Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 6 (2015), pp. 4639-4646

http://www.ripublication.com

Rotating Fluid Flow Of An Accelerated Horizontal Plate

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Abstract

The flow of an incompressible viscous fluid bounded by an infinite flat plate with angular velocity normal to the plate is considered. In this paper the effect of rotation parameter on the axial velocity, transverse velocity and skin friction in the rotating fluid is discussed. Perturbation method is used to obtain the expressions for the velocity profiles. Axial velocity profiles increases at various rotation parameters with fixed time T. But transverse velocity profile decreases with fixed time T. It is found that there is a reversal trend occured for transverse velocity.

Key words: Rotation parameter, Skin friction, Axial velocity, Transverse velocity

1 INTRODUCTION

The fluid flow generated by pulsatile motion of the boundary is found to have importance in aerospace science, nuclear fusion, astrophysics, and atmospheric sciences, cosmical gas dynamics, geophysics and physiological fluid dynamics. Fluid flow driven by a rotating disk has constituted a major field of study in fluid mechanics. This flow has technical applications in many areas, such as rotating machinery, lubrication, viscometry, computers storage devices and crystal growth processes.

In view of these applications the geophysical importance of the flows in the rotating frame of reference attracted the attention of a number of scholars. The flow of an incompressible viscous fluid near an infinite flat plate which is impulsively started from rest into motion in its own plane with a constant velocity was first investigated by Stokes[7]. Watson [6] studied how the velocity distribution gets modified in a rotating fluid. A semi infinite mass of an incompressible viscous fluid bounded by an infinite flat with angular velocity is considered by Deka el al [5]. Janusz Wojt Kowiak et al investigated the steady laminar flow and convective heat transfer in a pipe constricted by a coaxially rotating disk. Grote et al discussed numerical simulations of thermal convection in rapidly rotating spherical fluid shells have been carried out with and without magnetic fields generated by the dynamo process. Dikansky et al studied experimentally a drop of magnetic fluid in a rotating magnetic field with the additional influence of a stationary magnetic field. Gauthier. G et al [4] analyzed the instabilities in the flow between co and counter rotating disks. Nazar et al [3] investigated the unsteady boundary layer flow due to stretching surface in a rotating fluid. Hayat et al [2] examined the hydromagnetic flow in a semi-infinite expanse of electrically conducting rotating Johnson-Segalman fluid bounded by unconducting plate in the presence of transverse magnetic field. Von Karman et al [1] investigated the decay of the fluid flow due to a rotating disk when the disk is stopped suddenly. Nitin Shukla et al analyzed the instability of electromagnetic waves in self-gravitating rotating magnetized dusty plasma with opposite polarity grains.

Shahzad et al investigated the unsteady rotating flow of the third grade fluid over a suddenly moving plate in its own plane. Shamima Sultan studied the Energy Decay law of dusty fluid turbulent flow in a rotating system.

2 FORMULATION OF THE PROBLEM

Consider the follow of an incompressible viscous fluid bounded by an infinite flat plate occupying the plane z = 0. Initially the fluid and the plate rotate simultaneously with a uniform angular velocity Ω about the z-axis. We introduce a coordinate system (x, y, z) rotating with fluid.

Relative to the rotation of fluid, the plate is impulsively started from rest and then moves with uniform acceleration in its own plane along the axis. The horizontal homogeneity of the problem shows that the flow quantities depend on z & t, t being the time variable.

Denoting the velocity components of the fluid u, v in the z directions respectively, the problem is governed by the following equations.

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} + 2\Omega v \tag{1}$$

$$\frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial z^2} - 2\Omega u \tag{2}$$

The boundary conditions are

$u = v = 0$ at $t \le 0$ for all z

$u = ct$, $v = 0$ at $z = 0$ for t > 0

$u \to 0,\ \ v \to 0$ as $z \to \infty$ for t > 0.

where c > 0 is a constant. (3)

We introduce the following non dimensional variables

$$U = \frac{u}{(\nu c)^{1/3}},\ V = \frac{v}{(\nu c)^{1/3}},\ Z = z(c/\nu^2)^{1/3},$$

$$T = t(c^2/\nu)^{1/3},\ \Omega_1 = \Omega(\nu/c^2)^{1/3}$$

$$f = \frac{N_0 m}{\rho}, \ \frac{1}{\Lambda_1} = \frac{k}{m}(\nu / c^2)^{1/3}$$

The boundary conditions for the problem in the dimension less form are

$U = V = 0$ at $T \leq 0$ for all z

U = T, V = 0 at z = 0 for T > 0

$U \to 0, V \to 0$ as $z \to \infty$ for T > 0 (4)

6 SOLUTION OF THE PROBLEM

In order to solve these differential equations, we expand the velocity components in powers of $e^{i\omega T}$.

$$W = W_0 + e^{i\omega T} W_1 + ... \quad (5)$$

where higher powers are neglected because of its convergence.

