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Some General Results on Elegant Graphs

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Abstract:

A connected graph with on edges is called elegant if it is possible its vertices with distinct numbers mod (m+1) i.e. (0, 1, 2…m) in such a way that the values of on the edges obtained by sums mod(m+1) of their end points labeling are distinct and non zero. In this paper, the general results on elegant graphs is shown.

Key words: Elegant labeling, regular graph, pseudo odd edges labeling, functions, complete binary tree.

I **INTRODUCTION**

A sample graph a with E edges is called elegant if the vertices of G can be labeled with distinct integer (0, 1, 2, …. R) in such a way that the set of values on the edges obtained by the sums mod (E+1) of the labels of their end vertices is {1, 2, …..E)}
In this paper by a graph, we mean an undirected graph without loops or Multiple edges and we denote the cycle on n vertices by C_n and the path on n vertices by P_n (Also certain special classes of graphs are shown to the elegant and more general results for establishing elegantes. More over, some general results on elegant graphs, add the edge labeling and pseudo odd edge labeling are discussed with some theorems.

II PRELIMINARIES

Definition: 1

A connected graph with m edges is called elegant if it is possible to lable its vertices with distinct numbers mod (m+1) ie (0, 1, 2….m) in such a way that the values on the edges obtained by sums mod(m+1) of their end points labeling are distinct and non zero(1).

DEFINITION: 2

An edge coloring of a graph is said to be optimal (or classical coloring) if no two edges incident with the same vertex, have the same color, and the minimum number of colors is used.

DEFINITION: 3

An edge coloring of a graph is said to be omni color if every edge has a different color.

DEFINITION:4

The family M_n of graph G is such that there exists an optimal edge coloring of the complete graph K_n in which G appear as a omni colored subgraph (i.e) every edge of G has different color.

DEFINITION: 5

A connected graph with m edges is called harmonious if it is possible to lable its vertices with distinct numbers (mod m) in such a way that the values on the edges obtained by sums (mod m) of their end points lablings are also distinct (2).

III ELEGANT LABELINGS AND EDGE COLORING

In this first part of this paper, we prove the existence of a coloring at K_n with a omini colored path on n vertices as subgraph, we had been conjectectured by Hartman [2].
In the second part we prove that the cycle on n vertices is elegant if and only if $n \neq 1$ mod(4) and give a new construction of an elegant labeling of the path $P_{n,}$ $n \neq 4$

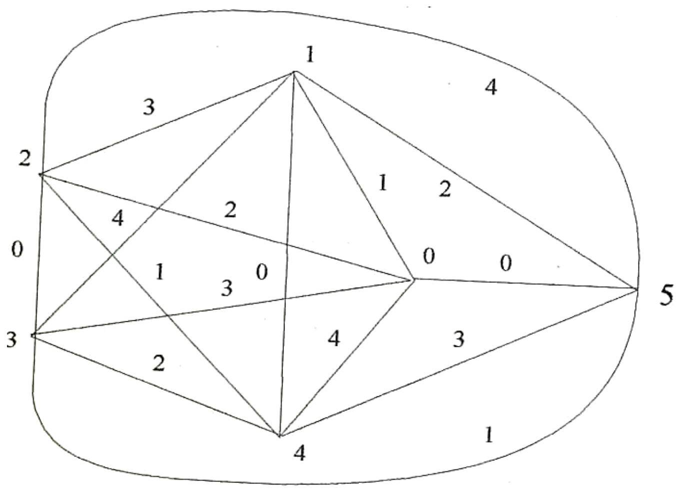
THEOREM: 2.1.1

The path on n vertices belongs to M_n if and only if $n \neq 4$, 6

PROOF:

Case I

In the classical coloring of the edges of $K_{\ p+1}$ with 2p+1, colors, we label the vertices 0, 1, 2… 2p and then color edges according to the sum mod (2p+1) of the labels of their end points. If the vertices form a regular polygon, 0, 1, 2…. 2p then a color class is a set of parallel edges. The boundary is a omni colored cycle. By deleting one edge we obtain a omni colored path on 2p+1 vertices (3).
In every optimal edge-coloring of $K_{\ p+1}$ exactly one color is missing at each vertex and all these missing colors are different. Extend the graph K_{2p+1} to K_{2p+2} thus we obtain a coloring of K_{2p+2} with 2p+1 colors by coloring the edges incident to the new vertex with these missing colors.



