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Solving Assignment Problems with fuzzy costs using circumcenter of centroids

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Abstract

Objective of this paper is to minimize the cost by using fuzzy numbers through fuzzy assignment problem. Considering each fuzzy cost as triangular fuzzy numbers the fuzzy assignment problem has been transformed into crisp values by using linguistic variables and solved by Hungarian technique. This new approx is proposed for ranking fuzzy numbers based on the Circumcenter of Centroids and an index of optimism is used to reflect the decision maker's optimistic attitude and also an index of modality that represents the neutrality of the decision maker. Triangular fuzzy numbers along with crisp numbers are ranked with the particularity that crisp numbers are to be considered as a particular case of fuzzy numbers.

Key words: Optimization, ranking of fuzzy numbers, triangular fuzzy numbers, Fuzzy Assignment problem, Circumcenter of Centroids.

Mathematics Subject Classification: 90C08, 90C10, 03E72,

1. Introduction

An assignment problem (AP) is a particular type of transportation problem where n tasks (jobs) are to be assigned to an equal number of n machines (workers) in one to one basis such that the assignment cost (or profit) is minimum (or maximum).Hence, it can be considered as a balanced transportation problem in which all supplies and demands are equal, and the number of rows and columns in the matrix are identical. Various ranking procedures have been developed since 1975 where the theories of fuzzy sets first introduced by Zadeh [21]. Ranking fuzzy numbers proposed by Jain [12] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [4] reviewed some of these ranking methods for ranking fuzzy subsets. Chen [6] presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois and Prade presented the mean value of fuzzy number. Lee and Li [15] presented a comparison of fuzzy numbers based on the probability measure of fuzzy events. Chen and Chen [7] derived a new method on ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations. F. Azman and L.Abdullah [11] and N. Ravi Shankar and P. Phani Bushan Rao[17] have given a review on Ranking Fuzzy Numbers Using the Centroid Point Method.

Most of the ranking procedures proposed in the literature use Centroid of trapezoid as reference point, as the Centroid is a balancing point of the trapezoid.The linguistic variable convert's qualitative data into quantitative data which is used to solve the fuzzy assignment problems. In section 2, a new method is proposed here which is based on Circumcenter to rank fuzzy quantities. Then calculate the Centroids of these three parts followed by the calculation of the Circumcenter of those Centroids. Finally, a ranking function is defined for the Euclidean distance between the Circumcenter point and the original point to rank fuzzy numbers. Most of the ranking procedures proposed in the literature use Centroid of trapezoid as reference point, as the Centroid is a balancing point of the trapezoid. But the Circumcenter of Centroids can be considered a much more balancing point as this point is equidistant from all the vertices which are Centroids. This method uses an index of optimism to reflect the decision maker's optimistic attitude and also uses an index of modality that represents the neutrality of the decision maker.

2. Preliminaries

Definition 2.1 The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in *X*.A function $\mu_{\tilde{a}}$ such that the value assigned to the element of the universal set *X* fall within a specified range i.e. $\mu_{\tilde{a}}$: X → [0,1] The assigned value indicates the membership grade of the element in the set *A*.The function $\mu_{\tilde{a}(x)}$ is called the membership function and the set $\hat{A}$ = $\{(x\ ,\ \mu_{\tilde{a}(x)}; x \in X\ )\}$defined by $\mu_{\tilde{a}(x)}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 A fuzzy number ậ = (a, b.c) is a triangular fuzzy number if its membership function $\mu_{\tilde{a}}$ is defined as

$$\mu_{ậ(x)} = \begin{cases} (x - a) / (b - a) \text{ if } a \leq x \leq b \\ (c - x) / (c - b) \text{ if } b \leq x \leq c \\ 0 \quad \text{otherwise} \end{cases}$$

Where a, b, c are real numbers

Definition 2.3 A fuzzy number ậ = (a, b.c, d) is a trapezoidal fuzzy number if its membership function $\mu_{\tilde{a}}$ is defined as

$$\mu_{\tilde{a}(x)} = \begin{cases} (x - a) / (b - a) \text{ if } a \le x \le b \\ 1 \quad \text{if } b \le x \le c \\ (x - d) / (c - d) \text{ if } c \le x \le d \\ 0 \quad \text{otherwise} \end{cases}$$