Equations (1), (2), (3) and equation (4) take the following forms

$$\frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial Z^2} + 2\Omega_1 V \quad (6)$$

$$\frac{\partial V}{\partial T} = \frac{\partial^2 V}{\partial Z^2} - 2\Omega_1 U \quad (7)$$

Assume $W(Z,T) = U + iV)$.

Equations (6) and (7) take the form

$$\frac{\partial W}{\partial T} = \frac{\partial^2 W}{\partial Z^2} - 2i\Omega_1 W \quad (8)$$

From equation (5), on differentiation we obtain

$$\frac{\partial W}{\partial Z} = \frac{dW_0}{dZ} + e^{i\omega T} \frac{dW_1}{dZ} \quad (9)$$

$$\frac{\partial W}{\partial T} = i\omega e^{i\omega T} W_1 \quad (10)$$

$$\frac{\partial^2 W}{\partial Z^2} = \frac{d^2 W_0}{dZ^2} + e^{i\omega T} \frac{d^2 W_1}{dZ^2} \quad (11)$$

Where higher powers are neglected.

Substitution of equation (9), (10) and equation (11) in equation (8) gives

$$i\omega e^{i\omega T} W_1 = \frac{d^2 W_0}{dZ^2} + e^{i\omega T} \frac{d^2 W_1}{dZ^2} - 2i\Omega_1 (W_0 + e^{i\omega T} W_1) \quad (12)$$

$$i\omega e^{i\omega T} W_1 = \frac{d^2 W_0}{dZ^2} - 2i\Omega_1 W_0 + e^{i\omega T} \left(\frac{d^2 W_1}{dZ^2} - 2i\Omega_1 W_1 \right)$$

$$\frac{d^2 W_0}{dZ^2} - 2i\Omega_1 W_0 = 0 \quad (13)$$

$$W_0 = A e^{(2i\Omega_1)^{1/2} Z} + B e^{-(2i\Omega_1)^{1/2} Z} \quad (14)$$

$$\frac{d^2W_1}{dZ^2} - 2i\Omega_1 W_1 - i\omega W_1 = 0 \quad (15)$$

$$\frac{d^2W_1}{dZ^2} - (2i\Omega_1 + i\omega)W_1 = 0 \quad (16)$$

$$W_1 = Ae^{(2i\Omega_1+i\omega)^{1/2} Z} + Be^{-(2i\Omega_1+i\omega)^{1/2} Z} \quad (17)$$

By boundary conditions $A = 0$ and $B = T$

$$W = T\left[\exp(-\sqrt{2i\Omega_1})Z + \exp(i\omega T)\exp(-\sqrt{2i\Omega_1 + i\omega})Z\right] \quad (18)$$

The dimensionless skin friction at the plate z = 0 is calculated from (18) as

$$\left(\frac{dW}{dZ}\right)_{Z=0} = \frac{T}{-\sqrt{2i\Omega_1}} + \frac{T\exp(i\omega T)}{-\sqrt{2i\Omega_1 + i\omega}} \quad (19)$$

6 Results and Discussion

Figure 2. 1 shows the axial velocity profiles of the fluid for several values of Ω_1 with T = 0. 4, the profile steadily decrease and becomes steady as height (i. e. z) increases. It is further noted that the axial velocity decreases with increase in rotation at a particular height.

Figure 2. 2 shows the axial velocity profiles of the fluid for a fixed $\Omega_1 = 0.8$, with various values of T. It is seen that the axial velocity increases with increase in time. It is found that the profiles are not widely separated. They all become steady as height increases.

Figure 2. 3 shows the transverse velocity profiles of the fluid for various values of Ω_1 with T = 0. 2. The profile steadily decrease with increase in Ω_1 up to the height Z = 0. 9 approximately and after that heights they steadily increase with a crossover at Z = 0. 9 approximately.

Figure 2. 4 shows that the transverse velocity profiles of the fluid for various values of T, fixed $\Omega_1 = 0.8$. It is found from the figure the velocity increases with increase in time and all become steady at a certain height

From the above table 1. 1 it can be seen that for a fixed value of Ω_1, both skin friction for axial direction (τ_x) and for transverse direction (τ_y) increase with an increase in time. It is to be seen that the skin friction in axial direction is less than that in the transverse direction.

