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Quantum Algorithm for Minimum Metric Bottleneck Wandering Salesperson Problem by Numbering Method

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Abstract

A quantum algorithm for the minimum metric bottleneck wandering salesperson problem by a numbering method and its example are reported. The shortest path from the initial city *s* to the final city *f* passing through *m* cities that are included *s* and *f* is decided. A computational complexity of a classical computation is (m-2)!. The computational complexity becomes about 3 $(log_2\ (m\text{-}2))^2\ (m\text{-}2)^2$ by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates and the numbering method. Therefore, a polynomial time process becomes possible.

AMS Subject Classification: Primary 81-08; Secondary 68R10, 68W40.

Keywords: Quantum algorithm, minimum metric bottleneck wandering salesperson problem, numbering method, computational complexity, polynomial time.

1. Introduction

A development of a quantum computer is advanced. Haroche and Wineland [1] made the very first steps towards building it. Quantum algorithms that have been started by Deutsch-Jozsa's algorithm for the rapid solution [2-4] are expanded the application range by Shor's algorithm for the factorization [3-5], Grover's algorithms for the database search [3, 6, 7] and so on. A quantum algorithm for the 3-SAT problem by a numbering method has recently been reported by Fujimura [8]. Its computational complexity becomes a polynomial time. The minimum metric bottleneck wandering salesperson problem [9] is examined by the numbering method this time. Therefore, its result is reported.

2. Minimum Metric Bottleneck Wandering Salesperson Problem

The shortest path from the initial city s to the final city f passing through m cities that are included s and f is decided [9]. A computational complexity of a classical computation is $(m\text{-}2)!$.

3. Quantum Algorithm

It is assumed that m cities of P_s (x_s, y_s), P_0 (x_0, y_0), P_1 (x_1, y_1), ⋯, $P_{m\text{-}3}$ $(x_{m\text{-}3}, y_{m\text{-}3})$ and P_f (x_f, y_f) are set, P_s and P_f are fixed, and a distance between P_i and P_j is L (i, j)[= L (j, i)]. Therefore, path of P_0, P_1, ⋯, $P_{m\text{-}4}$, $P_{m\text{-}3}$ is considered.

(1) The number of the repeated permutation of m-2 cities is $(m\text{-}2)^{m\text{-}2}$.

(2) The number of permutation of m-2 cities is $(m\text{-}2)!$.

When m-2 cities are P_0, P_1, ⋯, $P_{m\text{-}4}$ and $P_{m\text{-}3}$, a_0 $(m\text{-}2)^{m\text{-}3}$ + a_1 $(m\text{-}2)^{m\text{-}4}$ + ⋯+ $a_{m\text{-}4}$ $(m\text{-}2)^1$ + $a_{m\text{-}3}$ $(m\text{-}2)^0$ = U is the numbering datum from 0 to $(m\text{-}2)^{m\text{-}2}$-1 [The 0-th datum is 0, 0, ⋯, 0 and 0. The $((m\text{-}2)^{m\text{-}2}\text{-}1)$-th datum is $(m$-3), $(m$-3), ⋯, $(m$-3) and $(m$-3).] in (1). In (2), it is assumed that the first datum is 0, 1, ⋯, m-3, and the $(m$-2)!-th datum is $(m$-3), $(m$-4), ⋯, 0, the V-th datum is obtained from v_1 $(m\text{-}3)!$ + v_2 $(m\text{-}4)!$ + ⋯+ $v_{m\text{-}3}1!$. Each of t_i [$1 \le i \le m\text{-}2$. i is an integer.] is 1 piece of permutation from 0 to m-3. When v_i is 0 from i=1 to i=m-4 sequentially, t_i is the smallest number in remained numbers. When v_i isn't 0 from i=1 to i=m-4 sequentially, and v_{i+1}, v_{i+2}, ⋯, $v_{m\text{-}4}$ and $v_{m\text{-}3}$ are 0, t_i is the v_i-th small number in remained numbers, and $t_{i+1} > t_{i+2} > \cdots > t_{m\text{-}3} > t_{m\text{-}2}$ is selected in remained numbers. When v_i isn't 0 from i=1 to i=m-4 sequentially, and there are $v_{i+1} \neq 0$ or $v_{i+2} \neq 0$ or $\cdots$or $v_{m\text{-}4} \neq 0$ or $v_{m\text{-}3} \neq 0$, t_i is the (v_i+1)-th small number in remained numbers. When $v_{m\text{-}3}$ is 1, $t_{m\text{-}3}<t_{m\text{-}2}$ is selected in remained numbers. Therefore, t_1 $(m\text{-}2)^{m\text{-}3}$ + t_2 $(m\text{-}2)^{m\text{-}4}$ + $\cdots$+ $t_{m\text{-}3}$ $(m\text{-}2)^1$ + $t_{m\text{-}2}$ $(m\text{-}2)^0$ is U (V). This method is named the numbering method for this problem. g is the minimum integer that follows (m-2)!/1 $\leq 4^g = 2^{2g}$, because a number of combinations of an answer is at least 1. U (V=1), U (V= ((m-2)!/4)-1), U (V= ((m-2)!/16)-1), $\cdots$, U (V= ((m-2)!/$4^{g\text{-}1}$)-1) and U (V= (m-2)!/4^g) are computed. M_1 that is a starting distance value is decided at random. Next, a quantum algorithm is shown as the following.