Where a, b, c are real numbers

Definition 2.4 If the membership function $f_{\tilde{A}}(x)$ is piecewise linear, Then $\tilde{A}$ is said to be a trapezoidal fuzzy number. If $w = 1$, then $\tilde{A} = (a, b, c, d; 1)$ is a normalized trapezoidal fuzzy number and $\tilde{A}$ is a generalized or non normal trapezoidal fuzzy number if $0 < w < 1$. The membership function of a trapezoidal fuzzy number is given by

$$\mu_{\tilde{a}(x)} = \begin{cases} w(x - a) / (b - a) \text{ if } a \le x \le b \\ w \quad \text{if } b \le x \le c \\ w(x - d) / (c - d) \text{ if } c \le x \le d \\ 0 \quad \text{otherwise} \end{cases}$$

The image of $\tilde{A} = (a, b, c, d; w)$ is given by-$\tilde{A} = (-d, -c, -b, -a; w)$.As a particular case if $b = c$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $\tilde{A} = (a, b, d; w)$.The value of "b" corresponds with the mode or core and $[a, d]$ with the support. If $w = 1$, then $\tilde{A} = (a, b, d)$ is a normalized triangular fuzzy number $\tilde{A}$ is a generalized or non normal triangular fuzzy number if $0 < w < 1$.

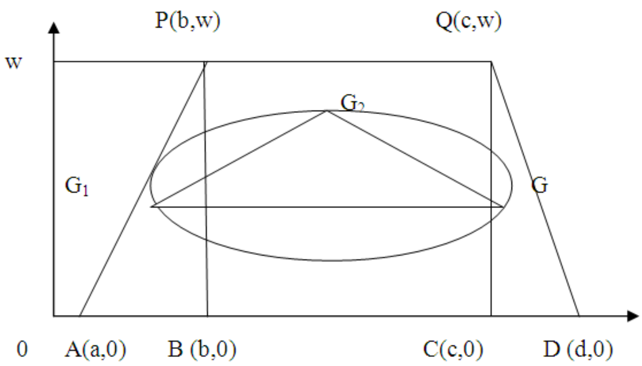

Figure 1 Circumcenter of centriod

Definition 2.5 Consider the Centroid of trapezoid as the balancing point of the trapezoid (Figure 1). Divide the trapezoid into three plane figures. These three plane figures are a triangle (*APB*), a rectangle (*BPQC*), and again a triangle (*CQD*), respectively. Each Centroid point (G_1 of *APB*, G_2 of *BPQC*, and G_3 of *CQD*) are balancing points, and the Circumcenter of Centroid points are equidistant from each vertex (which are Centroids). Therefore, these points are better reference points than the Centroid point of the trapezoid.

Suppose $\breve{A}$ = (a, b, c) be the triangular fuzzy number. Then the defuzzification of the fuzzy by grade mean integration method is $(\breve{A}) = \frac{(a+4b+c)}{6}$, Suppose $\breve{A}$ = (a, b, c, d) be the trapezoidal fuzzy number. Then the defuzzification of the fuzzy by grade mean integration method is $\rho(\breve{A}) = \frac{(a+2b+2c+d)}{6}$

A piecewise quadratic fuzzy function (PQFN) $\breve{A}$ = (a, b, c, d, e) is defined by the membership function as

$$\mu_{\breve{A}}(x) = \begin{cases} \frac{-(x-a)^2}{2(b-c)^2} + 1 & for \quad a \le x \le b \\ \frac{-(x-c)^2}{2(c-b)^2} + 1 & for \quad b \le x \le c \\ \frac{-(x-c)^2}{2(d-c)^2} + 1 & for \quad c \le x \le d \\ \frac{-(x-e)^2}{2(e-d)^2} + 1 & for \quad d \le x \le e \\ 0 \quad otherwise \end{cases}$$

The PQFN be a bell shapped function symmetric about the line x = a_3 has a supporting interval $\breve{a}$ = [a, e]. Moreover, c = $\frac{1}{2}$(a+e) and c-b = d-c, α-cut at level α =$\frac{1}{2}$ between the points (b, d) called cross over points also the interval of confidence at level is a = b{ a + 2 (b-c), e-2 (e-d) }

