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Graceful and Even Graceful Labeling of Some New Graphs

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Abstract

In this paper, we prove that union of two star graphs $K_{1,n}$ and $K_{1,n}$ is graceful, two copies of even cycle C_n sharing common edge, the tensor product of $K_{1,n}$ and P_2 are even graceful graphs. In addition to this we prove that the splitting graph and the shadow graph of bistar $B_{n,n}$ admit even graceful labeling.

Keywords: Graceful labeling, simple even graceful labeling, shadow graph, splitting graph

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary[10]. The symbols *V(G)* and *E(G)* will denote the vertex set and edge set of a graph *G*. The cardinality of the vertex set is called the *order* of *G* denoted by *p*. The cardinality of the edge set is called the *size* of *G* denoted by *q*. A graph with *p* vertices and *q* edges is called a (*p*, *q*) graph. In 1985, Lo[11] introduced the notion of *edge-graceful graphs*. In [9], Gayathri et al., introduced the *even edge-graceful graphs* and further studied in. In [12], Sin-Min Lee, Kuo-Jye Chen and Yung-Chin Wang introduced the *k-edge-graceful graphs*. Here we introduce *simple even edge-graceful graphs*. We provide a brief summary of definitions and other information which serves as prerequisites for the present investigations.

A simple graph $G(m,n)$ with m vertices and n edges is *graceful* if there is a labeling

l of its vertices with distinct integers from the set $\{0,1,2,...,n\}$ So that the induced edge labeling e defined by $e(uv) = |l(u - l(v))|$ assigns each edge a different label. A graph which admits graceful labeling is called a *graceful graph*. A graph $G = (V(G), E(G))$ is said to admit *simple even graceful labeling* if *f: V(G) → {0, 1, 2, ...2q }* is injective and the induced function *f *: E(G) →{2, 4, 6, ……2q}* defined as f* *(uv)* = *|f(u)-f(v)|* is bijective. A graph which admits simple even graceful labeling is called an *even graceful graph*. For a graph *G the splitting graph S*' of *G* is obtained by adding a new vertex *v*' corresponding to each vertex *v* of *G* such that *N(v) = N(v'). The shadow graph* $D_2(G)$ of a connected graph *G* is constructed by taking two copies of *G* say *G*' and *G*''. Join each vertex u_0 in G_0 to the neighbours of the corresponding vertex *v*' in *G*".

2. Results

Theorem 2.1 Union of $K_{1,n}$ and $K_{1,n}$ is graceful

Proof: Let $u_1, u_2, ..., u_n$ be the vertices of a star $K_{1,n}$ with u be the apex vertex and $v_1, v_2, ..., v_n$ be the vertices of another star $K_{1,n}$ with v be the apex vertex. Let G be the graph obtained by joining above star with their end vertices u_i and v_i for $i = 1,2,...,n$

Let w_i be the new vertex by joining u_i and v_i.

We define $f : V(G) \to \{0,1,2,...,q-1\}$ as follows

$f(u) = 0$

$f(v) = n$

$f(w_1) = 2n$

$f(w_2) = 2n-1$

$f(w_3) = 2n-2$

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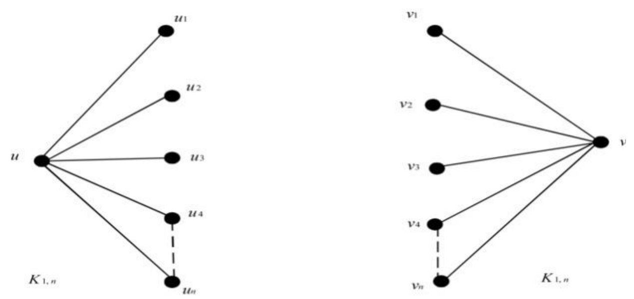
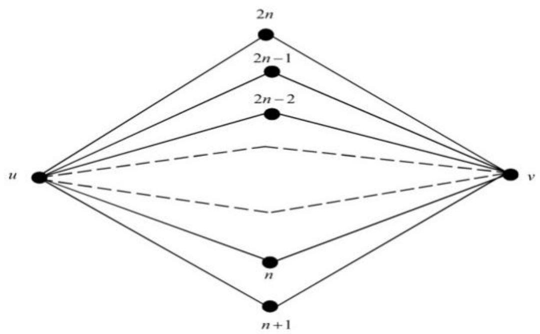
$f(wn) = n+1$

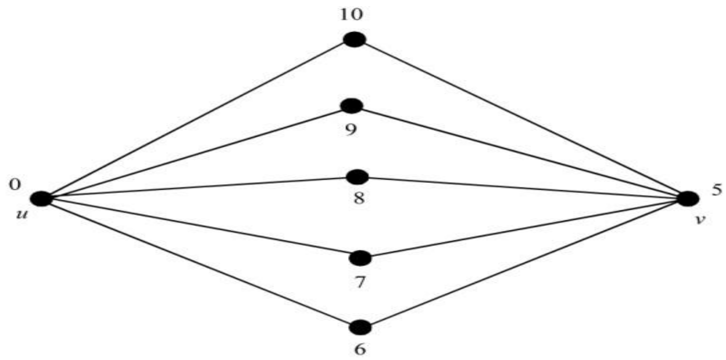
The above defined function f Provides a graceful labeling.