First of all, quantum registers $|a_0>$, $|a_1>$, $\cdots$, $|a_{m\text{-}3}>$, $|b>$, $|c_0>$, $|c_1>$, $\cdots$, $|c_{m\text{-}4}>$, $|d_1>$, $|d_2>$, $|e>$ and $|k>$ are prepared. When P is the minimum integer that is log_2 (m-2) or more, each of $|a_h>$ that h is an integer from 0 to m-3 is consisted of P quantum bits [=qubits]. States of $|a_0>$, $|a_1>$, $\cdots$, $|a_{m\text{-}3}>$, $|b>$, $|c_0>$, $|c_1>$, $\cdots$, $|c_{m\text{-}4}>$, $|d_1>$, $|d_2>$, $|e>$ and $|k>$ are a_0, a_1, $\cdots$, $a_{m\text{-}3}$, b, c_0, c_1, $\cdots$, $c_{m\text{-}4}$, d_1, d_2, e and k, respectively.

Step 1: Each qubit of $|a_0>$, $|a_1>$, $\cdots$, $|a_{m\text{-}3}>$, $|b>$, $|c_0>$, $|c_1>$, $\cdots$, $|c_{m\text{-}4}>$, $|d_1>$, $|d_2>$, $|e>$ and $|k>$ is set $|0>$.

Step 2: The Hadamard gate H [3, 4] acts on each qubit of $|a_0>$, $|a_1>$, $\cdots$, $|a_{m\text{-}4}>$ and $|a_{m\text{-}3}>$. It changes them for entangled states. The total states are $(2^p)^{m\text{-}2}$.

Step 3: It is assumed that a quantum gate (A) changes $|b>$ for $|1>$ in $a_h<m$-2, or it changes $|b>$ for $|0>$ in the others of a_h. As a target state for $|b>$ is 1, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [3,6,7] act on $|b>$. When Q is the minimum even integer that is $(2^p/ (m\text{-}2))^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is Q, because they are a couple. Next, an observation gate (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{m\text{-}3}>$. Therefore, each state of $|a_h>$ is 0, 1, $\cdots$, m-4 and m-3, and the total states become $(m\text{-}2)^{m\text{-}2}$.

Step 4: It is assumed that a quantum gate (B) changes $|c_0>$, $|c_1>$, $\cdots$, $|c_{m\text{-}5}>$ and $|c_{m\text{-}4}>$ for $|c_0 + 1>$, $|c_1 + 1>$, $\cdots$, $|c_{m\text{-}5} + 1>$ and $|c_{m\text{-}4} + 1>$ in a_h = 0, 1, $\cdots$, m-5 and m-4, respectively. This action repeats from $|a_0>$ to $|a_{m\text{-}3}>$. As the target state for $|c_0>$ is 1, (PI) and (IM) act on $|c_0>$. When R_1 is the minimum even integer that is $((m\text{-}2)/ (m\text{-}3))^{(m\text{-}3)/2}$ or more, the total number that (PI) and (IM) act on $|c_0>$ is R_1. Next, (OB) observes $|c_0>$. Therefore, only the paths that contain 1 piece of 0 remain. The number of data is (m-2) $(m\text{-}3)^{m\text{-}3}$. As the target state for $|c_1>$ is 1, (PI) and (IM) act on $|c_1>$. When R_2 is the minimum even integer that is $((m\text{-}3)/ (m\text{-}4))^{(m\text{-}4)/2}$ or more, the total number that (PI) and (IM) act on $|c_1>$ is R_2. Next, (OB) observes $|c_1>$. Therefore, only the paths that contain 1 piece of 1 remain. The number of data is (m-2) (m-3) $(m\text{-}4)^{m\text{-}4}$.