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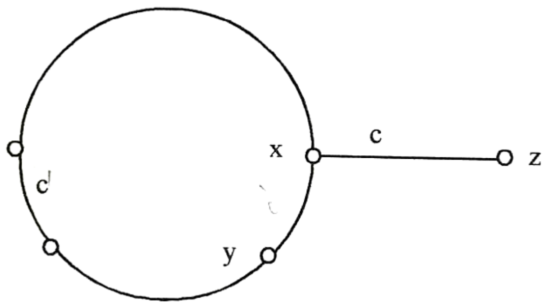
Assume we are able to color K_{2p+1} in such a way that

i) In the boundary every color appears exactly once expect one color c which does not appear and one color c^1 which appears twice.

(ii) the vertex x where is missing is incident to an edge {x, y} of the boundary colored with Extend this coloring to $K_{2p+2.}$ Let z be the new vertex then {z, x} is colored with c and by deleting the edge {x, y} from the boundary and adding the edge {x, z} we obtain a omni colored path {y………x.z}



We are now going to exhibit such a coloring of K_{2p+1} by some changes on the classical coloring defined above.

Sub case 1:

P= 0 (mod 3)

The cycle defined by the vertices 1, 0, 2, 2p, 3, 2p-1…. $2p/^{3+1,}$ $4p/^{3+1,}$ 1 has its edges colored 1, 2, 1, 2…………………..2, 1, $4p/^{3+2,\,2}$. We change this coloring to 2, 1, 2, 1….$4p/^{3+2,\,1}$

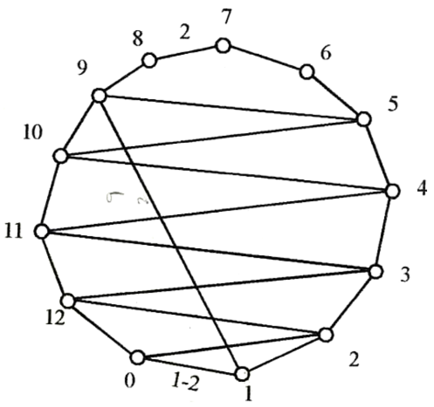
We have a required coloring K_{2p+1} with c =1, c^1=2, x= p+1, Y=p+2

In the new coloring

The color missing at vertex 1 is now 4p/3+2 instead of 2.

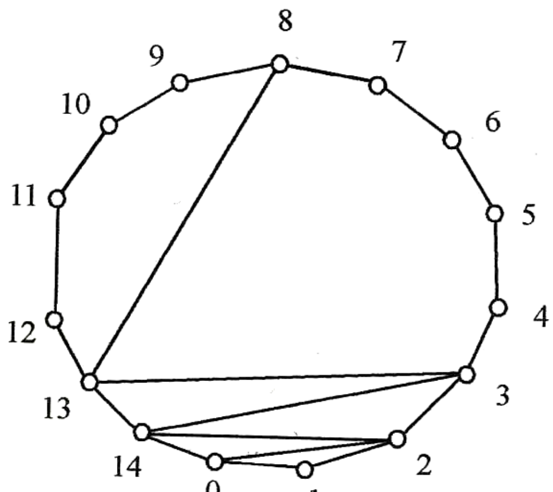
The color missing at vertex 2p/3+1 is now instead of 4p/3+2

EXAMPLE: P=6



The cycle 1, 0, 2, 12, 3, 11, 4, 10, 5, 9, 1 is colored 1, 2, 1, 2….1, 10. We change the coloring to 2, 1, 2, 1…….10, 1. Required coloring of K_{13} with c=1, c=2, x=7, y=8.
In the new coloring
The color missing at vertex 1 is now 10 instead of 2
The color missing at vertex 5 is now 2 instead of 10.

