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Estimation of Parameters for Order Statistics of Lehmann-Type Laplace Distribution Type I and Type II

V.S.Akilandeswari[1], J.Priyadarshini[2], Y.Yazh devi[3] and V.Saavithri[4]

1.Assistant Professor, Department of Mathematics,
Saranathan College of Engineering, Trichy
2, 3. Research Scholars, Department of Mathematics, Nehru Memorial College, Trichy
4. Assistant Professor, Department of Mathematics, Nehru Memorial College, Trichy

Abstract

In this paper, the parameters of order statistics of Lehmann –Type Laplace distributions both Type – I and Type – II are estimated by using profile likelihood method. Two sets of data are analyzed for estimating the parameters at all orders.

Keywords: Order statistics, Lehmann – Type Laplace Distribution – Type I, Lehmann – Type Laplace Distribution Type II, Profile likelihood.

1. Introduction

R.Poornima and V.Saavithri proposed two new families of distributions called Lehmann – Type Laplace Distribution – Type I (LLD – I) and Lehmann – Type Laplace Distribution – Type II (LLD–II) with three parameters[4].
The first one is distribution function of Type–I distribution, called Lehmann–Type Laplace Distribution (LLD–I) given by

$$F\ \ x;\alpha,\theta,\phi\ \ = \begin{cases} \dfrac{1}{2^{\alpha}} e^{\left[\alpha\left(\frac{x-\theta}{\phi}\right)\right]} & if\ x\le\theta \\ \left[1-\dfrac{1}{2} e^{\left[-\left(\frac{x-\theta}{\phi}\right)\right]}\right]^{\alpha} & if\ x\ge\theta \end{cases}$$

where α > 0 is a shape parameter, ϕ > 0 is a scale parameter, $-\infty < \theta < \infty$ is a location parameter.
The probability density function of LLD – I is

$$f_X(x) = \begin{cases} \dfrac{\alpha}{2\phi} e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)}, & x \le \theta \\ \dfrac{\alpha}{\phi}\left(1 - \dfrac{1}{2} e^{\left(\frac{\theta - x}{\phi}\right)}\right) e^{\alpha\left(\frac{\theta - x}{\phi}\right)}, & x \ge \theta \end{cases}$$

It is worthwhile to note that the two parameter Laplace family is a special case of the family of Laplace distributions when α = 1.
The second new type of distribution is density of LLD – II given by

$$f_X(x) = \begin{cases} \dfrac{\alpha}{2\phi} e^{(\alpha+1)\left(\frac{x-\theta}{\phi}\right)} & x \le \theta \\ \dfrac{\alpha}{\phi}\left(1 - \dfrac{1}{2} e^{\left(\frac{\theta - x}{\phi}\right)}\right) e^{\alpha\left(\frac{\theta - x}{\phi}\right)} & x \ge \theta \end{cases}$$

When α = 1, this family coincides with the family of skewed Laplace distribution obtained by Balakrishnan and Ambagaspitiya (1994). The parameter 'α' controls the skewness and kurtosis of the distribution.
Suppose that $(X_1, X_2, \ldots\ldots, X_n)$ are n jointly distributed random variables. The corresponding order statistics [1, 2, 3, 5] are the $X_i{}'s$ arranged in non – decreasing order. The smallest of $X_i{}'s$ are denoted by $X_{1:n}$, the second smallest is denoted by $X_{2:n}$ and finally the largest is denoted by $X_{n:n}$. Thus $X_{1:n} \le X_{2:n} \le \ldots\ldots\le X_{n:n}$. The joint density of order statistics corresponding to independent and identically distributed sample from an absolutely continuous distribution with density is

$$f_{X_{1:n}, X_{2:n}, \ldots\ldots X_{n:n}}(x_1, x_2, \ldots\ldots x_n) = n! \prod_{r=1}^{n} f(x_r) \qquad -\infty < x_r < \infty$$

The cumulative distribution function of $X_{r:n}$ may be obtained from

$$F_{r:n}(x) = \sum_{i=r}^{n} \binom{n}{i} (F(x))^i \, (1 - F(x))^{(n-i)}$$

Then the probability density function of $X_{r:n}$ is

$$f_{r:n}(x) = \frac{d}{dx} F_{r:n}(x)$$

In this paper, the parameters for the order statistics of Lehmann – Type Laplace distribution – Type I and Lehmann – Type Laplace distribution – Type II are estimated using profile likelihood method.

