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Use of the GARCH Models to Energy Markets: Oil Price Volatility

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Abstract

This paper focuses on the profit of Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) models and their applications to the Value-At-Risk. More precisely, we give an extensive bibliographic overview of the developments of the ARCH-GARCH. To this end, this work add another interest of GARCH models which it made an application relates to exchange rate volatility of oil price. The study relates to the period 01 January 2009 to 31 December 2014, a total of 2192 observations given.
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Keywords: GARCH models, Volatility, energy prices.

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Introduction

"The recent volatility in oil prices presents an excellent opportunity for traders to make a profit if they are able to predict the right direction. Volatility is measured as the expected change in the price of an instrument in either direction. For example, if oil volatility is 15% and current oil prices are US $100, it means that within the next year traders expect oil prices to change by 15% (either reach $85 or $115). If the current volatility is more than the historical volatility, traders expect higher volatility in prices going forward. If the current volatility is lower than the long-term average, traders expect lower volatility in prices going forward." Investopedia (2015)
High price volatility is a long-standing characteristic of world oil markets and, more recently, of natural gas and electricity markets. However, there is no widely accepted answer to what the best models and measures of price volatility are because of the complexity of distribution of energy prices where this problem motivated several researchers.

The ARCH model proposed by Engle (1982) let these weights be parameters to be estimated. Thus, the model allowed the data to determine the best weights to use in forecasting the variance. A useful generalization of this model is the GARCH parameterizations introduced by Bollerslev (1986). "GARCH models have been developed to modeling the volatility of finance data. To this end, it plays important role in financial decisions. Volatility is one of the principal parameters employed to describe and measure the fluctuations of asset prices. It plays a important role in the modern financial analysis of which risk management, option valuation and asset allocation." Khindanova (2004)

Moreover, GARCH models include a study of efficiency of crude oil markets, an investigation of the effect of resource price uncertainty on rates of economic growth studies of the links between related products, testing the theory of storage and the leverage effect analysis of the effects of regulatory changes and the introduction of futures trading, and forecasting of energy prices. GARCH models specify conditional variance as a function of both past value of squared residuals and past values of the variance. Adrangi et al. (2001b) use GARCH to analyze the nature of non-linearity in energy prices. By the non linearity the authors mean second and higher order dependence between the energy prices. Adrangi et al. (2001a) use bivariate GARCH to investigate the dynamics of crude oil and diesel prices. Bivariate GARCH is also employed by Ng and Pirrong (1996) to study the behavior of refined petroleum spot and futures prices.

In addition, univariate GARCH models have been used to model oil price volatility since the early 1990s, and have become standard practice. Despite the explosion of new types of GARCH models, including multivariate GARCH Bollerslev (1986), fractionally integrated GARCH Baillie et al. (1996), nonparametric GARCH (Bühlmann and McNeil, (2002), and multiplicative component GARCH Engle and Sokalska (2012), simple models of the GARCH(1, 1) type remain very useful because they converge much faster to a local maximum in quasi-maximum likelihood estimation, while delivering forecasting performance that is not obviously inferior to multivariate models Wang and Wu (2012).

Recently, Efimova, and Serletis (2014) investigate the empirical properties of oil, natural gas, and electricity price volatilities using a range of univariate and multivariate GARCH models and daily data from wholesale markets in the United States for the period from 2001 to 2013.

Moreover, there are several studies confirm this results. As mentioned before early to this topic are and Zeghdoudi et.al (2013a, 2013b, 2014) and Chan and Grant (2015).

The goal of this paper is to add another profit of GARCH models whose this work contains an application relates to exchange rate volatility of oil price.

The structure of the paper is as follows. Section 2 considers representing several GARCH models. Finally, we make an application example for energy markets forecast: oil price using GARCH models.

Materials and Methods

GARCH Models

In the next subsection, we shall present several GARCH models. (see Zakoian(1991, 1994)).