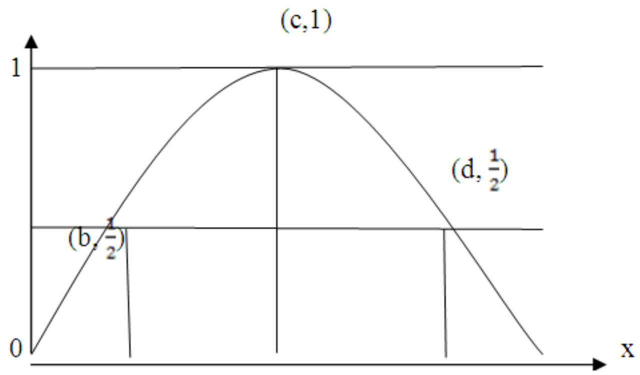

Fig 2.Picewise Quadratic Fuzzy Function(PQFN)

Consider a generalized trapezoidal fuzzy number $\tilde{A}$= (*a*, *b*, *c*, *d*; *w*).The Centroids of the three plane figures are $G_1 = ((a+2b)/3, w/3)$, $G_2 = ((b+c)/2, w/2)$, and $G_3 = ((2c+d)/3, w/3)$ respectively. Equation of the line G_1G_3 is $y = w/3$ and G_2 does not lie on the lineG_1G_3. Therefore, G_1, G_2 and G_3 are non-collinear and form a triangle. The Circumcenter of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number is denoted by $S_{\tilde{A}}(\overline{x_0}, \overline{y_0})$

As a special case, for triangular fuzzy number $\tilde{A}$ = (*a*, *b*, *d*; *w*), when *c* = *b* the Circumcenterr of Centroids is given by

$$S_{\tilde{A}}(\overline{x_0}, \ \overline{y_0}) = \left(\frac{a+4b+d}{6}, \ \frac{4\,(a-b)(d-b)+5w^2}{12w}\right)$$

Definition 2.6 For any decision maker whether pessimistic (α = 0), optimistic (α = 1), or neutral (α = 0.5), the ranking function of the trapezoidal fuzzy number $\tilde{A}$ = (*a*, *b*, *c*, *d*;*w*) which is mapped from the set of all fuzzy numbers to a set of real numbers is defined as $R(\tilde{A}) = \sqrt{\overline{x_0}^2 + \overline{y_0}^2}$ which is the Euclidean distance from the Circumcenter of the Centroids by the definition 2.5 and the original point.

Definition 2.7
Linguistic Variable
In more specific terms, a linguistic variable is characterized by a quintuple (ν, T(ν), U, G, M) in which ν is the name of the variable; T(ν) the term-set of ν, that is, the collection of its linguistic values; U is an universe of discourse; G is a syntactic rule which generates the terms in T(ν) and M is a semantic rule which associates with each linguistic value X its meaning, M(X***)***, where M(X) denotes a fuzzy subset of U. The meaning of a linguistic value X is characterized by a compatibility function**, *C:*** $U \rightarrow$ [0, I], which associates with each u in U its compatibility with X

3. Assignment Method

The assignment problem can be stated in the form of n x n cost matrix [c_{ij}] of real numbers as given in the following table

Jobs→ Persons ↓	1	2	3	---j---	n
1	C_{11}	C_{12}	C_{13}	--C_{1j}--	C_{1n}
2	C_{21}	C_{22}	C_{23}	--C_{2j}--	C_{2n}
-	-	-	-	-	-
-	-	-	-	-	-
I	C_{i1}	C_{i2}	C_{i3}	--C_{ij}--	C_{in}
-					
N	C_{n1}	C_{n2}	C_{n3}	--C_{nj}--	C_{nn}

Mathematically assignment problem can be stated as
Minimize $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \quad x_{ij}$ where i = 1, 2, 3….n, j = 1, 2, 3….n

Subject to
$\sum_{i=1}^{n} x_{ij} = 1$, i = 1, 2, 3….n ………….. (1)
$\sum_{j=1}^{n} x_{ij} = 1$ j = 1, 2, 3….n $x_{ij} \in \{0,1\}$

where $x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person is assigned the } j^{th} \text{ job} \\ 0 & otherwise \end{cases}$

is the decision variable denoting the assignment of the person i to job j, Cij is the cost of assigning the j^{th} job to the i^{th} person.