The paper is organized as follows. Section 2 details the order statistics of LLD – I and LLD – II. Section 3 gives the parameter estimation. Section 4 illustrates the parameter estimation using profile likelihood method. Conclusion is given in section 5.

2. Order statistics

2.1 Lehmann – Type Laplace Distribution – Type I

Let $X_{r:n}$ be the r^{th} order statistics connected with a sample of size n from the LLD – I. Then the probability density function of LLD – I is

$$f_{r:n}(x) = \begin{cases} r\binom{n}{r}\frac{\alpha}{2^{\alpha}\varphi}\left(\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x-\theta}{\varphi}\right)}\right)^{(r-1)}\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x-\theta}{\varphi}\right)}\right)^{(n-r)}e^{\alpha\left(\frac{x-\theta}{\varphi}\right)}, & x \le \theta \\ r\binom{n}{r}\frac{\alpha}{2\varphi}\left(\left(1-\frac{1}{2}e^{-\left(\frac{x-\theta}{\varphi}\right)}\right)^{(\alpha r-1)}\right)\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{x-\theta}{\varphi}\right)}\right)^{\alpha}\right)^{(n-r)}e^{-\left(\frac{x-\theta}{\varphi}\right)}, & x \ge \theta \end{cases} \tag{2.1.1}$$

2.2 Lehmann – Type Laplace Distribution – Type II

Let $X_{r:n}$ be the r^{th} order statistics connected with a sample of size n from the LLD – II. Then the probability density function of LLD – I is

$$f_{r:n}(x) = \begin{cases} r\binom{n}{r}\left(\frac{\alpha}{2(\alpha+1)}\right)^{(r-1)}\frac{\alpha}{2\varphi}e^{r(\alpha+1)\left(\frac{x-\theta}{\varphi}\right)}\left(1-\left(\frac{\alpha}{2(\alpha+1)}\right)e^{(\alpha+1)\left(\frac{x-\theta}{\varphi}\right)}\right)^{(n-r)}, & x \le \theta \\ r\binom{n}{r}\frac{\alpha}{2\varphi}\left(1-e^{\alpha\left(\frac{\theta-x}{\varphi}\right)}+\frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x}{\varphi}\right)}\right)^{(n-r)}\left(e^{\alpha\left(\frac{\theta-x}{\varphi}\right)}-\frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x}{\varphi}\right)}\right)^{(n-r)} \\ \quad \frac{\alpha}{\varphi}\left(1-\frac{1}{2}e^{\left(\frac{x-\theta}{\varphi}\right)}\right)e^{\alpha\left(\frac{x-\theta}{\varphi}\right)}, & x \ge \theta \end{cases} \tag{2.2.1}$$

3. Parameter Estimation

3.1 Lehmann – Type Laplace Distribution – Type I

The log – likelihood function of LLD – I is

$$\log l = n\log r + n\log\binom{n}{r} + n\log\alpha - (n_2 + n_1\alpha r)\log 2 - n\log\varphi + \sum_{i=1}^{n_1} r\alpha\left(\frac{x_i-\theta}{\varphi}\right) - \sum_{i=1}^{n_2}\left(\frac{x_i-\theta}{\varphi}\right) + \sum_{i=1}^{n_1}(n-r)log\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x_i-\theta}{\varphi}\right)}\right) + \sum^{n_2}(\alpha r-1)\,log\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right) + \sum^{n_2}(n-r)log\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right)^{\alpha}\right) \tag{3.1.1}$$

Partially differentiating (3.1.1) with respect to the parameters α and ϕ and equating them to zero,