GARCH (p, q) Models

A process $X\{t\}$ satisfies a representation GARCH (p, q) if:
$X_t = Z_t\sqrt{h_t}$ where $h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i X_{t-i}^2 + \sum_{i=1}^{q} \beta_i X_{t-i}^2$
where $Z_t \rightarrow N(0, \sigma^2)$
$\alpha_0 > 0$, $\alpha_i \geq 0$, $\forall i=1,.., q$ and $\beta_i \geq 0$, $\forall i=1,.., p$.
X_t admit the conditional moments:
$\mathbb{E}(X_t \setminus X_{t-1}) = 0$

$$V(X_t \setminus X_{t-1}) = h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i X_{t-i}^2 + \sum_{i=1}^{q} \beta_i X_{t-i}^2$$

GARCH (p, q) with errors Model

We consider a regressive linear model expressed under following form:
$Y_t = \mathbb{E}((Y_t \setminus Y_{t-1}) + \varepsilon_t$
where $\varepsilon_t \rightarrow N(0, \sigma^2)$ and $\mathbb{E}((\varepsilon_t \varepsilon_s) = 0$ if $s \neq t$, satisfying the condition with difference of martingale $\mathbb{E}(\varepsilon_t \setminus \varepsilon_{t-1}) = 0$. It is always supposed that the process ε_t can be written in the form:
$\varepsilon_t = Z_t\sqrt{h_t}$
where $Z_t \rightarrow N(0, \sigma^2)$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i X_{t-i}^2 + \sum_{i=1}^{q} \beta_i X_{t-i}^2$$

with
$\alpha_0 > 0$, $\alpha_i \geq 0$, $\forall i=1,..., q$ and $\beta_i \geq 0$, $\forall i=1,..., p$
sufficient to guarantee the positivity of h_t.

ARMA-GARCH models

Modeling GARCH can be applied not to the initial process, but to the process of innovation. This then makes it possible to introduce various additional effects of explanatory variables either into the conditional mean, or in the conditional variance. For example, we can consider a linear model of regression with errors GARCH:
$Y_t = X_t b + \varepsilon_t$ $\varepsilon_t \rightsquigarrow$ GARCH (p, q)
We can also consider a model ARMA with errors GARCH:
$\Phi(L) Y_t = \Theta(L)\varepsilon_t$ $\varepsilon_t \rightsquigarrow$ GARCH (p, q)
This model is called model ARMA-GARCH.
Then, we can design a model ARMA in which the non conditional variance of Y_t can have an effect on the conditional variance:
$\Phi(L) Y_t = \Theta(L)\varepsilon_t$
$\mathbb{E}(\varepsilon_t \setminus \varepsilon_{t-1}) = 0$

$$V(\varepsilon_t \setminus \varepsilon_{t-1}) = c + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-1} + \gamma_0 [\mathbb{E}(Y_t \setminus Y_{t-1})]^2 + \sum_{j=1}^{p} \gamma_i Y_{t-j}^2$$

GARCH-M model

Engle-Lilien-Robbins(1987) proposed models GARCH-M (General Autoregressive Conditional Heteroscedasticity in Mean) where the conditional variance is an explanatory variable of the conditional mean. These processes seem more adapted thus to a description of the influence of volatility on the output of the titles.

Moosa and Al-Loughani (1994) use GARCH-M to test the hypothesis of unbiasedness and efficiency of energy futures prices as forecasters of spot prices.Antoniou and Foster (1992) use GARCH-M to investigate the effect of introduction of futures trading on crude oil spot prices.

Definition The writing of model GARCH-M relates to non stationarity of its process of conditional variance and by an infinite non conditional variance. Let be a process Y_t, where $(Y_t) = 0$ This process is written in the following form

$$Y_t = X_t b + \delta h_t + Z_t \sqrt{h_t} = X_t b + \delta V(\varepsilon_t \setminus \varepsilon_{t-1}) + Z_t \sqrt{h_t}$$

$$\varepsilon_t = Z_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}$$

$$\mathbb{E}(\varepsilon_t \setminus \varepsilon_{t-1}) = 0$$

$$V(\varepsilon_t \setminus \varepsilon_{t-1}) = V(Y_t \setminus Y_{t-1}) = h_t$$

Besides the linear form of the writing of Y_t above: we can consider various alternatives of the relation between the dependent variable Y_t and the conditional variance. For example, we can consider the following cases:

Log-Linear form	$X_t = X_t b + \delta log h_t + \varepsilon_t$
Square Root form	$Y_t = X_t b + \delta \sqrt{h_t} + \varepsilon_t$

Asymmetric GARCH

Asymmetric GARCH models allow the volatility of energy prices to depend on the direction of a price shock:

$$\sigma_t^2 = \omega + (\alpha + \gamma d_{t-1})\varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2,$$

where d_t=1 if $\varepsilon_t < 0$ and zero otherwise and γ is the "leverage term". Morana (2001) uses this model to evaluate the performance of one-month ahead forecasts of daily Brent crude oil prices on the basis of their GARCH properties. Asymmetric GARCH has been found to have a superior fit compared to symmetric GARCH.

the ex-ante volatility is estimated on the basis of the generalized error distribution.

the estimated value of the scaling factor υ of the generalized error distribution is below two, the value associated with the normal distribution, thus indicating thick tails. Morana (2001) compares the performance of one-month ahead forecasts with the random walk model using the decomposition of the mean squared forecast error.

EGARCH

Similarly to the asymmetric GARCH, exponential GARCH (EGARCH) may be used to test for the presence of the leverage effect. It is used by Adrangi et al.(2001b) to show that conditional heteroskedasticity is the source of non-linearities in energy price data. In their study of crude oil, heating oil, and unleaded gasoline futures Adrangi et al.(2001b) find that crude oil and unleaded gasoline series may be modeled by EGARCH(1, 1) process:

$$\log(h_t) = \alpha_0 + \alpha_1 + \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \alpha_2 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| + + \beta_1 \log(h_{t-1}) + \beta_2 \mathrm{TTM},$$

where h_t is the conditional variance, ε_t is the disturbance term, and TTM is the time to maturity. The time to maturity is introduced in order to test the maturity effect, which implies that the volatility of a futures contract increases as the delivery date approaches. The parameter estimate α_2 is negative and significant, which provides evidence in support of the leverage effect.

FIGARCH

According to the FIGARCH model, conditional variance is fractionally integrated. In other words, the price shocks have persistent but not permanent effects on the conditional variance. Brunetti and Gilbert (2000) use the bivariate FIGARCH to study the interaction between the NYMEX and the IPE crude oil markets and their volatilities.

They find that the volatilities of both series are long memory processes. Also their respective volatility processes are fractionally integrated of the same order. Thus a bivariate FIGARCH and fractional cointegration model may be used by market participants who seek arbitrage opportunities between the two markets.

To define the bivariate GARCH, recall that a univariate GARCH(p, q) process is given by $[1-\beta(L)]\sigma_t^2 = \omega+\alpha(L)\,\varepsilon_t^2$, where α(L) and β(L) are lag polynomials of orders q and p, respectively.

By defining the innovations as $v_t = \varepsilon_t^2 - \sigma_t^2$, GARCH (p, q) process may be expressed in ARMA(m, p) terms:

$$[1-\alpha(L)-\beta(L)]\,\varepsilon_t^2 = \omega+[1-\beta(L)]\,v_t$$

SWARCH

Switching ARCH (SWARCH) models are developed for modeling conditional heteroskedasticity if the regimes governing the volatility process change during the time period under consideration. Susmel and Thompson (1997) find that the SWARCH model is best suited to modeling natural gas prices, which were largely deregulated since the end of the 1980s-beginning of the 1990s.

Stable GARCH

The study by Rachev and Mittinik (2000) incorporates both the time-varying conditional heteroskedasticity of energy returns and the non-normal distribution of the error term. The authors evaluate the performance of a GARCH model that is based on stable Paretian distributions of the error term. Rachev and Mittnik (2000) consider alternative models for the American Exchange (AMEX) oil index.They use the maximized likelihood value (ML) and Kolomogorov distance (KD) to make comparisons across the models. The likelihood function in the case of Pareto stable disturbances is defined as follows:

Max $L(\theta)=\prod_{t=1}^{T} S_{\alpha,\beta}\left(\frac{r_{t-\delta}}{c}\right)c^{-1}$

Where **α** is the index of stability, **β** is the skewness parameter, **c** is the scale parameter and δ is the location parameter of a stable Paretian distribution. The ML value is the overall measure of fit that allows one to compare different model specifications.