The objective is to minimize the total cost of assigning all the jobs to the available persons. (One job to one person). When the costs $\tilde{c}_{ij}$ are fuzzy numbers, then the fuzzy assignment problem becomes

$$Y(\tilde{z}) = \sum_{i=1}^{n} \sum_{j=1}^{n} Y(\tilde{c}_{ij}) \quad x_{ij} \ \ldots\ldots\ldots \qquad (2)$$

Subject to the same conditions (1)

4. Algorithm for the proposed method: (Balanced /Unbalanced assignment problem)

If the number of rows is not equal to the number of columns then the problem is termed as unbalanced assignment problem then this problem into change balanced assignment problem as follows necessary number of dummy row (s) / column(s) are added such that the cost matrix is a square matrix the values for the entries in the dummy row (s) / column(s) are assumed to be zero.

By defuzzifying the fuzzy cost coefficients into crisp coefficients by ranking fuzzy numbers based on the Circumcenter of Centroids method. This problem is obviously the crisp assignment problem of the form (1) which can be solved by Hungarian Method.

The algorithm for the proposed method are

Step 1: First test whether the given fuzzy cost matrix of an fuzzy assignment problem is a balanced one or not, ie, If the number of row (s) / column(s) is not equal to the number of column(s)/row(s) then the problem is termed as unbalanced assignment problem then.Change this unbalanced assignment problem by adding the number of dummy row (s) / column(s) such that the cost matrix is a square matrix, the values for the entries in the dummy row (s) / column(s) are assumed to be zero. If it is a balanced one (i.e, number of persons are equal to the number of works) then go to step 2. If it is an unbalanced one then convert it into a balanced one and then go to step 2.

Step 2: Replace the cost matrix C_{ij} with linguistic variables by triangular or trapezoidal fuzzy numbers.

Step 3: Find Ranking by Circumcenter of Centroids method.

Step 4: Replace Triangular or Trapezoidal numbers by their respective ranking indices.

Step 5: Find the effectiveness matrix. Subtract the minimum element of each row of the given cost matrix from all of the elements of the row. Examine if there is at least one zero in each row and in each column. If it is so, stop here, otherwise subtract the minimum element of each column from all the elements of the column. The resulting matrix is the starting effectiveness matrix

Step6: Assign the zeroes:

(a). Examine the rows of the current effective matrix successively until a row with exactly one unmarked zero is found. Mark this zero, indicating that an assignment will be made there. Mark all other zeroes lying in the column of above encircled zero. The cells marked will not be considered for any future assignment. Continue in this manner until all the rows have taken care of

(b). Similarly for columns

Step 7: Check for Optimality. Repeat step 5, successively till one of the following occurs.

(a). There is no row and no column without assignment. In such a case, the current assignment is optimal.

(b). There may be some row or column without an assignment. In this case the current solution is not optimal.

Proceed to next step

Step 8: Draw minimum number of lines crossing all zeroes as follows.

If the number of lines is equal to the order of the matrix, then the current solution is optimal, otherwise it is not optimal.

Go to the next step

Step9: Examine the elements that do not have a line through them. Select the smallest of these elements and subtract the same from all the elements that do not have a line through them and add this element to every element that lies in the intersection of the two lines.

Step 10: Repeat this until an optimal assignment is reached

5. Numerical Example

Balanced Assignment Problem:

The sales manager of a car manufacturer currently has four sales people on the road meeting buyers. The sales people are in Austin (W), Boston(X), San Francisco(Y) and Chicago (Z) they have to fly to three other cities Denver (Job1), Edmonton(Job2), Alberta(Job3) and Fargo (Job4)with assignment cost varying between 0$ to 50$. The cost matrix elements are linguistic variables which are replaced in terms of fuzzy numbers. The table below shows the cost of airplane tickets in dollars between these cities. Where should they send each of their salespeople in order to minimize airfare?

Now let us consider Fuzzy Assignment Problem with row wise representing W, X, Y, Z and column wise representing Job1, Job2, Job3 and Job4 with assignment cost varying between 0$ to 50$. The cost matrix $[C_{ij}]$ whose elements are linguistic variables which are replaced by fuzzy numbers. The problem is solved by Hungarian method to find the optimal solution.

