=2 and λ=1.

Once the tsunameter receives the request ($p_5$)), it searches the computer database for the requested data. The probability of finding the response is denoted by α.The time unit is negligible. If there is required information in the data base, then searching time is expressed by uniform distribution over the time interval [0,b).Hence the expected searching time provided there is required information in the data base is equal to b/2. Searching time is equal to b with probability 1- α [12].

Let us consider the sub model-Tsunameter processing the result [13]. In the sub model tsunameter prepares the response that is been requested.

**Sub model:**

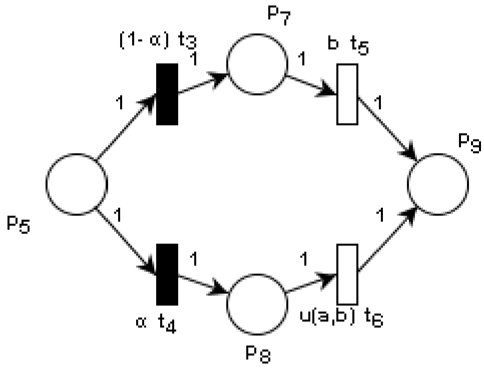


**FIGURE 9. Sub model-Tsunameter processing the result**

$p_5$- Tsunameter with the request(if token presents) $t_3$-fires with probability (1- α)
$p_7$- Ready to search with the probability (1- α) $t_4$-fires with probability α
$p_8$- Ready to search with the probability α $t_5$-searching the data with deterministic time b
$p_9$- Tsunameter with the result $t_6$-searching the data with random delay
The tsunameter processing time in the computer database is represented by a random variable denoted by $\mathbf{U_{b,\alpha}}$
The random variable has the probability density function

$$f_{U_{b,\alpha}}(t) = \begin{cases} \frac{\alpha}{b} \text{ for } t \in [0, b) \\ (1-\alpha).\delta(t-b) \text{ for } t = b \\ 0 \text{ otherwise} \end{cases}$$

where δ(t-b) is a dirac delta distribution in point b.
Expected value, variance, and coefficient of variation for the random variable are given by the following expressions;

$$E(U_{b,\alpha}) = \frac{b(2-\alpha)}{2}$$

$$V(U_{b,\alpha}) = \frac{b^2\alpha(4\text{-}3\alpha)}{12}$$

$$SCV(U_{b,\alpha}) = \frac{\alpha(4\text{-}3\alpha)}{3(2\text{-}\alpha)^2}$$

GSPN with the sub model:

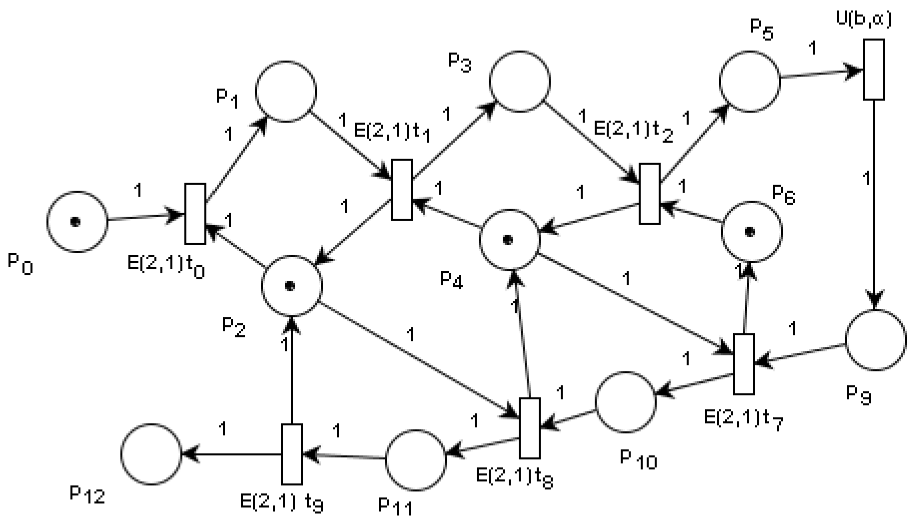


**FIGURE 10. GSPN with sub model**

Let us consider the II step.
It illustrates the probability distribution of the random variable X of the length of the time interval between the time instant the warning station sends the request and the time instant it receives the request.(i.e) warning station to satellite, satellite to buoy, buoy to tsunameter, tsunameter to buoy, buoy to satellite, satellite to warning station.
This random variable is given by the expression.
$X=Sum(E_k(\lambda), E_k(\lambda), E_k(\lambda), U_{b,\alpha,} E_k(\lambda), E_k(\lambda), E_k(\lambda))$
Random variables of the transmission times and random variable of the searching process are independent.