α

$$= n\left\{\begin{array}{c} n_1\, r\log 2 - \sum_{i=1}^{n_1}\frac{r}{\varphi}(x_i - \theta) + (n-r)\sum_{i=1}^{n_1}\left[\frac{\frac{e^{\alpha\left(\frac{x_i-\theta}{\varphi}\right)}}{2^\alpha}\left(\frac{x_i-\theta}{\varphi} - \log 2\right)}{\left(1-\frac{1}{2^\alpha}e^{\alpha\left(\frac{x_i-\theta}{\varphi}\right)}\right)}\right] \\ -\sum_{i=1}^{n_2} r\, log\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right) + (n-r)\sum_{i=1}^{n_1}\left[\frac{\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right)^\alpha \log\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right)}{\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right)^\alpha\right)}\right] \end{array}\right\}^{-1} \quad (3.1.2)$$

φ

$$= n\left\{\begin{array}{c} \sum_{i=1}^{n_1}\frac{r\alpha}{\varphi^2}(x_i - \theta) + \sum_{i=1}^{n_1}\left[\frac{\frac{(n-r)\alpha}{2^\alpha\varphi^2}(x_i-\theta)e^{\alpha\left(\frac{x_i-\theta}{\varphi}\right)}}{\left(1-\frac{1}{2^\alpha}e^{\alpha\left(\frac{x_i-\theta}{\varphi}\right)}\right)}\right] - \sum_{i=1}^{n_2}\left[\frac{\frac{(\alpha r-1)}{2\varphi^2}(x_i-\theta)e^{-\left(\frac{x_i-\theta}{\varphi}\right)}}{\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right)}\right] \\ +\sum_{i=1}^{n_2}\left[\frac{\frac{(n-r)\alpha}{2\varphi^2}(x_i-\theta)e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right)^{\alpha-1}}{\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\varphi}\right)}\right)^\alpha\right)}\right] \end{array}\right\}^{-1} \quad (3.1.3)$$

3.2 Lehmann – Type Laplace Distribution – Type II

The log – likelihood function of LLD – II is given by

$$\log l = n\log r + n\log\binom{n}{r} + (n_1 r - n_1)\log\left(\frac{\alpha}{\alpha+1}\right) - n_1 r\log 2 - n\log\left(\frac{\alpha}{\varphi}\right)$$
$$+\sum_{i=1}^{n_1} r(\alpha+1)\left(\frac{x_i-\theta}{\varphi}\right) - \sum_{1}^{n_1}(n-r)log\left(1-\frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{x_i-\theta}{\varphi}\right)}\right)$$
$$+\sum_{1}^{n_2}(r-1)log\left(1-e^{\alpha\left(\frac{\theta-x_i}{\varphi}\right)}\right) + \frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}$$
$$+(n-r)\sum_{i=1}^{n_2} log\left(e^{\alpha\left(\frac{\theta-x_i}{\varphi}\right)} - \frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}\right)$$
$$+\sum_{i=1}^{n_2} log\left(1-\frac{1}{2}e^{\left(\frac{\theta-x_i}{\varphi}\right)}\right) + \sum_{i=1}^{n_2}\log\alpha\left(\frac{\theta-x_i}{\varphi}\right) \quad (3.2.1)$$

Partially differentiating (3.2.1) with respect to the parameters α and ϕ and equating them to zero,