Summary of Existing Literature on Energy GARCH

There is an extensive literature on modeling conditional heteroskedasticity of energy prices. These models range from the basic ARCH to complex models of SWARCH, FIGARCH, and stable GARCH. the majority of the reviewed energy GARCH models are of low order the most common being GARCH(1, 1).Many of the applications favor modifications of GARCH, which allow for the asymmetric effects, i.e., asymmetric GARCH and EGARCH. Several authors find that GARCH-M models have best fitting properties. Some researchers employ the bivariate GARCH to study the links between closely related markets. More complex studies involve models based on fractional integration, switching regimes, and stable distributions of energy returns.

Most of the studies use daily or monthly spot and/or futures data on crude oil. GARCH modeling of price changes of petrochemicals is commonly carried out within the framework of error correction models. There are only single cases of studies of natural gas and coal. The only research indirectly related to electricity, deals with share prices of electricity companies. In our research, we use a range of energy commodities, including two crude oil series, two natural gas series, a series of electricity prices, and a series of Gas oil prices.

Results

The main results of this paper are the following modeling and application using software EViews. While applying to the data of the rate of exchange of the oil prices.

The data represent the rate of daily exchange of the oil prices(S_t). The study relates to the period 01 January 2009 to 31 December 2014, a total of 2192 observations given.

The first step of this application is study the stationary of the series. To this end, we used the unit root test of Dickey-Fuller (ADF) and Philips Péron test (PP).

Unit Root tests and Descriptive Analysis

In this section, we summarized unit root tests and descriptive analysis results of prix lnprix Dlnprix Dprix

Table 1 unit root test with α=1%

Test	**ADE**	**PP**
Prix	-1.758414	-2.065765
Lnprix	-2.178748	-2.981970
Dlnprix	-40.25772	-40.23678
Dprix	-47.08854	-47.12848

Table 1 confirm the stationarity of the series Dln prix and Dprix. Now, we use only the series Dprix

Table 2

	Mean	Median	Std.Dev	Skewness	Kurtosis	Jarque-B
Prix	0.095152	0.063664	1.851917	0.005052	2.938286	0.357179

According to Table 2 the skewness of oil price is 0.005052, so it is positive and the mean is larger than median, and there are right skewed distribution-most values are concentrated on left of the mean, with extreme values to the right. Kurtosis is 2.938286 is less than 3-Platykurtic distribution, flatter than a normal distribution with a wider peak. The probability for extreme values is less than for a normal distribution, and the values are wider spread around the mean. Jarque-Bera is 0.357179.

Table 3

Models	Adju R^2 R^2 R^2 R^2 R^2	SEE	BIC	RMSE	MAE	MAPE
GARCH(1, 1)	1.000000	1, 20^E-15	-65, 68464	1, 20^E-15	8.40^E-16	5, 48^E-14
GARCH(1, 2)	1.000000	1, 38^E-15	-65, 43098	1, 37^E-15	9, 71^E-16	6, 30^E-14
GARCH(2, 1)	1.000000	1.85^E-15	-64.92211	1, 84^E-15	1, 17^E-15	7, 55^E-14
GARCH(2, 2)	1.000000	4.28^E-15	-63.24255	4.27^E-15	2, 73^E-15	1, 77^E-13
GARCH(1, 3)	1.000000	4.90^E-15	-53.69498	4.89^E-15	3, 35^{E}13	2, 31^E-11

Discussion

The nonlinear GARCH-class models, which are capable of capturing long-memory and/or asymmetric volatility, exhibit greater forecasting accuracy than the linear ones, especially in volatility forecasting over longer time horizons, such as five or twenty days. Our results indicated that the degree of persistence of volatility was reduced by incorporating the variance changes into the volatility model.

According to Table 3 and Fig 1(Part A, B) that GARCH (1, 1) are clearly the best performing models as they receive the lowest score on fitting metrics whilst representing the lowest MAE, RMSE, MAPE, SEE and BIC among all models. They are closely followed by GARCH (1, 2) which is placed comfortably lower than GARCH(2, 1), GARCH(2, 2) and GARCH(1, 3). However the GARCH(1, 1) model is simple and easy to handle. The results also show that GARCH (1, 1) model improve the forecasting performance and confirm the work of Efimova and Serletis (2014) thus of Chan and Grant (2015).