From the above table 1. 2 it is seen that for a fixed time both skin friction for axial direction (τ_x) and for transverse direction (τ_y) decrease with an increase in the rotation parameter Ω_1. It is to be seen that the skin friction in axial direction is less than that in the transverse direction.

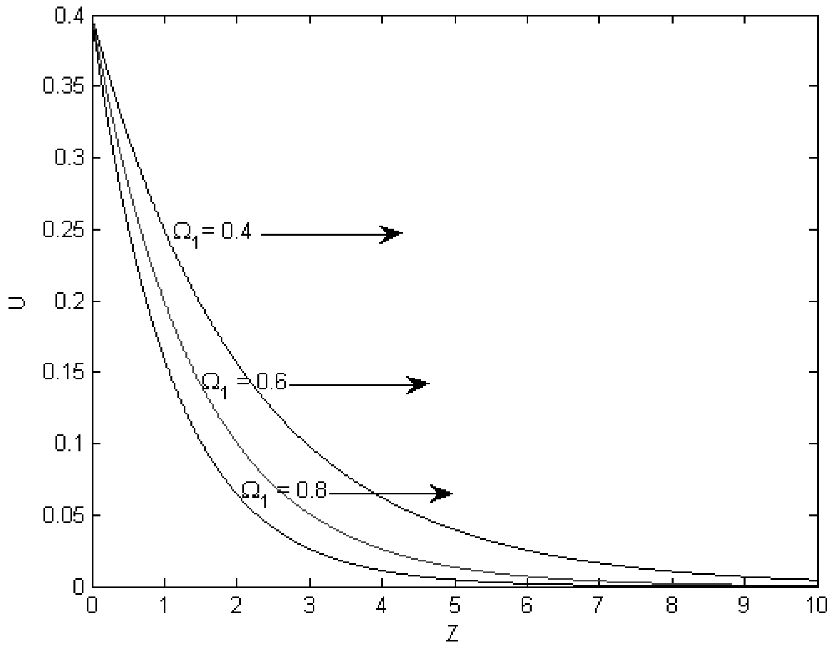

Figure 2. 1 Axial velocity profiles for fluid with T= 0. 4 and various values of rotation parameters

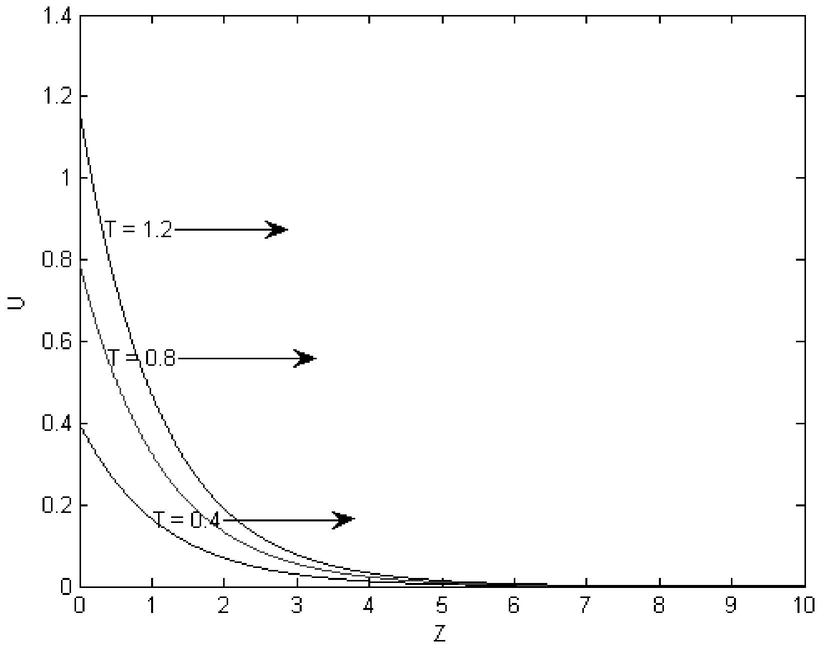

Figure 2. 2 Axial velocity profiles for fluid with $\Omega_1 = 0.8$ and various values of T.