$$\frac{n_1 r - n_1}{\alpha(\alpha+1)} + \frac{n}{\alpha} + \sum^{n_1} r\left(\frac{x_i-\theta}{\varphi}\right) + \sum^{n_2}\left(\frac{\theta - x_i}{\varphi}\right)$$
$$- \sum_{i=1}^{n_1}\left[\frac{(n-r)\left\{\frac{e^{(\alpha+1)\left(\frac{x_i-\theta}{\varphi}\right)}}{2(\alpha+1)^2} + \frac{\alpha}{2(\alpha+1)}\left(\frac{x_i-\theta}{\varphi}\right)e^{(\alpha+1)\left(\frac{x_i-\theta}{\varphi}\right)}\right\}}{\left(1-\frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{x_i-\theta}{\varphi}\right)}\right)}\right]$$
$$+ \sum_{i=1}^{n_2}\left[\frac{(n-r)\left\{\frac{e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}}{2(\alpha+1)^2} + \frac{\alpha}{2(\alpha+1)}\left(\frac{\theta-x_i}{\varphi}\right)e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)} - e^{\alpha\left(\frac{\theta-x_i}{\varphi}\right)}\left(\frac{\theta-x_i}{\varphi}\right)\right\}}{\left(1-e^{\alpha\left(\frac{\theta-x_i}{\varphi}\right)} + \frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}\right)}\right]$$
$$+ \sum_{i=1}^{n_2}\left[\frac{(n-r)\left\{e^{\alpha\left(\frac{\theta-x_i}{\varphi}\right)}\left(\frac{\theta-x_i}{\varphi}\right) - \frac{\alpha}{2(\alpha+1)}\left(\frac{\theta-x_i}{\varphi}\right)e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}\left(\frac{\theta-x_i}{\varphi}\right) - \frac{1}{2(\alpha+1)^2}e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}\right\}}{\left(e^{\alpha\left(\frac{\theta-x_i}{\varphi}\right)} + \frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}\right)}\right] = 0 \quad (3.2.2)$$

$$-\frac{n}{\varphi} + \sum_{i=1}^{n_1} r(\alpha+1)\left(\frac{x_i-\theta}{\varphi^2}\right) - \sum_{i=1}^{n_2}\alpha\left(\frac{\theta-x_i}{\varphi^2}\right)$$
$$- \sum_{i=1}^{n_1}\left[\frac{(n-r)\left\{\frac{\alpha}{2}\left(\frac{x_i-\theta}{\varphi^2}\right)e^{(\alpha+1)\left(\frac{x_i-\theta}{\varphi}\right)}\right\}}{\left(1-\frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{x_i-\theta}{\varphi}\right)}\right)}\right]$$
$$+ \sum_{i=1}^{n_2}\left[\frac{(r-1)\left\{\frac{e^{\alpha\left(\frac{\theta-x_i}{\varphi}\right)}\alpha\left(\frac{\theta-x_i}{\varphi^2}\right)}{2(\alpha+1)^2} + \frac{\alpha(\theta-x_i)}{2\varphi^2}e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}\right\}}{\left(1-e^{\alpha\left(\frac{\theta-x_i}{\varphi}\right)} + \frac{\alpha}{2(\alpha+1)}e^{(\alpha+1)\left(\frac{\theta-x_i}{\varphi}\right)}\right)}\right]$$
$$+ \sum_{i=1}^{n_2}\left[\frac{(n-r)\left\{\alpha\left(\frac{\theta-x_i}{\varphi^2}\right)\left(1-\frac{1}{2}e^{\left(\frac{\theta-x_i}{\varphi}\right)}\right)\right\}}{\left(1-\frac{\alpha}{2(\alpha+1)}e^{\left(\frac{\theta-x_i}{\varphi}\right)}\right)}\right] + \sum_{i=1}^{n_2}\left[\frac{\frac{1}{2}\left(\frac{\theta-x_i}{\varphi^2}\right)e^{\left(\frac{\theta-x_i}{\varphi}\right)}}{\left(1-\frac{1}{2}e^{\left(\frac{\theta-x_i}{\varphi}\right)}\right)}\right] = 0 \quad (3.2.3)$$

Keeping θ fixed, α and ϕ can be calculated by solving the equations (3.2.2) and (3.2.3) in MATLAB.

4. Numerical example

4.1 Dataset 1

Bain and Engelhart (1973) considered the following data set, consisting of 33 differences in flood levels between stations on a river:
1.96, 1.97, 3.60, 3.80, 4.79, 5.66, 5.76, 5.78, 6.27, 6.30, 6.76, 7.65, 7.84, 7.99, 8.51, 9.18, 10.13, 10.24, 10.25, 10.43, 11.45, 11.48, 11.75, 11.81, 12.34, 12.78, 13.06, 13.29, 13.98, 14.18, 14.40, 16.22, 17.06.
Table 4.1 gives the parameter values for all the orders of order statistics of LLD-I and LLD-II of this dataset.

Table 4.1 gives the parameter values for all the orders of order statistics of LLD-I and LLD-II.