**Note:**
For independent random variables the following equations for expected values and variations are true.
$E(Sum(x_1,x_2,x_3..x_m)=\sum_{n=1}^{m} E(Xn)$
$Var(Sum(x_1,x_2,x_3,\ldots x_m))=\sum_{n=1}^{m} Var(Xn)$
According to the above expressions the expected value,the variance and squared coefficient of variation(SCV) for the random variable X are expressed by the following formulae.
$E(X) = (6k/\lambda) + b-(b\,\alpha/2)$
$Var(X) = (6k/\lambda^2)+(b^2\alpha/3)-(\alpha^2b^2/4)$
$SCV = (72k+\alpha\lambda^2b^2(4-3\alpha)) / (432+3b^2\lambda^2(4-4\alpha+\alpha^2)+72kb\lambda(2-\alpha))$

**Calculation of Mean Response Time.**

To calculate the mean response time [11], let us assume $\alpha$ as.9.( $\alpha$=.9).The maximal searching time in the computer data base by the tsunameter is assumed as 10seconds.(b=10).The transmission time between the tools is given by the random variable $E_2(1)$.

The mean transmission time between the tools is

$E(E_2(1)) = k/\lambda = 2/1 = 2$.

Therefore the transmission time between warning station-satellite-buoy-tsunameter-buoy-satellite-warning station is given by $6\ E(E_2(1)) = 6.2 = 12$

E(Transmission)=12

The random variable of the length of time interval between the time instant when Warning station sends the request to and the time interval it receives the response is

$\mathbf{X_{b,\alpha}}$=Sum($6E_k(\mu),u_{b,\alpha}$)

The Expected value of the random variable $X_{b,\alpha}$

$$
\begin{aligned}
E(X_{b,\alpha}) &= E(\,Sum(6E_k(\lambda),u_{b,\alpha})) \\
&= Sum(E(6E_k(\lambda),u_{b,\alpha})) \\
&= E(6E_k(\lambda)+E(u_{b,\alpha}) \\
&= 6k/\lambda + \alpha b/2 + (1-\alpha)b
\end{aligned}
$$

The expected value of the searching process with b=10 secs and $\alpha$=.9 k=2, $\lambda$=1

$$
\begin{aligned}
E(u_{b,\alpha}) &= \alpha b/2 + (1-\alpha)b \\
&= (\alpha b/2)+b-\alpha b \\
&= b(2-\alpha)/2
\end{aligned}
$$

Hence

$$
\begin{aligned}
E(u_{10,.9}) &= 10(2-.9)/2 \\
&= 10(1.1)/2=5.5
\end{aligned}
$$

As a result

$$E(X_{10,.9}) = (6x2) + 5.5 = 12+5.5 = 17.5$$

The mean response time is 17.5 seconds.

When a warning station sends a request for data it receives it after 17.5 seconds with the searching time of 10 seconds. This tells us how the quickly the DART II buoy can respond back in case of an adhoc data request for oceanic data and this can be used to analyse the readiness of the system and also to place the buoy in such a location where we can have enough lead time for effective prior warning

## 7. Conclusion

In this work, we have defined a DART II system in terms of tasks and resources and then we have mapped it onto Stochastic Petri Net. We have constructed a model of the Tsunami warning system based on that mapping. We have done the modelling of DART II based Tsunami warning system in two parts representing the different activities as in the real world system. Structural properties of the model have been checked to ensure the reliability of this model. Mean response time to fetch data is found. Mean response time is an important metric that can be used to analyse the efficiency of the data request cycle of the modelled tsunami warning system. Based on the response time, steps can be taken to further optimize thereby improving the efficiency of the overall system and it can be validated using the same technique.

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