Similarly, these actions are repeated sequentially from $|c_2>$ to $|c_{m\text{-}4}>$. Only the paths that contain 1 piece of number from 0 to m-3, respectively, remain. The number of data is $(m\text{-}1)!$ [$=W_0$].

Step 5: It is assumed that a quantum gate (C (0, 1)) changes $|d_1>$ and $|d_2>$ for $|L\ (s,\ a_0) + L\ (a_0,\ a_1)>$ and $|\ (m\text{-}2)^{m\text{-}3}a_0 + (m\text{-}2)^{m\text{-}4}a_1>$, respectively, from $|a_0>$ and $|a_1>$. Similarly, (C (i, i+1)) [$1 \le i \le m\text{-}4$. i is an integer.] changes $|d_1>$ and $|d_2>$ for $|d_1 + L\ (a_i,\ a_{i+1})>$ and $|d_2 + (m\text{-}2)^{m\text{-}3\text{-}(i+1)}a_{i+1}>$, respectively, from $|a_i>$ and $|a_{i+1}>$. This action is repeated sequentially from $|a_1>$ to $|a_{m\text{-}4}>$. (C (m-4, m-3)) changes $|d_1>$ and $|d_2>$ for $|d_1 + L\ (a_{m\text{-}4},\ a_{m\text{-}3}) + L\ (a_{m\text{-}3},\ f)>$ and $|d_2 + (m\text{-}2)^0 a_{m\text{-}3}>$, respectively, from $|a_{m\text{-}4}>$ and $|a_{m\text{-}3}>$. Therefore, $|d_1>$ and $|d_2>$ become $|L_{\text{total}} = L\ (s,\ a_0) + L\ (a_0,\ a_1) + \cdots + L\ (m\text{-}4,\ m\text{-}3) + L\ (m\text{-}3,\ f)>$ and $|U>$, respectively.

Step 6: It is assumed that a quantum gate (D) changes $|e>$ for $|e + d_1>$ in $d_1 \le M_1$, or it changes $|e>$ for $|e + M_1 + d_2>$ in the others of d_1.

Step 7: It is assumed that a quantum gate (E_1) changes $|k>$ for $|1>$ in $e \le M_1$ or $M_1 + U\ (V{=}1) \le e \le M_1 + U\ (V{=}\ ((m\text{-}2)!/4)\text{-}1)$, or it changes $|k>$ for $|0>$ in the others of e. As the target state for $|k>$ is 1, (PI) and (IM) act on $|k>$. The number of the data that is included in $e \le M_1$ or $M_1 + U\ (V{=}1) \le e \le M_1 + U\ (V{=}\ ((m\text{-}2)!/4)\text{-}1)$ is $W_1 \approx (m\text{-}2)!/4$. When T_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|k>$ is $T_1 \approx 2$. Next, (OB) observes $|k>$, and the data of W_1 remain. Similarly, (E_i) [$2 \le i \le g\text{-}1$. i is the integer.] changes $|k>$ for $|1>$ in $e \le M_1$ or $M_1 + U\ (V{=}1) \le e \le M_1 + U\ (V{=}\ ((m\text{-}2)!/4^i)\text{-}1)$, or it changes $|k>$ for $|0>$ in the others of e. As the target state for $|k>$ is 1, (PI) and (IM) act on $|k>$. The number of the data that is included in $e \le M_1$ or $M_1 + U\ (V{=}1) \le e \le M_1 + U\ (V{=}\ ((\mathrm{m}\text{-}2)!/4^i)\text{-}1)$ is $W_i \approx (m\text{-}2)!/4^i$. When T_i is the minimum even integer that is $(W_{i\text{-}1}/W_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|k>$ is $T_i \approx 2$. Next, (OB) observes $|k>$, and the data of W_i remain. These actions are repeated sequentially from 2 to g-1 at i. (E_g) changes $|k>$ for $|1>$ in $e \le M_1$, or it changes $|k>$ for $|0>$ in the others of e. As the target state for $|k>$ is 1, (PI) and (IM) act on $|k>$. The number of the data that is included in $e \le M_1$ is $W_g \approx 1$. When T_g is the minimum even integer that is $(W_{g\text{-}1}/W_g)^{1/2}$ or more, the total number that (PI) and (IM) act on $|k>$ is $T_g \approx 2$. Next, (OB) observes $|a_0>$, $|a_1>$, $\cdots$, $|a_{m\text{-}3}>$, $|b>$, $|c_0>$, $|c_1>$, $\cdots$, $|c_{m\text{-}4}>$, $|d_1>$, $|d_2>$, $|e>$ and $|k>$, and one of the data of W_g remains. Therefore, one example of paths that are $L_{\text{total}} \le M_1$ is obtained.