Order	LLD – I				LLD - II			
	θ	α	ϕ	L	θ	α	ϕ	L
1	17.5	$6.4251X10^{-5}$	$1.1534\ X10^{-5}$	$-1.3284X10^{3}$	18	$5.7431\ X10^{7}$	$1.8915\ X10^{8}$	-113.9887
2	17.5	$4.5363X10^{-5}$	$2.2938\ X10^{-5}$	-830.5779	18	$2.0978\ X10^{7}$	$7.1259\ X10^{8}$	-105.9329
3	17.5	$3.0774X10^{-5}$	$2.2938\ X10^{-5}$	-783.6694	17.5	$5.6878X10^{7}$	$1.8998X10^{8}$	-120.9183
4	**17.5**	**$1.5511X10^{-4}$**	**$1.4690\ X10^{-5}$**	**-778.7525**	17	$5.7681\ X10^{7}$	$1.9223\ X10^{8}$	-142.6265
5	17.5	$1.3135X10^{-4}$	$1.4690\ X10^{-5}$	-793.3918	16	$5.7923\ X10^{7}$	$1.9287\ X10^{8}$	-165.5653
6	17.5	$1.1598X10^{-4}$	$1.4690\ X10^{-5}$	-818.7974	15	$5.8035X10^{7}$	$1.9315X10^{8}$	-188.3865
7	17.5	$1.0475X10^{-6}$	$1.4765\ X10^{-5}$	-840.7250	14.5	$5.8099\ X10^{7}$	$1.9331\ X10^{8}$	-209.5945
8	17.5	$9.6787X10^{-7}$	$1.4765\ X10^{-5}$	-878.7638	14	$5.8140\ X10^{7}$	$1.9341\ X10^{8}$	-231.0944
9	17.5	$9.0645X10^{-7}$	$1.4765\ X10^{-5}$	-922.1373	13.5	$5.8168\ X10^{7}$	$1.9348\ X10^{8}$	-251.9056
10	17.5	$8.576X10^{-7}$	$1.4765\ X10^{-5}$	-969.9989	13	$5.8189\ X10^{7}$	$1.9352\ X10^{8}$	-271.3685
11	17.5	$8.2130X10^{-5}$	$1.4690\ X10^{-5}$	$-1.0361X10^{3}$	12.5	$5.8205\ X10^{7}$	$1.9356\ X10^{8}$	-289.3976
12	17.5	$7.8812X10^{-5}$	$1.4690\ X10^{-5}$	$-1.0927\ X10^{3}$	12	$5.8217X10^{7}$	$1.9359X10^{8}$	-305.3845
13	17.5	$7.6012X10^{-5}$	$1.4690\ X10^{-5}$	$-1.5131\ X10^{3}$	11.5	$5.8227\ X10^{7}$	$1.9361\ X10^{8}$	-324.3180
14	17.5	$7.3617X10^{-5}$	$1.4690\ X10^{-5}$	$-1.2171\ X10^{3}$	11	$5.8236\ X10^{7}$	$1.9363\ X10^{8}$	-331.3521
15	17.5	$7.1546X10^{-5}$	$1.4690\ X10^{-5}$	$-1.2848\ X10^{3}$	10.5	$5.8243\ X10^{7}$	$1.9364\ X10^{8}$	-340.0777
16	17.5	$6.9736X10^{-5}$	$1.4690\ X10^{-5}$	$-1.3560\ X10^{3}$	10	$5.8249\ X10^{7}$	$1.9366\ X10^{8}$	-346.3707
17	17.5	$6.8141X10^{-5}$	$1.4690\ X10^{-5}$	$-1.4307\ X10^{3}$	9.5	$5.8254X10^{7}$	$1.9367X10^{8}$	-349.8447
18	17.5	$6.6726X10^{-5}$	$1.4690\ X10^{-5}$	$-1.5091\ X10^{3}$	9	$5.8258\ X10^{7}$	$1.9368X10^{8}$	-349.7640
19	17.5	$6.5461X10^{-5}$	$1.4690\ X10^{-5}$	$-1.5911\ X10^{3}$	8.5	$5.8262\ X10^{7}$	$1.9369X10^{8}$	-346.1899
20	17.5	$6.4323X10^{-5}$	$1.4690\ X10^{-5}$	$-1.6769\ X10^{3}$	88	$5.8265X10^{7}$	$1.9369X10^{8}$	-339.2867
21	17.5	$6.3296X10^{-5}$	$1.4690\ X10^{-5}$	$-1.7665\ X10^{3}$	7	$5.8268\ X10^{7}$	$1.9370\ X10^{8}$	-329.3763
22	17.5	$6.2362X10^{-5}$	$1.4690\ X10^{-5}$	$-1.8600\ X10^{3}$	6.5	$5.8271\ X10^{7}$	$1.9371\ X10^{8}$	-315.8421
23	17.5	$6.1510X10^{-5}$	$1.4690\ X10^{-5}$	$-1.9577\ X10^{3}$	66	$5.8274X10^{7}$	1.9371×10^{8}	-299.7085
24	17.5	$6.0730\ X10^{-5}$	$1.4690\ X10^{-5}$	$-2.0598\ X10^{3}$	5.5	$5.8276\ X10^{7}$	$1.9372X10^{8}$	-281.8240
25	17.5	$6.0013\ X10^{-5}$	$1.4690\ X10^{-5}$	$-2.1666\ X10^{3}$	5	$5.8278\ X10^{7}$	$1.9372\ X10^{8}$	-262.7596
26	17.5	$5.9352\ X10^{-5}$	$1.4690\ X10^{-5}$	$-2.2784\ X10^{3}$	4.5	$5.8280\ X10^{7}$	$1.9373\ X10^{8}$	-241.9975
27	17.5	$5.8740\ X10^{-5}$	$1.4690\ X10^{-5}$	$-2.3957\ X10^{3}$	4	$5.8281\ X10^{7}$	$1.9373\ X10^{8}$	-219.7213