Solution: The Linguistic variables showing the qualitative data is converted into quantitative data using the following table.

As the assignment cost varies between 0$ to 50$ the minimum possible value is taken as 0 and the maximum possible value is taken as 50. The Assignment matrix is assigned as below,

$$\begin{bmatrix} extremelylow & low & fairlyhigh & extremelyhigh \\ low & verylow & high & veryhigh \\ medium & extremelyhigh & verylow & extremelylow \\ veryhigh & low & fairlylow & fairlylow \end{bmatrix}$$

Extremely low	(0, 2, 5)
Very low	(1, 2, 4)
Low	(4, 8, 12)
Fairly low	(15, 18, 20)
Medium	(23, 25, 27)
Fairly High	(28, 30, 32)
High	(33, 36, 38)
Very High	(37, 40, 42)
Extremely High	(44, 48, 50)

The linguistic variables are represented by triangular fuzzy numbers
Now

$$\begin{matrix} & 1 & 2 & 3 & 4 \\ W \\ X \\ Y \\ Z \end{matrix} \begin{bmatrix} (0,2,5;1) & (4,8,12;1) & (28,30,32;1) & (44,48,50;1) \\ (4,8,12;1) & (1,2,4;1) & (33,36,38;1) & (37,40,42;1) \\ (23,25,27;1) & (44,48,50;1) & (1,2,4;1) & (0,2,5;1) \\ (37,40,42;1) & (4,8,12;1) & (15,18,20;1) & (15,18,20;1) \end{bmatrix} \quad (3)$$

Calculate Y(0, 2, 5) using Circumcenter of Centroids method (by Definition 2.5)
$S(\tilde{c}_{11}) = (2.17,-1.58)$, $S(\tilde{c}_{12}) = (8,-4.92)$, $S(\tilde{c}_{13}) = (30,-0.92)$, $S(\tilde{c}_{14}) = (47.6,-2.25)$,
$S(\tilde{c}_{21}) = (8,-4.92)$, $S(\tilde{c}_{22}) = (2.16,-0.25)$, $S(\tilde{c}_{23}) = (35.8,-1.58)$, $S(\tilde{c}_{24}) = (39.8,-1.58)$,
$S(\tilde{c}_{31}) = (25,-0.91)$, $S(\tilde{c}_{32}) = (47.6,-2.25)$, $S(\tilde{c}_{33}) = (2.16,-0.25)$, $S(\tilde{c}_{34}) = (2.16,-1.58)$,
$S(\tilde{c}_{41}) = (39.8,-1.58)$, $S(\tilde{c}_{42}) = (8,-4.9)$, $S(\tilde{c}_{43}) = (17.8,-3.75)$, $S(\tilde{c}_{44}) = (17.8,-3.75)$,
Then calculate the Rank of the matrix (by Definition 2.6)
$R(\tilde{c}_{11}) = 2.68$, $R(\tilde{c}_{12}) = 9.39$, $R(\tilde{c}_{13}) = 30$, $R(\tilde{c}_{14}) = 47.6$,
$R(\tilde{c}_{21}) = 9.39$, $R(\tilde{c}_{22}) = 2.17R(\tilde{c}_{23}) = 35.83$, $R(\tilde{c}_{24}) = 39.83$,
$R(\tilde{c}_{31}) = 25.0$, $R(\tilde{c}_{32}) = 47.6$, $R(\tilde{c}_{33}) = 2.17$, $R(\tilde{c}_{34}) = 2.67$,
$R(\tilde{c}_{41}) = 39.83$, $R(\tilde{c}_{42}) = 9.38$, $R(\tilde{c}_{43}) = 18.19$, $R(\tilde{c}_{44}) = 18.19$
Now the above calculated values are replaced in (3) and the resulting assignment problem is solved by using Hungarian method.

$$\begin{matrix} & 1 & 2 & 3 & 4 \\ W \\ X \\ Y \\ Z \end{matrix} \begin{bmatrix} 2.68 & 9.39 & 30 & 47.6 \\ 9.39 & 2.17 & 35.83 & 39.83 \\ 25.0 & 47.6 & 2.17 & 2.67 \\ 39.83 & 9.38 & 18.19 & 18.19 \end{bmatrix}$$