Step 8: When the state of $|k>$ is 1 or 0, M_1 is assumed to be M_2 [$< M_1$] or M_2 [$> M_1$], respectively, these computations from step 1 to step 8 are repeated. It is assumed that the minimum distance $M_{\min}$ obtains by repeating about $log_2\,(m\text{-}2)!$ [10].

4. Numerical Computation

It is assumed that there are m = 11, P_s (0, 0), P_0 (1,-2), P_1 (3,-1), P_2 (4, 1), P_3 (2, 3), P_4 (1,-1), P_5 (3,-2), P_6 (4, 0), P_7 (0, 1), P_8 (2, 2), P_f(-4, 3), g = 10 [9!/1 = 362880 $\leq 4^{10}$ = 1048576], U (V=1) = 6053444, U (V= (9!/4) – 1 = 90719) = 95584620 [for example, V = 90719 = 2·8! + 1·7! + 6·6! + 5·5! + 4·4! + 3·3! + 2·2! +1·1!, U = 95584620 = $2\cdot 9^8 + 1\cdot 9^7 + 8\cdot 9^6 + 7\cdot 9^5 + 6\cdot 9^4 + 5\cdot 9^3 + 4\cdot 9^2 + 0\cdot 9^1 + 3\cdot 9^0$], U (V = (9!/16)-1 = 22679) = 26275844, U (V = (9!/64) – 1 = 5669 = 10598820, U (V = (9!/256) – 1 ≈ 1417) = 6897444, U (V = (9!/1024) – 1 ≈ 353) = 6198556, U (V = (9!/4096) – 1 ≈ 88) = 6073780, U (V = (9!/16384) – 1 ≈ 21) = 6055612, U (V = (9!/65536) – 1 ≈ 5) = 6053596, U (V = (9!/262144) – 1 ≈ 1) = 6053444 and M_1 = 30.

First of all, $|a_0>$, $|a_1>$, ···, $|a_8>$, $|b>$, $|c_0>$, $|c_1>$, ···, $|c_7>$, $|d_1>$, $|d_2>$, $|e>$ and $|k>$ are prepared. When P is the minimum integer that is log_2 (m-2) = log_2 9 ≈ 3.170 ≤ 4 = P, each of $|a_h>$ that h is the integer from 0 to 8 is consisted of P = 4 qubits. States of $|a_0>$, $|a_1>$, ···, $|a_8>$, $|b>$, $|c_0>$, $|c_1>$, ···, $|c_7>$, $|d_1>$, $|d_2>$, $|e>$ and $|k>$ are a_0, a_1, ···, a_8, b, c_0, c_1, ···, c_7, d_1, d_2, e and k, respectively.

Step 1: Each qubit of $|a_0>$, $|a_1>$, ···, $|a_8>$, $|b>$, $|c_0>$, $|c_1>$, ···, $|c_7>$, $|d_1>$, $|d_2>$, $|e>$ and $|k>$ is set $|0>$.

Step 2: $\boxed{H}$ acts on each qubit of $|a_0>$, $|a_1>$, ···, $|a_7>$ and $|a_8>$. It changes them for entangled states. The total states are $(2^p)^{m-2} = (2^4)^9$.

Step 3: (A) changes $|b>$ for $|1>$ in a_h<9, or it changes $|b>$ for $|0>$ in the others of a_h. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When Q is the minimum even integer that is $(2^p/ (m\text{-}2))^{1/2} = (2^4/9)^{1/2} \approx 1.333 \leq 2 = Q$, the total number that (PI) and (IM) act on $|b>$ is Q. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_8>$. Therefore, each state of $|a_h>$ is 0, 1, ···, 7 and 8, and the total states become $(m\text{-}2)^{m-2} = 9^9$.