Join of $K_{1,n}$ and $K_{1,n}$ with their end points is graceful

Example



Join of $K_{1,5}$ and $K_{1,5}$ with their end points is graceful

Theorem 2.2 Two copies of even cycle C_n sharing a common edge is a simple even graceful graph.

Proof: Let $v_1, v_2, ..., v_n$ be the vertices of cycle C_n of even order. Consider two copies of cycle C_n.Let G be a graph with |V(G)|=2n-2 and |E(G)|=2n-1 denote the graph for two copies of even cycle C_n sharing a common edge. Without loss of generality let this edge be $e = v_{\frac{n+2}{2}} v_{\frac{3n}{2}}$. To define $f : V(G) \to \{0,1,2,...,2q\}$ two cases are to be considered.

Case 1: $n \equiv 0(\mathrm{mod}\,4)$.

For $1 \le i \le n+1$

$f(v_i) = i - 1$ if i is odd

$= 4n - i$ if i is even

For $n + 2 \le i \le \frac{3n}{2}$

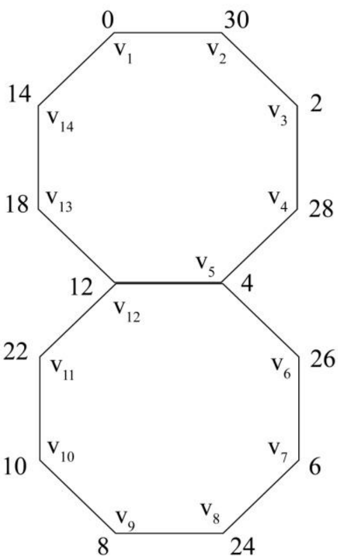
$f(v_i) = i$ if i is even

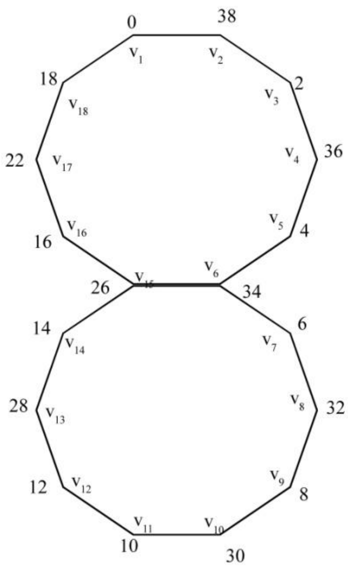
$= 4n - (i+1)$ if i is odd

For $\frac{3n+2}{2} \le i \le 2n-2$
$f(v_i) = i$ if i is even
$= 4n-(i+1)$ if i is odd

Case 2: $n \equiv 2 (\mathrm{mod}\, 4)$.
For $1 \le i \le n$
$f(v_i) = i-1$ if i is odd
$= 4n-i$ if i is even
For $n+2 \le i \le \frac{3n}{2}$
$f(v_i) = i$ if i is even
$= 4n-i+1$ if i is odd
$f(v_i) = n-1$ if $i = n+1$
For $\frac{3n+2}{2} \le i \le 2n-2$
$f(v_i) = i$ if i is even
$= 4n-(i+1)$ if i is odd

Above defined labeling pattern exhausts all possibilities and in each case the graph under consideration admits graceful labeling.





Two copies of cycle C_8 sharing common edge Two copies of cycle C_{10} sharing common edge

Theorem 2.3 $K_{1,n}(T_p)P_2$ is a simple even graceful labeling

Proof: Let $u_{1,}u_2,\ldots,u_n,u_{n+1}$ be the vertices of star $K_{1,n}$ with u_1 be the apex vertex. Let $v_{1,}v_2$ be the vertices of P_2. Let G be the graph $K_{1,n}(T_p)P_2$. We divide the vertices of G into two disjoint sets