28	17.5	5.8172 X10^{-5}	1.4690 X10^{-5}	-2.5192 X10^{3}	3.5	5.8283X10^{7}	1.9373X10^{8}	-196.2492
29	17.5	5.7644 X10^{-5}	1.4690 X10^{-5}	-2.6498 X10^{3}	2.5	5.8284 X10^{7}	1.9374 X10^{8}	-171.6412
30	17.5	5.7151 X10^{-5}	1.4690 X10^{-5}	-2.7887 X10^{3}	2	5.8286 X10^{7}	1.9374 X10^{8}	-146.9487
31	17.5	5.6691 X10^{-5}	1.4690 X10^{-5}	-2.9382 X10^{3}	1	5.8287X10^{7}	1.9374X10^{8}	-124.0128
32	17.5	5.6259 X10^{-5}	1.4690 X10^{-5}	-3.1019 X10^{3}	**0**	**5.8288 X10^{7}**	**1.9374 X10^{8}**	**-105.8046**
33	17.5	5.5854 X10^{-5}	1.4690 X10^{-5}	-3.2895 X10^{3}	0	5.8289 X10^{7}	1.9375 X10^{8}	-106.5143

From the table 4.1 it is seen that the likelihood function value of order statistics of LLD-I is maximum when the order is 4. Similarly, for order statistics of LLD-II, the maximum likelihood value is -105.8046 when the order is 32.

4.2 Dataset2

This dataset of 21 data is from a study in Ohba84. The system is a small on-line data entry control software package used in Japan since 1980.The size of the software is approximately 40, 000 lines of code.

2, 3, 4, 5, 9, 11, 12, 19, 21, 22, 24, 26, 30, 31, 37, 38, 41, 42, 45, 46. Table 4.2 gives the parameter values for all the orders of order statistics of LLD-I and LLD-II of this dataset.

Table 4.2 The parameter values for all the orders of order statistics of LLD-I and LLD-II.