Step 4: (B) changes $|c_0>$, $|c_1>$, ···, $|c_6>$ and $|c_7>$ for $|c_0 + 1>$, $|c_1 + 1>$, ···, $|c_6 + 1>$ and $|c_7 + 1>$ in a_h = 0, 1, ···, 6 and 7, respectively. This action repeats from $|a_0>$ to $|a_8>$. As the target state for $|c_0>$ is 1, (PI) and (IM) act on $|c_0>$. When R_1 is the minimum even integer that is $(9/8)^{8/2} \approx 1.602 \leq 2 = R_1$, the total number that (PI) and (IM) act on $|c_0>$ is R_1. Next, (OB) observes $|c_0>$. Therefore, only the paths that contain 1 piece of 0 remain. The number of data is $9\cdot 8^8$. As the target state for $|c_1>$ is 1, (PI) and (IM) act on $|c_1>$. When R_2 is the minimum even integer that is $(8/7)^{7/2} \approx 1.596 \leq 2 = R_2$, the total number that (PI) and (IM) act on $|c_1>$ is R_2. Next, (OB) observes $|c_1>$. Therefore, only the paths that contain 1 piece of 1 remain. The number of data is $9\cdot 8\cdot 7^7$. Similarly, these actions are repeated sequentially from $|c_2>$ to $|c_7>$. Only the paths that contain 1 piece of number from 0 to 8, respectively, remain. The number of data is 9! [$=W_0$].

Step 5: (C (0, 1)) changes $|d_1>$ and $|d_2>$ for $|L\ (s,\ a_0) + L\ (a_0,\ a_1)>$ and $|9^8 a_0 + 9^7 a_1>$, respectively, from $|a_0>$ and $|a_1>$. Similarly, (C (i, i+1)) [$1 \leq i \leq 7$. i is an integer.] changes $|d_1>$ and $|d_2>$ for $|d_1 + L\ (a_i,\ a_{i+1})>$ and $|d_2 + 9^{7-i} a_{i+1}>$, respectively, from $|a_i>$ and $|a_{i+1}>$. This action is repeated sequentially from $|a_1>$ to $|a_7>$. (C (7, 8)) changes $|d_1>$ and $|d_2>$ for $|d_1 + L\ (a_7,\ a_8) + L\ (a_8,\ f)>$ and $|d_2 + 9^0 a_8>$, respectively, from $|a_7>$ and $|a_8>$. Therefore, $|d_1>$ and $|d_2>$ become $|L_{\mathrm{total}} = L\ (s,\ a_0) + L\ (a_0,\ a_1) + \cdots + L\ (a_7,\ a_8) + L\ (a_8,\ f)>$ and $|U>$, respectively.

Step 6: (D) changes $|e>$ for $|e + d_1>$ in $d_1 \leq M_1 = 30$, or it changes $|e>$ for $|e + 30 + d_2>$ in the others of d_1.