$T_1 = \{(u_{i,}v_1) \mid i = 1,2,\ldots,n+1\}$

$T_2 = \{(u_i, v_2) \mid i = 1,2,\ldots,n+1\}$

Define $f : V(G) \to \{0,1,2,\ldots,2q\}$ as follows

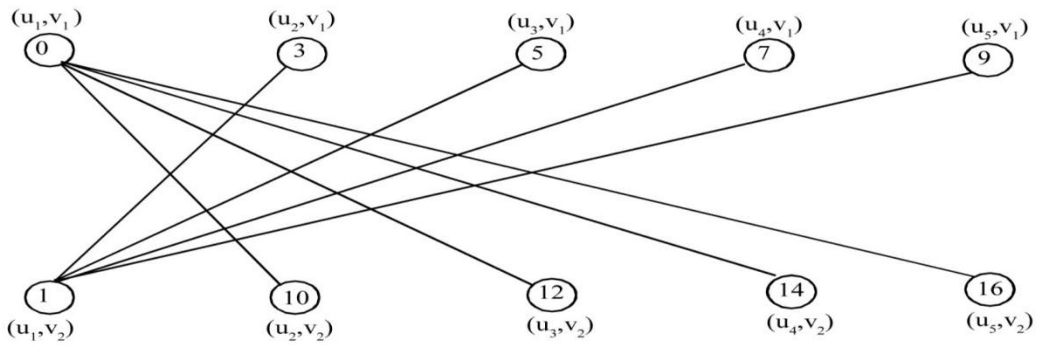
$f(u_{1,}v_1) = 0$

$f(u_i, v_1) = 2i-1$ for $2 \le i \le n+1$

$f(u_{1,}v_2) = 1$

$f(u_i, v_2) = 2(n+i)-2$ for $2 \le i \le n+1$

The above defined function f provides labeling for tensor product of $K_{1,n}$ and path P_2. That is $K_{1,n}(T_p)P_2$ is an even graceful graph.



Tensor product of $K_{1,4}$ and P_2

Theorem 2.4 $S'(B_{n,n})$ is a simple even graceful graph.

Proof. Consider $B_{n,n}$ with the vertex set $\{u, v, u_i, v_i, 1 \le i \le n\}$ where u_i, v_i the pendant vertices are.

In order to obtain $S'(B_{n,n})$ add u', v', u_i' , v_i' vertices corresponding to u, v, u_i, v_i where $1 \le i \le n$

If $G = S'(B_{n,n})$ then

$|V(G)| = 4(n+1)$ and $|E(G)| = 3(2n+1)$. We define vertex labeling

$f = V(G) \to \{0,1,2,\ldots,12n+6\}$ as follows.

$f(u) = 0$

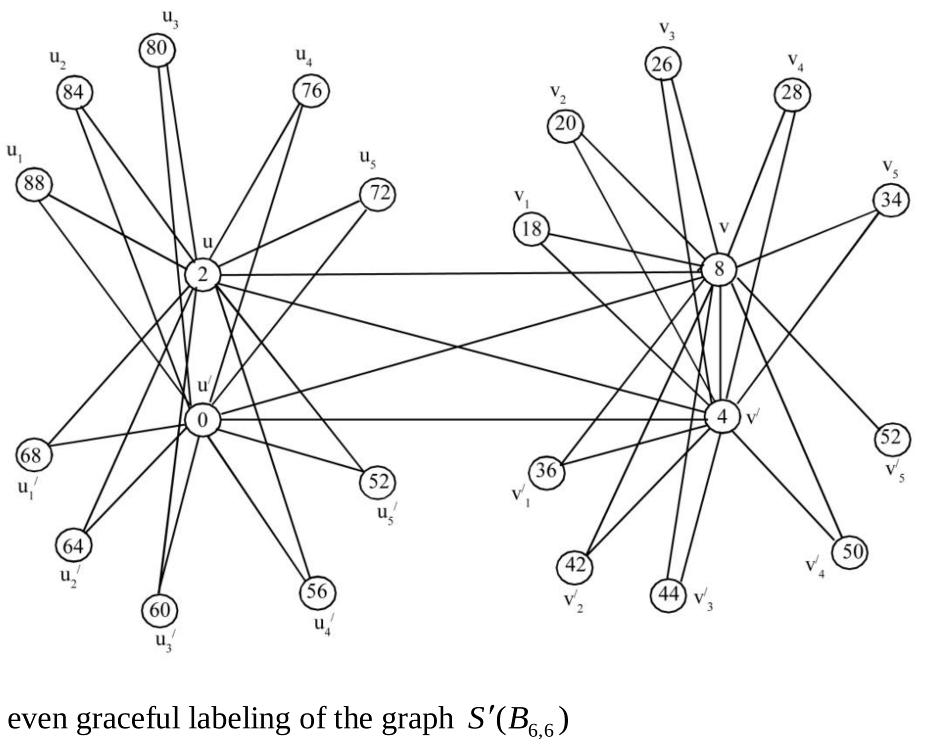
$f(v) = 4$

$f(u') = 2$
$f(v') = 5$
$f(u_i) = 6 + 4i$, $1 \le i \le n$
$f(u'_i) = 12n + 8 - 2i$, $1 \le i \le n$
$f(v'_1) = f(u'_n) + 2$
$f(v'_{1+i}) = f(v'_1) - 2i$, $1 \le i \le n$-1
$f(v'_1) = f(v'_n) - 2$
$f(v_{1+i}) = f(v'_1) - 4i$ $1 \le i \le n$-1

