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Computation of a Multi-Choice Fuzzy Goal Programming Problem

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Abstract

The important activity of a manager is Decision Making. Decision Making is getting complex day by day due to incomplete information and conflicting criteria. Multi Choice Goal Programming (MCGP) is considered as an important tool in multi criteria decision making to solve such problems. However in real world problems, finding accurate targets for a goal is a difficult task. In order to deal with this type of uncertainty, we consider multi-choice fuzzy target values. The concept of multiple number of fuzzy target values lead to multi-choice fuzzy goal programming (MCFGP) problem. MCFGP cannot be solved by existing methods. In order to solve MCFGP, its equivalent mathematical model is presented by using three different techniques. Finally the equivalent models become mixed integer non linear mathematical models. In order to solve the mixed integer non linear programming model, the help of existing optimization software have been taken. To illustrate the methodology a numerical example is provided.

AMS subject classification:
Keywords: Multi-choice goal programming, Fuzzy Goal Programming, Fuzzy target values.

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1. Introduction

In decision making problem the popular decision making technique is Goal Programming (GP). For the first time GP was introduced by [15]. Which helps in further development of GP in different decision making problems. A few years later [17, 18] contributed more in this field and he used GP for media planning. Applications of GP was carried out by [16]. [11] used GP for real time reservoir operations. Some application of GP carried out by [47]. Impressive books on the subject have been written by [22, 26, 31] and [23]. An up to date review of the subject is given by [41]. The contributions of [25, 37, 42] and many others [26, 38, 48] are remarkable. Few years later some books are published by [27, 30]. [35] presented equivalent model of the multi-choice goal programming by using Vandermonde's interpolating polynomial, binary variables and least square approximation method.

In 1970, [7] introduced some applications of fuzzy theories to the various decision making process in fuzzy environment. In [49]'s work, he presented a fuzzy optimization technique to linear programming problem with single and multiple objective functions. [34] proposed a fuzzy goal programming to specify the imprecise aspiration levels of the fuzzy goals also [21] used the same. [24] written on the rediscovery of fuzzy goal programming. [43] used arithmetic addition to aggregate the fuzzy goals for construction of the relevant decision function. [46] used non linear membership function to formulate the FGP Problem. [14] used FGP approach for the optimal planning of metropolitan solid waste management. [5] used FGP approach to portfolio selection. [10] applied and modeled FGP for use of land-planning in agricultural system. [1] used FGP approach for multi-objective transportation problems. [36] carried out work on fuzzy goal programming approach to multilevel programming problems. [40] used FGP approach in collaborative Production-Distribution planning in Supply Chain. Theory and application with recent developments of fuzzy multi-criteria decision making can be found in [28]. A comprehensive review of the state-of-the-art in fuzzy goal programming can be found in [4].

[12] proposed a mathematical model for programming the multi-choice aspiration level in the name of multi-choice goal programming. He used the multiplicative terms of binary variables in order to tackle with multi choice aspiration levels for each goal. [13], replaced the binary variables with continuous type variables. [8], used binary variables with restrictions in transformation of multi-choice linear programming problem. [9] used interpolating polynomial to solve multi-choice linear programming problem. Fuzzy multi-choice goal programming problem was first addressed by [6]. [2] generalized the transformation technique proposed by [8]. [3] proposed fully fuzzy multi-choice multi-objective linear programming problem, in which, each right hand side constraint has two choices. [33] reformulated the MCGP model to tackle all types functions by introducing the DM's preferences.

The paper is organized as follows: Section 2 contains Fuzzy Goal Programming followed by Mathematical Model in Section 3. In Section 4 Solution Procedure are presented. In Section 5, a numerical example is provided to justify the methodology. In Section 6, concluding remarks are made with supporting references.

2. Fuzzy Goal Programming

Consider the multi-objective linear programming problem as:

$$\text{Optimize}: Z_k(X) = \sum_{j=1}^{n} c_j^k x_j, k = 1, 2, \ldots, K \tag{2.1}$$

subject to

$$X \in F, \text{ (F is a feasible set)} \tag{2.2}$$

The word 'Optimize' may refer to three different type of optimization problems, namely: minimization, maximization, and both minimization and maximization. Depending on the nature of optimization, flexible targets can be assigned to objective functions in three different way. After assigning fuzzy target (flexible target) values to the objective function following different models of fuzzy goal programming problem are formulated ([19],[45]).