Performing row reductions

$$\begin{matrix} W \\ X \\ Y \\ Z \end{matrix}\begin{bmatrix} 0 & 6.71 & 27.32 & 44.92 \\ 7.22 & 0 & 33.63 & 37.66 \\ 22.83 & 45.43 & 0 & 0.5 \\ 30.45 & 0 & 8.81 & 8.81 \end{bmatrix}$$

Performing column reductions

$$\begin{matrix} W \\ X \\ Y \\ Z \end{matrix}\begin{bmatrix} 0 & 6.71 & 27.32 & 44.42 \\ 7.22 & 0 & 33.63 & 37.16 \\ 22.83 & 45.43 & 0 & 0 \\ 30.45 & 0 & 8.81 & 8.31 \end{bmatrix}$$

The optimal Assignment matrix is

$$\begin{matrix} W \\ X \\ Y \\ Z \end{matrix}\begin{bmatrix} 0 & 6.71 & 27.32 & 44.42 \\ 7.22 & 0 & 25.32 & 28.85 \\ 22.83 & 53.74 & 0 & 0 \\ 30.45 & 0 & 0.5 & 0 \end{bmatrix}$$

The optimal assignment schedule is $W \rightarrow 1$, $X \rightarrow 2$, $Y \rightarrow 3$, $Z \rightarrow 4$

Unbalanced Assignment Problem:

Let us consider a fuzzy unbalanced assignment problem as above with rows representing 4 area A, B, C, D and columns representing the salesman's the cost matrix is given whose elements are triangular fuzzy numbers. The problem is to find the optimal assignment so that the total cost of area assignment becomes minimum.

$$\begin{matrix} \\ W \\ X \\ Y \\ Z \end{matrix}\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ (3,6,9;1) & (6,9,11;1) & (3,6,12;1) & (3,9,12;1) & (3,6,9;1) \\ (3,9,12;1) & (3,6,9;1) & (6,9,11;1) & (6,9,18;1) & (3,9,12;1) \\ (6,9,11;1) & (9,12,15;1) & (3,6,9;1) & (3,9,12;1) & (6,9,18;1) \\ (3,9,12;1) & (6,9,11;1) & (6,9,18;1) & (3,6,9;1) & (3,12,15;1) \end{matrix}$$

Solution: The given problem is a fuzzy unbalanced assignment problem so to change into the fuzzy balanced assignment problem as follows

$$\begin{matrix} \\ V \\ W \\ X \\ Y \\ Z \end{matrix}\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ (3,6,9;1) & (6,9,11;1) & (3,6,12;1) & (3,9,12;1) & (3,6,9;1) \\ (3,9,12;1) & (3,6,9;1) & (6,9,11;1) & (6,9,18;1) & (3,9,12;1) \\ (6,9,11;1) & (9,12,15;1) & (3,6,9;1) & (3,9,12;1) & (6,9,18;1) \\ (3,9,12;1) & (6,9,11;1) & (6,9,18;1) & (3,6,9;1) & (3,12,15;1) \\ (0,0,0;1) & (0,0,0;1) & (0,0,0;1) & (0,0,0;1) & (0,0,0;1) \end{matrix} \quad (4)$$

we calculate Y$(3,6,9;1)$ using Circumcenter of Centroid method (by Definition 2.5)

$S(\tilde{c}_{11}) = (6,-2.5)$, $S(\tilde{c}_{12}) = (8.8,-0.75)$, $S(\tilde{c}_{13}) = (6.5,-5.58)$,

$S(\tilde{c}_{14}) = (8.5,-5.5)$, $S(\tilde{c}_{15}) = (6,-2.5)$

$S(\tilde{c}_{21}) = (8.5,-5.5)$, $S(\tilde{c}_{22}) = (6,-2.5)$, $S(\tilde{c}_{23}) = (8.8,-0.75)$,

$S(\tilde{c}_{24}) = (10,-8.5)$, $S(\tilde{c}_{25}) = (8.5,-5.5)$

$S(\tilde{c}_{31}) = (8.8,-0.75)$, $S(\tilde{c}_{32}) = (12,-2.5)$, $S(\tilde{c}_{33}) = (6,-2.5)$,