Sub Case 2
P=1 (mod(3))
The path 1, 0, 2, 2p, 3, 2p-1, …. (p+2) /3 (5p+4) /3, p+1 is colored 1, 2, 1, 2…./2, 1(2p+4)/3 and the color and the color 1 is missing at p+1. We change this coloring to 2, 1, 2, 1…. 1 (2p+4)/3, 1. Now the color 1 is missing in 1 so that we have a required coloring of K_{p+1} with c=1, c^1=2, x=1 and y=0
In the new coloring
The color missing at vertex p+1 is now (2p+4)/3 instead of 1.
The color missing at vertex (p+2)/3 is now 2 instead of (2p+4) / 3
EXAMPLE: P=7



Subcase 3

P=2 (mod3)

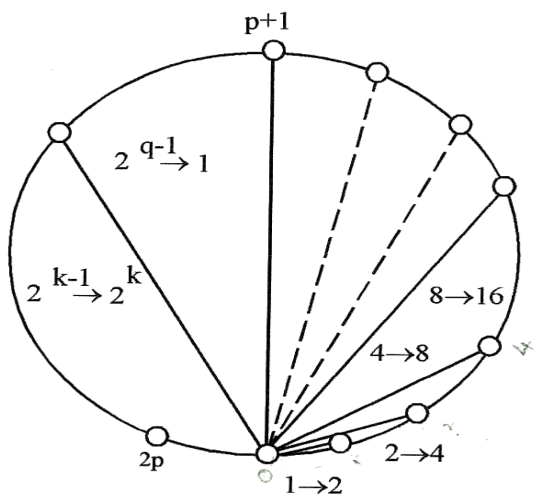
Consider the sequence of vertices 1, 2, 4…2^k, …. (taken mod(2p+1)}. All these vertices are different until 1 is met again (because 2p+1 is odd)

Sub case 3.1

Assume first that 2p does not belong to the sequence 1, 2, 4…2^k, ….2^q = p+1, 1. Then all the edges between the vertex 0 and 1, 2, …… 2^k, …. p+1 except the first one are not in the boundary.

In the classical coloring for each k the (0, 2^k) is colored with 2^k, i.e. the sequence of edges{0, 1}, (0, 2), …is colored 1, 2, 4, ….2^q… and the color 2^{k+1} is missing at vertex 2^k. Color this edge with 2^{k+1}. At the vertex 0 we did nothing else than a circular permutation on the colors used for all these adjacent edges. Thus we still have a coloring.

This is a required coloring with c=1, c^1=2, x=1 and y=0



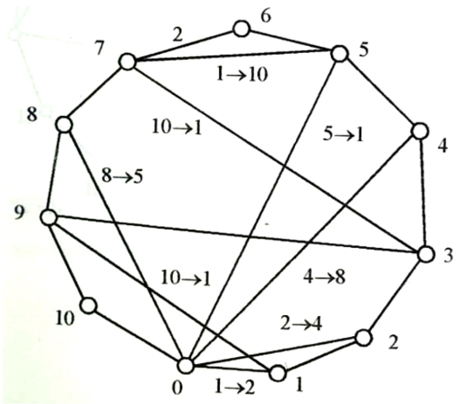
Subcase 3.2

If we are not in the above case, we have a sequence of different vertices 1, 2, 4..2^k, …p, 2p. In this case we cannot use the same change because the edge {0, 2p} is in the boundary.

Subcase 3.2.1

P=5 (mod 6)

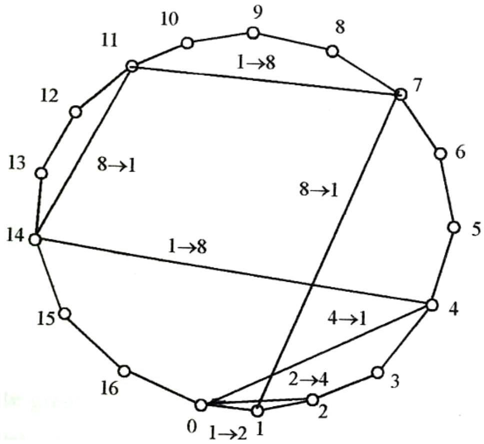
There is a path (p, p+2, p-2, p+4, p-4, ….3 2p-1, 1) the edges of which are colored 1, 2p, 1, 2p, …1, 2p