Model I(a):

$$\text{Optimize}: \quad Z_k(X) \preceq g_k, k = 1, 2, 3, \ldots, K \tag{2.3}$$

$$\text{subject to} \quad X \in F, \text{ (F is a feasible set)} \tag{2.4}$$

Model I(b):

$$\text{Optimize}: \quad Z_k(X) \succeq g_k, k = 1, 2, 3, \ldots, K \tag{2.5}$$

$$\text{subject to} \quad X \in F, \text{ (F is a feasible set)} \tag{2.6}$$

Model I(c):

$$\text{Optimize}: \quad Z_k(X) \cong g_k, k = 1, 2, 3, \ldots, K \tag{2.7}$$

$$\text{subject to} \quad X \in F, \text{ (F is a feasible set)} \tag{2.8}$$

The aim of the goal programming is to minimize the deviation between aspiration level (target values) and objective functional values.

Mathematically it is represented as:

$$\min: \quad |Z_k(X) - \tilde{g}_k| \tag{2.9}$$

$$\text{subject to} \quad X \in F, \text{ (F is a feasible set)} \tag{2.10}$$

The mathematical model for fuzzy goal programming problem given in (2.9-2.10) can be solved using either of weighted fuzzy goal programming method, lexicographic Fuzzy goal programming method, minmax fuzzy goal programming method or Interactive fuzzy goal programming method.

3. Mathematical Model

In traditional fuzzy goal programming models the decision maker is required to specify an imprecise aspiration level for each of the objectives. Decision maker(s) faces difficulty in case of large-scale problems. Applying concept of multi-choice aspiration level and fuzzy set theory in goal programming problem, the decision maker is allowed to specify multiple imprecise aspiration levels. An objective with multiple imprecise aspiration level can be treated as a multi-choice fuzzy goal. The following multi-choice fuzzy goal programming model contains three kinds of fuzzy goals.

We present a general multi-choice fuzzy goal programming model after assigning multiple aspiration level to the multi-objective linear programming problem as:

Model II(a):

$$\text{Optimize}: \quad Z_k(X) \preceq g_k,\ g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, k = 1, 2, \ldots, K \tag{3.1}$$

$$\text{subject to} \quad X \in F,\ \text{(F is a feasible set)} \tag{3.2}$$

Model II(b):

$$\text{Optimize}: \quad Z_k(X) \succeq g_k,\ g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, k = 1, 2, 3, \ldots, K \tag{3.3}$$

$$\text{subject to} \quad X \in F,\ \text{(F is a feasible set)} \tag{3.4}$$

Model II(c):

$$\text{Optimize}: \quad Z_k(X) \cong g_k, g_k,\ g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, k = 1, 2, 3, \ldots, K \tag{3.5}$$

$$\text{subject to} \quad X \in F,\ \text{(F is a feasible set)} \tag{3.6}$$

where OPTIMIZE means finding an optimal decision X such that all fuzzy goals are satisfied. F is an optional set of hard constraints (rigid goals). The symbol $\tilde{}$ is the representation of fuzzifier of the imprecise target values.

The aim of the goal programming is to minimize the deviations between target values and objective functional values. Mathematically it is represented as:

$$\min: \quad |Z_k(X) - \tilde{g}_k|, \tilde{g}_k \in \{\tilde{g}_k^{(1)}, \tilde{g}_k^{(2)}, \tilde{g}_k^{(3)}, \ldots, \tilde{g}_k^{(R_k)}\}, k = 1, 2, 3, \ldots, K \tag{3.7}$$

$$\text{subject to} \quad X \in F,\ \text{(F is a feasible set)} \tag{3.8}$$

4. Solution Procedure

The solution procedure completes in two steps. In first step, attempt has been made to deal with fuzzy goals using membership functions. In Second step, multi-choice parameters have been handled using binary variable technique, interpolating polynomial, and least square approximation techniques. Finally, the equivalent mathematical model is a non-linear mixed integer programming, which is solved using existing technique or software.