Order	LLD – I				LLD - II			
	θ	α	ϕ	L	θ	α	ϕ	L
1	**50**	**2.1991X10^{-4}**	**7.1244X10^{-4}**	**-142.7461**	47	5.7555 X10^{7}	1.9237 X10^{8}	-139.4244
2	48	2.1227X10^{-4}	7.0212 X10^{-4}	-233.4475	46	5.7644 X10^{7}	1.9261 X10^{8}	-231.1841
3	47	2.0999 X10^{-4}	7.0057 X10^{-4}	-332.0362	43	5.7667 X10^{7}	1.9267 X10^{8}	-325.6257
4	47	2.0884X10^{-4}	7.0008X10^{-4}	-433.7099	43	5.7678 X10^{7}	1.9270 X10^{8}	-413.9674
5	47	2.0813X10^{-4}	6.9987X10^{-4}	-542.0821	41	5.7684 X10^{7}	1.9271 X10^{8}	-494.99
6	47	2.0766X10^{-4}	6.9976 X10^{-4}	-656.0598	39	5.7688 X10^{7}	1.9272 X10^{8}	-566.1103
7	47	2.0731X10^{-4}	6.9970 X10^{-4}	-774.9733	37	5.7691 X10^{7}	1.9273 X10^{8}	-619.2721
8	47	2.0684X10^{-4}	6.9966 X10^{-4}	-898.3925	30	5.7693 X10^{7}	1.9274 X10^{8}	-655.8475
9	47	2.0667X10^{-4}	6.9964 X10^{-4}	-1.0260X10^{3}	27	5.7695 X10^{7}	1.9274 X10^{8}	-679.9679
10	47	2.0653X10^{-4}	6.9962 X10^{-4}	-1.1578 X10^{3}	24	5.7696 X10^{7}	1.9274 X10^{8}	-689.0835
11	47	2.0642X10^{-4}	6.9961 X10^{-4}	-1.2935 X10^{3}	22	5.7697 X10^{7}	1.9274 X10^{8}	-687.7837
12	46	2.0624X10^{-4}	6.9960 X10^{-4}	-1.4286 X10^{3}	20	5.7698 X10^{7}	1.9275 X10^{8}	-677.9138
13	46	2.0616X10^{-4}	6.9959 X10^{-4}	-1.5582 X10^{3}	18	5.7699 X10^{7}	1.9275 X10^{8}	-658.0327
14	46	2.0610X10^{-4}	6.9959 X10^{-4}	-1.6921 X10^{3}	13	5.7699 X10^{7}	1.9275 X10^{8}	-618.8504
15	46	2.0604X10^{-4}	6.9958 X10^{-4}	-1.8306 X10^{3}	11	5.7700 X10^{7}	1.9275 X10^{8}	-558.4329
16	46	2.0604X10^{-4}	6.9958 X10^{-4}	-1.9741 X10^{3}	8	5.7700 X10^{7}	1.9275 X10^{8}	-488.7892
17	46	2.0604X10^{-4}	6.9958 X10^{-4}	-2.1233 X10^{3}	5	5.7701 X10^{7}	1.9275 X10^{8}	-406.9604
18	46	2.0599 X10^{-4}	6.9958 X10^{-4}	-2.2795 X10^{3}	3	5.7701 X10^{7}	1.9276 X10^{8}	-315.1110
19	46	2.0594 X10^{-4}	6.9958 X10^{-4}	-2.4449X10^{3}	2	5.7701 X10^{7}	1.9276 X10^{8}	-222.3320
20	46	2.0590 X10^{-4}	6.9958 X10^{-4}	-2.6253 X10^{3}	**0**	**5.7702 X10^{7}**	**1.9276 X10^{8}**	**-135.3126**

From the table 4.2 it is seen that the likelihood function value of order statistics of LLD-I is maximum when the order is 1 with value -142.7461. Similarly, for order statistics of LLD-II, the maximum likelihood value is -135.3126 when the order is 20.

5. Conclusion

In this paper, the parameters α, θ and ϕ of LLD – I and LLD – II are estimated by using profile likelihood method. Two sets of data are analyzed to estimate the parameters of order statistics of LLD – I and LLD – II at all possible orders. It is inferred that the maximum likelihood value is obtained for the order statistics of LLD-I at the initial stage of the orders and for the order statistics of LLD-II at the final stage of the orders.

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