Step 7: (E_1) changes $|k>$ for $|1>$ in $e \leq 30$ or $30 + U\ (V=1) = 30 + 6053444 = 6053474 \leq e \leq 30 + U\ (V = (9!/4) - 1 = 90719) = 30 + 95584620 = 95584650$, or it changes $|k>$ for $|0>$ in the others of e. As the target state for $|k>$ is 1, (PI) and (IM) act on $|k>$. The number of the data that is included in $e \leq 30$ or $6053474 \leq e \leq 95584650$ is $W_1 \approx 9!/4$. When T_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|k>$ is $T_1 \approx 2$. Next, (OB) observes $|k>$, and the data of W_1 remain. Similarly, (E_i) [$2 \leq i \leq 9$. i is the integer.] changes $|k>$ for $|1>$ in $e \leq 30$ or $6053474 \leq e \leq 30 + U\ (V = (9!/4^i)-1)$, or it changes $|k>$ for $|0>$ in the others of e. As the target state for $|k>$ is 1, (PI) and (IM) act on $|k>$. The number of the data that is included in $e \leq 30$ or $6053474 \leq e \leq 30 + U\ (V = (9!/4^i)-1)$ is $W_i \approx 9!/4^i$. When T_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|k>$ is $T_i \approx 2$. Next, (OB) observes $|k>$, and the data of W_i remain. These actions are repeated sequentially from 2 to 9 at i. (E_{10}) changes $|k>$ for $|1>$ in $e \leq 30$, or it changes $|k>$ for $|0>$ in the others of e. As the target state for $|k>$ is 1, (PI) and (IM) act on $|k>$. The number of the data that is included in $e \leq 30$ is $W_{10} \approx 1$. When T_{10} is the minimum even integer that is $(W_9/W_{10})^{1/2}$ or more, the total number that (PI) and (IM) act on $|k>$ is $T_{10} \approx 2$. Next, (OB) observes $|a_0>$, $|a_1>$, $|a_2>$, $|a_3>$, $|a_4>$, $|a_5>$, $|a_6>$, $|a_7>$, $|a_8>$, $|b>$, $|c_0>$, $|c_1>$, $|c_2>$, $|c_3>$, $|c_4>$, $|c_5>$, $|c_6>$, $|c_7>$, $|d_1>$, $|d_2>$, $|e>$ and $|k>$, and one of the data of W_{10} remains. For example, when a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, b, c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, d_1, d_2, e and k are 6, 1, 5, 0, 4, 2, 3, 7, 8, 1, 1, 1, 1, 1, 1, 1, 1, 1, 27, $6 \cdot 9^8 + 1 \cdot 9^7 + 5 \cdot 9^6 + 0 \cdot 9^5 + 4 \cdot 9^4 + 2 \cdot 9^3 + 3 \cdot 9^2 + 7 \cdot 9^1 + 8 \cdot 9^0 = 265748516$, 27 and 1, respectively.

Step 8: In the example, the state of $|k>$ is 1. Therefore, M_1 is assumed to be $M_2 = 15$ [$< 27 < M_1 = 30$], and these calculations from step 1 to step 8 are repeated. It is assumed that the state of $|k>$ is 0. When the states of $|k>$ are 1, 1, 0 and 1 at $M_3 = 23$, $M_4 = 19$,

M_5 = 17, and M_6 = 18, respectively, M_{min} is 18 [= M_6]. Therefore, a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, b, c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, d_1, d_2, e and k are 4, 0, 5, 1, 6, 2, 8, 3, 7, 1, 1, 1, 1, 1, 1, 1, 1, 1, 18, $4\cdot9^8 + 0\cdot9^7 + 5\cdot9^6 + 1\cdot9^5 + 6\cdot9^4 + 2\cdot9^3 + 8\cdot9^2 + 3\cdot9^1 + 7\cdot9^0$ = 174944644, 18 and 1, respectively. As a result, the shortest path $Ps \to P_4 \to P_0 \to P_5 \to P_1 \to P_6 \to P_2 \to P_8 \to P_3 \to P_7 \to P_f$ is obtained.

5. Discussion and Summary

The computational complexity of this quantum algorithm [= S] becomes the following. In the order of the actions by the gates, the number of them is P (m-2) at $\boxed{H}$, m-2 at (A), Q (m-2) ≈ 2 (m-2) at (PI) and (IM), m-2 at (OB), m-2 at (B), $\Sigma_{i=0\to m-4} R_i$ ≈ 2 (m-3) at (PI) and (IM), (m-3) at (OB), 2 (m-3) at (C (i, i+1)) [$0 \le i \le m$-4. i is the integer.], 2 at (D), g at (E_i) [$1 \le i \le$ g. i is the integer.], $\Sigma_{i=1\to g} T_i$ ≈ 2g at (PI) and (IM), and g at (OB). These processes repeated about log_2 (m-2)!. Therefore, S becomes ((P + 10) (m-2) – 3 + 4g)log_2 (m-2)!. In the example of the numerical computation, S is 978. The computational complexity of the classical computation [= Z] is (m-2)! = 9! = 362880. After all, S/Z becomes about 1/371. When m is large enough, S becomes about 3 $(log_2 (m-2))^2 (m-2)^2$, where P is about log_2 (m-2), g is about ((m-2)/2)log_2 (m-2) and n! is about $n^n e^{-n} (2\pi n)^{1/2}$ [Stirling's formula], and S/Z is about 3 $(log_2 (m-2))^2 (m-2)^2/ (m-2)!$. For example, as for m = 50, S/Z is about $1/10^{56}$. Therefore, a polynomial time process becomes possible.

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