4.1. Multi-choice fuzzy goal programming

The fuzzy goals are recognized in fuzzy set theory with the membership functions, which is defined over the feasible region. Linear membership functions are used in the literature [21, 34, 44, 46] and [29, 32] have used non-linear membership functions. Linear membership functions are used for the proposed fuzzy goals. We present the linear membership function for k-th goal as follows:

$$\mu_k^{(l)}(Z) = \begin{cases} 1, & Z_k \le g_k^{(l)} \\ 1 - \dfrac{Z_k - g_k^{(l)}}{\Delta_{kR}^{(l)}} & g_k^{(l)} \le Z_k \le g_k^{(l)} + \Delta_{kR}^{(l)}, l = 1, 2, \ldots, R_k \\ 0 & Z_k \ge g_k^{(l)} + \Delta_{kR}^{(l)} \end{cases}$$

$$\mu_k^{(l)}(Z) = \begin{cases} 1, & Z_k \ge g_k^{(l)} \\ 1 - \dfrac{g_k^{(l)} - Z_k}{\Delta_{kL}^{(l)}} & g_k^{(l)} - \Delta_{kL}^{(l)} \le Z_k \le g_k^{(l)}, k = 1, 2, \ldots, K \\ 0 & Z_k \le g_k^{(l)} - \Delta_{kL}^{(l)} \end{cases}$$

$$\mu_k^{(l)}(Z) = \begin{cases} 0 & Z_k \le g_k^{(l)} - \Delta_{kL}^{(l)} \\ 1 - \dfrac{g_k^{(l)} - Z_k}{\Delta_{kL}^{(l)}} & g_k^{(l)} - \Delta_{kL}^{(l)} \le Z_k \le g_k^{(l)}, l = 1, 2, \ldots, R_k \\ 1 - \dfrac{Z_k - g_k^{(l)}}{\Delta_{kR}^{(l)}} & g_k^{(l)} \le Z_k \le g_k^{(l)} + \Delta_{kR}^{(l)} \\ 0 & Z_k \ge g_k^{(l)} + \Delta_{kR}^{(l)} \end{cases}$$

where $\Delta_{kL}^{(l)}$ and $\Delta_{kR}^{(l)}$ are left spread and right spread of the fuzzy number, which are the maximum admissible violations from the aspiration level g_k to left and right respectively. These spreads are either subjectively selected by the decision maker or tolerances in a technical process.

[34] was the first mathematician to attempt for solving fuzzy goal programming problems. He considered a fuzzy goal programming model with all fuzzy type target values (Fig-3). He used the concept of fuzzy set theory in order to solve fuzzy goal programming problems by linear programming techniques. [21] suggested a single objective linear programming model with only 2K constraints for the same fuzzy goal programming model. In this paper, we propose a minimax approach to solve multi-choice fuzzy goal programming problem.

Minimax goal programming technique was first suggested by [20]. The aim of this approach is to minimize the maximum deviation from goal. By using minimax goal programming approach, the solution obtained is the most balanced optimal solution among the maximum satisfaction of different goals.

The mathematical model for multi-choice fuzzy goal is presented in equations (3.7-3.8). The proposed model cannot be solved directly. Therefore the equivalent mathematical models are proposed. During the transformation of the fuzzy goal programming model to its deterministic equivalent, two different criteria has been taken into consideration.

4.1.1 Criteria I: Left spread and right spread are equal

In this section, we consider the spreads (left spread and right spread) of fuzzy goals of k-th objective function are equal. Mathematically, $\Delta_{kL} = \Delta_{kL}^{(l)}, l = 1, 2, 3, \ldots, R_k$ and $\Delta_{kR} = \Delta_{kR}^{(l)}, l = 1, 2, 3, \ldots, R_k$. Now, we present the equivalent model as follows:

Model III:

$$\begin{aligned}
\min :\quad & \lambda \\
\text{subject to}\quad & \alpha_k \eta_k + \beta_k \rho_k \leq \lambda \qquad k = 1, 2, \ldots, K \\
& Z_k(X) + \eta_k - \rho_k (\cong / \preceq / \succeq) \tilde{g}_k, \tilde{g}_k \in \{\tilde{g}_k^{(1)}, \tilde{g}_k^{(2)}, \tilde{g}_k^{(3)}, \ldots, \tilde{g}_k^{(R_k)}\}, \\
& \qquad k = 1, 2, 3, \ldots, K \\
& \eta_k \times \rho_k = 0 \\
& X \in F \\
& \lambda, \eta_k, \rho_k \geq 0 \qquad k = 1, 2, \ldots, K
\end{aligned}$$

where

$$\tilde{g}_k \in \{\tilde{g}_k^{(1)}, \tilde{g}_k^{(2)}, \tilde{g}_k^{(3)}, \ldots, \tilde{g}_k^{(R_k)}\}, k = 1, 2, 3, \ldots, K$$

is the precise multiple aspiration level for the k-th goal ($k = 1, 2, \ldots, K$). η_k, ρ_k are negative and positive deviations from aspiration value of the k-th goal respectively.