The vertex function f defined above induces a bijective edge function $f^* : E(G) \to \{2,4,\ldots,2n+6\}$

Thus f is a simple even graceful labeling of $G = S'(B_{n,n})$

Hence $S'(B_{n,n})$ is an even graceful graph**.**



Simple even graceful labeling of the graph $S'(B_{6,6})$

Theorem 2.5 $D_2(B_{n,n})$ is a simple even graceful graph.

Proof. Consider two copies of $B_{n,n}$ Let $\{u, v, u_i, v_i : 1 \le i \le n\}$ and $\{u', v', u'_i, v'_i : 1 \le i \le n\}$ be the corresponding vertex sets of each copy of $B_{n,n}$. Let G be the graph $D_2(B_{n,n})$. Then $|V(G)| = 4(n+1)$ and $|E(G)| = 4(2n+1)$

We define vertex labeling $f = V(G) \rightarrow \{0,1,2,\ldots,16n+8\}$ as follows

$f(u) = 2$

$f(v) = 8$

$f(u') = 4$

$f(v') = 4$

$f(u_i) = 16n + 12 - 4i$, $1 \leq i \leq n$

$f(u_i') = f(u_n) - 4i$, $1 \leq i \leq n$

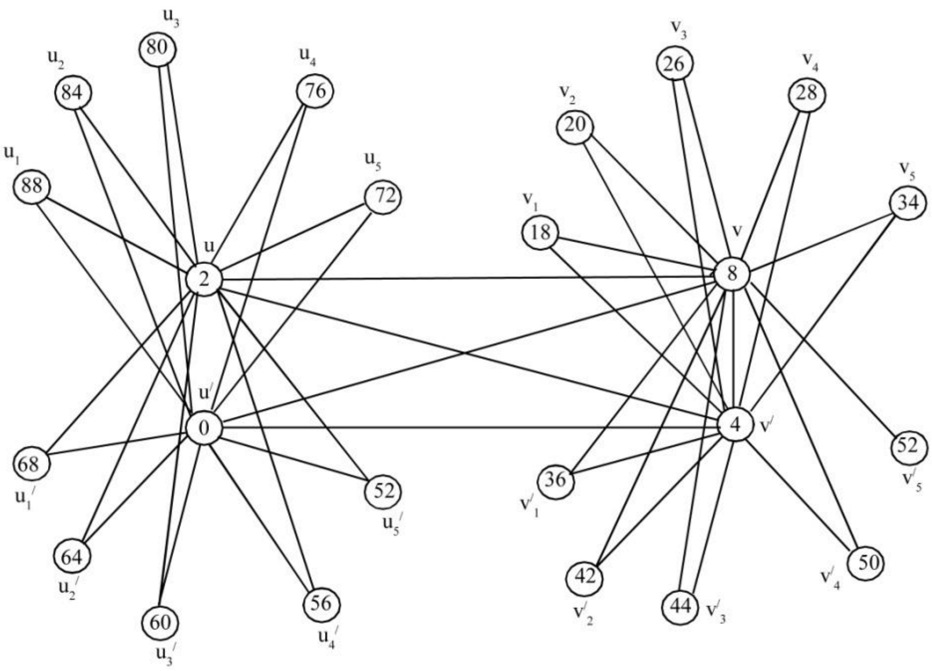
$f(v_{1+2i}) = 18 + 8i$, $0 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1$

$f(v_{2+2i}) = 20 + 8i$, $0 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor - 1$

$f(v_{1+2i}') = f(v_{n-1}) + 8(i+1)$, $0 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1$

$f(v_{2+2i}') = f(v_n) + 8(i+1)$, $0 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor - 1$

The vertex function f defined above induces a bijective edge function $f^* : E(G) \rightarrow \{2,4,\ldots,16n+8\}$. Thus, f is a simple even graceful labeling for $G = D_2(B_{n,n})$. Hence, $D_2(B_{n,n})$ is an even graceful graph.



Simple even graceful labeling of the graph $D_2(B_{5,5})$

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