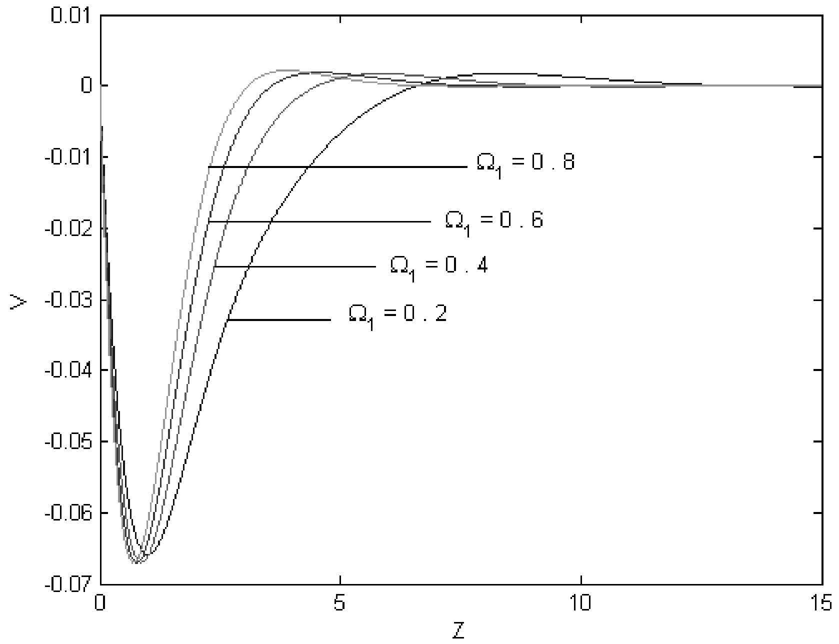

Figure 2. 3 Transverse velocity profiles for fluid with T =0. 2 and various values of the rotation parameter

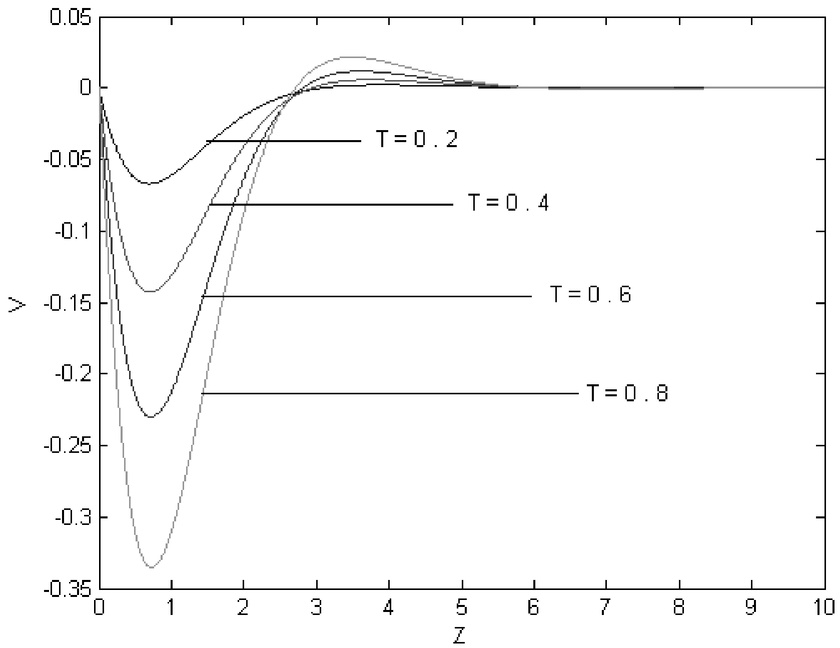

Figure 2. 4 Transverse velocity profiles for fluid with , $\Omega_1 = 0.8$ and various values of time T.

Table-1. 1: Variation of the skin-friction components τ_x, τ_y for $\Omega_1 = 0.8$ with various values of T

T	Ω_1	τ_x	τ_y
0. 2	0. 8	0. 0047	0. 4852
0. 4	0. 8	0. 0188	0. 9702
0. 6	0. 8	0. 0423	1. 4546
0. 8	0. 8	0. 0752	1. 9382
1	0. 8	0. 1175	2. 4206

Table-1. 2: Variation of the skin-friction components τ_x, τ_y for time T = 0. 2 with various values Ω_1 (Rotation parameter)

T	Ω_1	τ_x	τ_y
0. 2	0. 4	0. 0089	0. 9444
0. 2	0. 6	0. 0062	0. 6410
0. 2	0. 8	0. 0047	0. 4852
0. 2	1	0. 0038	0. 3904
0. 2	3	0. 0032	0. 1323

6 CONCLUSION

It has been found that the velocity profiles for varying times are not similar. For a fixed time the velocity profiles along the direction of motion of the plate decreases with an increase in Ω. But in the case of transverse velocity there is a reverse trend taking place at z = 2. 5. The velocity profiles attains a steady state at a particular height due the fact that the plate and the fluid rotate together with angular velocity normal to the plate. It can be seen that at a fixed instant both τ_x, τ_y increase with increase in rotation parameter Ω_1. It causes a gradual thinning of the boundary layer which develops on the plate.

7 References

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