Where the parameters $\alpha_k(\beta_k)$ represent the preferential and normalized weights and defined as:

$$\alpha_k(\beta_k) = \begin{cases} \dfrac{w_k}{v_k} & if\ \eta_k(\rho_k)\ is\ unwanted\ deviations,\ k = 1, 2, \ldots, K \\ 0 & Otherwise \end{cases}$$

In case of fuzzy goals (Fig-1, Fig-2, Fig-3), it is clear that $\eta_k \leq \Delta_{kL}$ and $\rho_k \leq \Delta_{kR}$ should be satisfied in order to provide feasible results. Therefore, the values of η_k and ρ_k should be restricted. Using minimax goal programming approach to solve multi-choice fuzzy goal programming problems requires only one extra constraint is to be added to Model 3.

Now, The mathematical model for optimization type ($\cong$i.e. Type C, Fig-3) is expressed as:

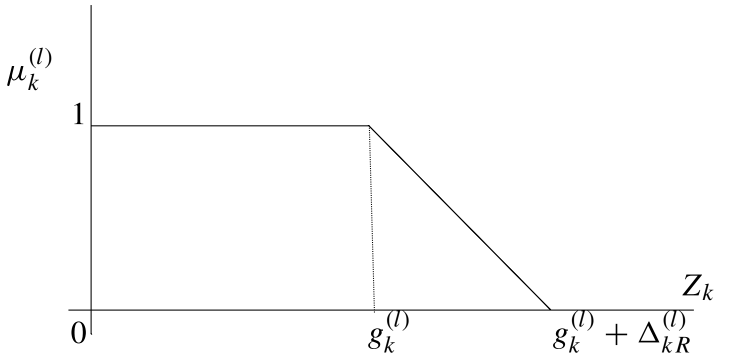

Figure 1: Membership function for l-th choice of k^{th} goal(Minimization: Type-A)

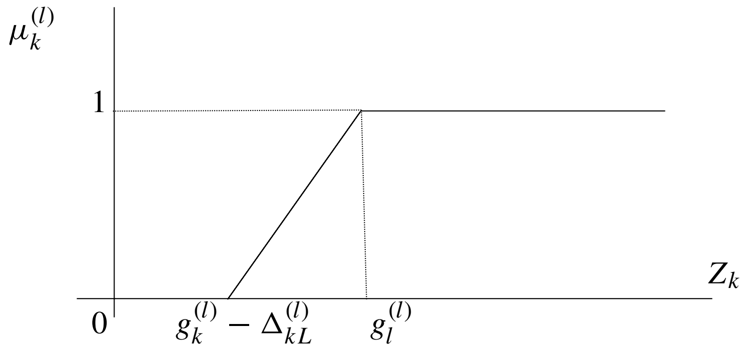

Figure 2: Membership function for l-th choice of k^{th} goal(Maximization: Type-B)

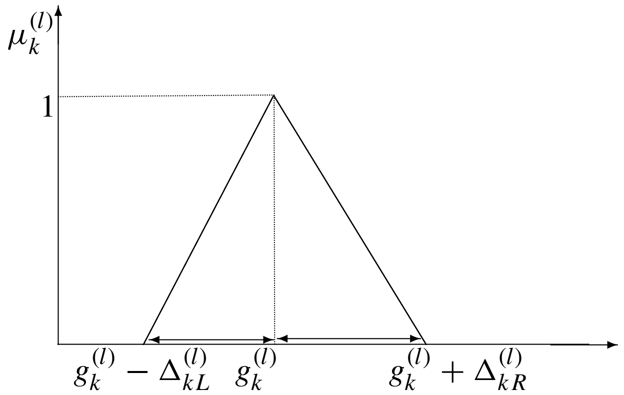

Figure 3: Membership function for l-th choice of k^{th} goal(Optimization: Type-C)