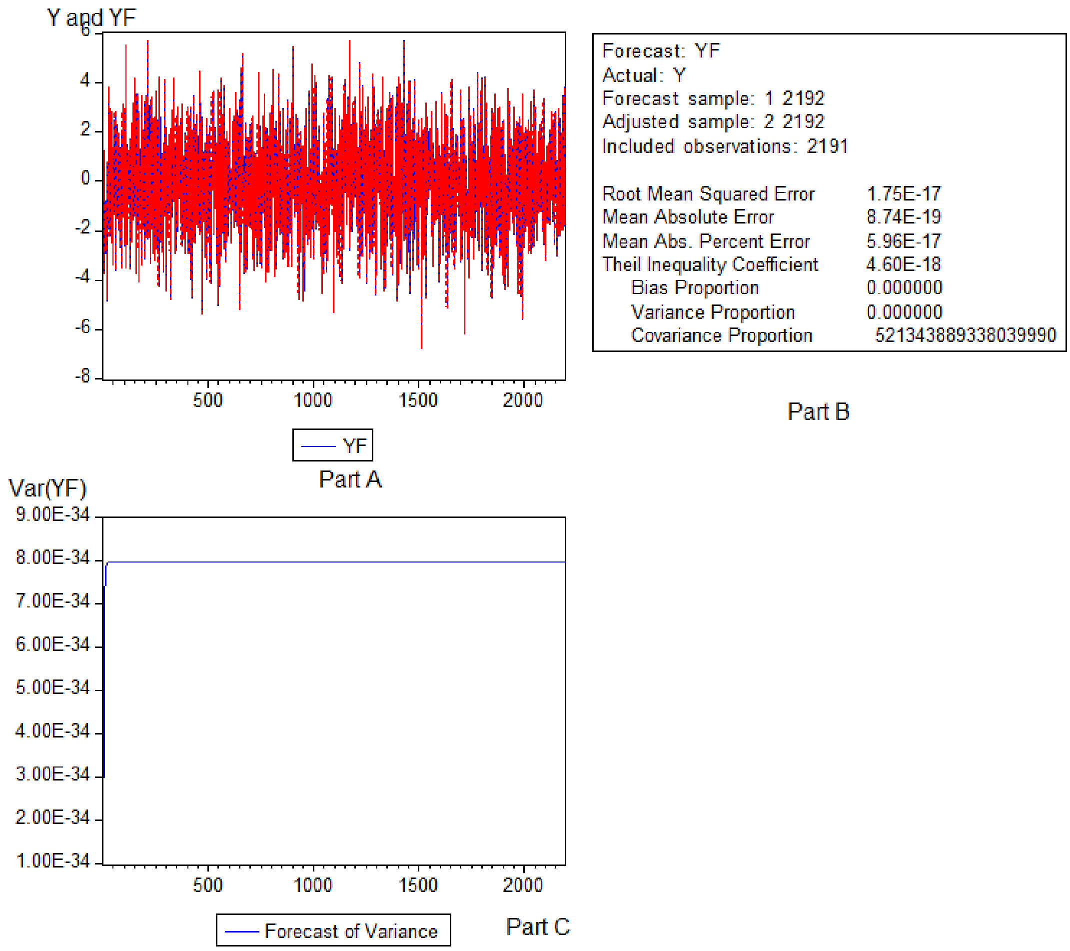

Fig1. GARCH (1, 1) oil price forecasting and variance forecast

Conclusion

This paper presented an empirical application of a range of univariate GARCH models to daily oil price data for the period 01 January 2009 to 31 December 2014, a total of 2192 observations given.

Our conclusion is that normal GARCH models explain some of the non-normality of the distribution of energy prices. When they do, the error term still exhibits skewness and leptokurtosis. At higher confidence levels, normal GARCH based estimates of

energy VAR perform marginally better than the ones commonly used by energy companies. To account for non-Gaussian distribution of energy returns and changing volatility, using the stable GARCH.
In future prospects this work can generalize on natural gas and electricity data. Also, using multivariate GARCH models.

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