$S(\tilde{c}_{34}) = (8.5,-5.5)$, $S(\tilde{c}_{35}) = (10,-8.5)$

$S(\tilde{c}_{41}) = (8.5,-5.5)$, $S(\tilde{c}_{42}) = (8.8,-0.75)$, $S(\tilde{c}_{43}) = (10,-8.5)$,
$S(\tilde{c}_{44}) = (6,-2.5)$, $S(\tilde{c}_{45}) = (10.5,-8.5)$
$S(\tilde{c}_{41}) = (0, 0)$, $S(\tilde{c}_{42}) = (0, 0)$, $S(\tilde{c}_{43}) = (0, 0)$,
$S(\tilde{c}_{44}) = (0, 0)$, $S(\tilde{c}_{45}) = (0, 0)$
We calculate the Rank (by Definition 2.6)
$R(\tilde{c}_{11}) = 6.5$, $R(\tilde{c}_{12}) = 8.83$, $R(\tilde{c}_{13}) = 8.51$, $R(\tilde{c}_{14}) = 10.12$, $R(\tilde{c}_{15}) = 6.5$
$R(\tilde{c}_{21}) = 10.12$, $R(\tilde{c}_{22}) = 6.5$, $R(\tilde{c}_{23}) = 8.83$, $R(\tilde{c}_{24}) = 13.12$, $R(\tilde{c}_{25}) = 6.5$
$R(\tilde{c}_{31}) = 8.83$, $R(\tilde{c}_{32}) = 12.25$, $R(\tilde{c}_{33}) = 6.5$, $R(\tilde{c}_{34}) = 8.51$, $R(\tilde{c}_{35}) = 13.12$
$R(\tilde{c}_{41}) = 10.12$, $R(\tilde{c}_{42}) = 8.83$, $R(\tilde{c}_{43}) = 13.12$, $R(\tilde{c}_{44}) = 6.5$, $R(\tilde{c}_{45}) = 13.5$
$R(\tilde{c}_{41}) = 0$, $R(\tilde{c}_{42}) = 0$, $R(\tilde{c}_{43}) = 0$, $R(\tilde{c}_{44}) = 0$, $R(\tilde{c}_{45}) = 0$
We replace these values for their corresponding C_{ij} in (4) and solve the resulting assignment problem by using Hungarian method

$$\begin{array}{c} \\ V \\ W \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & \end{array} \\ \begin{bmatrix} 6.5 & 8.83 & 8.51 & 10.12 & 6.5 \\ 10.12 & 6.5 & 8.83 & 13.12 & 10.12 \\ 8.83 & 12.25 & 6.5 & 8.51 & 13.12 \\ 10.12 & 8.83 & 13.12 & 6.5 & 13.50 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Performing row reductions

$$\begin{array}{c} V \\ W \\ X \\ Y \\ Z \end{array} \begin{bmatrix} 0 & 2.33 & 2.01 & 3.71 & 0 \\ 3.71 & 0 & 2.33 & 6.62 & 3.71 \\ 2.33 & 5.75 & 0 & 2.01 & 6.62 \\ 3.71 & 2.33 & 6.62 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The optimal Assignment matrix is

$$\begin{array}{c} V \\ W \\ X \\ Y \\ Z \end{array} \begin{bmatrix} 0 & 2.33 & 2.01 & 3.71 & 0 \\ 3.71 & 0 & 2.33 & 6.62 & 3.71 \\ 2.33 & 5.75 & 0 & 2.01 & 6.62 \\ 3.71 & 2.33 & 6.62 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The optimal assignment schedule is $V \rightarrow 1$, $W \rightarrow 2$, $X \rightarrow 3$, $Y \rightarrow 4$, $Z \rightarrow 4$.

6. Conclusions:

In this paper, the assignment costs are considered as linguistic variables represented by fuzzy numbers. Thus the fuzzy assignment problem has been transformed into crisp balanced and unbalanced assignment problem using Circumcenter of Centroid method. Here we have shown that the fuzzy assignment problems of qualitative nature can be solved in an effective way. This technique can also be tried in solving the problems like balanced and unbalanced Transportation problems, Transshipment, project scheduling problems, network flow problems etc. This research work is under process for hexagonal and octagonal fuzzy numbers.

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