P=5

Consider now the sequence of edges {0, 1}, (0, 2), (0, 4)… (0, 2^k), …(O, p} (p, p+2), {p+2, p-2} {p-2, p+4}…. (3, 2p-1}, {2p-1, 1} {2p-1, 1} colored 1, 2, 4…2^k, p, 1, 2p, 1, 2p….1, 2p
This yields a required coloring with c=1, c^1=2, x=p+1 and y=p+2

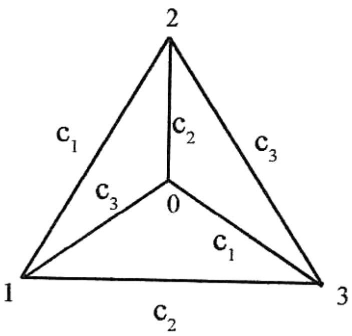
Sub case: 3.2.2
P=2, (mod 6)
We have P/2 = (mod 3)
Consider the path P = $u_{0,}$ $u_{1,}$ ……..u_1…..$u_{(2p-4)/3}$
u_1 = p/2 – 3i/4 if I =0 (mod4)
u_1 = 3p/2+2+3(i-1)/4 if i=1(mod 4);
u_1 = 3p/2-1-3(i-2)/4 if i=2 (mod 4)
u_1 = p/2+3+3(i-3)/4 if i=3 (mod 4)
u_0 = p/2, $u_{1\,=}$ 3p/2+2, $u_{2\,=}$ 3p/2-1, $u_{1\,=}$ p/2+3
u_4 = $u_{1\,=}$ p/2-3 $u_{1\,=}$ 3p/2+5
$_{(2p-4)/3}$ =p/2-{3(2p-4) /3}/4
= p/2- p/2+1
= 1



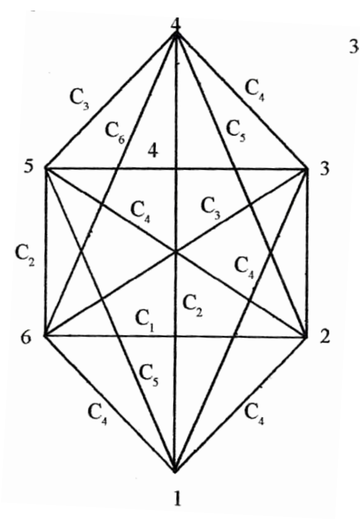
Therefore the path P=(p/2, 3p/2+2, 3p/2-1. p/2+3, p/2-3, 3p/2+5, 3p/2-4, p/2+6, p/2-6….4, 2p-2, p+3, p-1.1)

Therefore, edges of P are colored 1, p, 1, p…………1, p. Consider now the sequence of edges {0, 1}, (0, 2), (0, 4)… {0, p/2}, {p/2, 3p/2+2}, {3p/2+2, 3p/2-1) ….(p-1, 1) Colored 1, 2, 4…2^k…….p/2.1.p, 1, p….1, p1.
We have a required coloring with c=1, c^1=2, x=p+1 and y=p+2
The edges of P are colored 1, p, 1, p……1, p. consider now the sequence of edge [0, 1](0, 2), (0, 4)…. (0, p/2}, {p/2, 3p/2+2}, (3p/2/2+2, 3p/2-1)….(p-1, 1) Colored 1, 2, 4… 2^k ………..p/2, 1, p, 1, p….1, p. we change these colors to 2, 4, …… 2^k ……..p/2, 1, p, 1….1, p, 1.
We have a required coloring with c=1, c^1=2, x=p+1 and y=p+2

Case 2:
Sub case 2.1
In the complete graph $K_{4,}$ if we take any path P_4, it does not have a classical color.
P_4 € M_4. Since the edges $v_{1,}$ v_2 and v_3 v_4 being non-adjacent have the same color. i.e. the paths on 4 vertices does not belong to M_4



Sub case 2.2
In the complete graph K_6 we take any path P_6 they have atleast two edges are parallel. Therefore by classical coloring, they get the same color.
Therefore P_6 € M_6
i.e. the paths on 6 vertices does not belong to M_6



Thus we conclude that the paths on n vertices belong to M_n if and only if $n \neq 4, 6$ Hence the theorem.

IV CONCLUSION

Hence some general results on elegant graphs, add the edge labeling and pseudo odd edge labeling are proved.

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