Model IV(a):

$$\begin{aligned}
\text{min :}\quad & \lambda \\
\text{subject to}\quad & \frac{1}{\Delta_{kL}}\eta_k + \frac{1}{\Delta_{kR}}\rho_k \leq \lambda \\
& Z_k(X) + \eta_k - \rho_k = g_k,\ g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, k = 1, 2, \ldots, K \\
& \eta_k \times \rho_k = 0 \\
& X \in F \\
& \lambda, \eta_k, \rho_k \geq 0
\end{aligned}$$

However, fuzzy goals of Type A, ($\preceq$, Fig-1) and Type B, ($\succeq$, Fig-2) are handled accordingly. The mathematical models for Type A and Type B are represented in Model 4(b) and Model 4(c) respectively as:

Model IV(b):

$$\begin{aligned}
\text{min :}\quad & \lambda \\
\text{subject to}\quad & \frac{1}{\Delta_{kL}}\eta_k \leq \lambda \\
& Z_k(X) + \eta_k - \rho_k = g_k,\ g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, k = 1, 2, \ldots, K \\
& \eta_k \times \rho_k = 0 \\
& X \in F \\
& \lambda, \eta_k, \rho_k \geq 0
\end{aligned}$$

Model IV(c):

$$\begin{aligned}
\text{min :}\quad & \lambda \\
\text{subject to}\quad & \frac{1}{\Delta_{kR}}\rho_k \leq \lambda \\
& Z_k(X) + \eta_k - \rho_k = g_k,\ g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, k = 1, 2, \ldots, K \\
& \eta_k \times \rho_k = 0 \\
& X \in F \\
& \lambda, \eta_k, \rho_k \geq 0
\end{aligned}$$

From the above models it is observed that $\lambda \leq 1$ is the default bound for λ. Let's replace λ by ν i.e. $\nu = 1 - \lambda$. Since $\lambda \leq 1$ and 1-$\lambda \geq 0$, thus $\nu \geq 0$ should be added to the models. Therefore minimizing $1 - \lambda$ is same as maximizing ν. Now the mathematical models for Type A, Type B and Type C are represented as:

Model V(a):

$$\begin{aligned}
\text{min}: \quad & \nu \\
\text{subject to} \quad & Z_k + \eta_k - \rho_k \leq g_k,\ g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, \qquad k = 1, 2, \ldots, K \\
& \nu + \frac{1}{\Delta_{kR}}\rho_k \leq 1 \qquad k = 1, 2, \ldots, K \\
& \eta_k \times \rho_k = 0 \\
& X \in F \\
& \nu, \eta_k, \rho_k \geq 0 \qquad k = 1, 2, \ldots, K
\end{aligned}$$

Model V(b):

$$\begin{aligned}
\text{min}: \quad & \nu \\
\text{subject to} \quad & Z_k + \eta_k - \rho_k = g_k, g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, k = 1, 2, \ldots, K \\
& \nu + \frac{1}{\Delta_{kL}}\eta_k \leq 1 \\
& \eta_k \times \rho_k = 0 \\
& X \in F \\
& \nu, \eta_k, \rho_k \geq 0,\ k = 1, 2, \ldots, K
\end{aligned}$$

Model V(c):

$$\begin{aligned}
\text{min}: \quad & \nu \\
\text{subject to} \quad & Z_k + \eta_k - \rho_k \leq g_k, g_k \in \{g_k^{(1)}, g_k^{(2)}, g_k^{(3)}, \ldots, g_k^{(R_k)}\}, k = 1, 2, \ldots, K \\
& \nu + \frac{1}{\Delta_{kL}}\eta_k + \frac{1}{\Delta_{kR}}\rho_k \leq 1 \\
& \eta_k \times \rho_k = 0 \\
& X \in F \\
& \nu, \eta_k, \rho_k \geq 0
\end{aligned}$$

The resultant mathematical models cannot be solved directly due to presence of multi-choice parameters. In order to deal with multi-choice parameters, following transformation techniques are proposed.

4.1.2 Criteria II: Left spread and right spread are not equal

In this section, we consider the spreads (left spread and right spread) of fuzzy goals of k-th objective function are unequal. Mathematically, $\Delta_{kL}^{(l)}$ and $\Delta_{kR}^{(l)}, l = 1, 2, 3, \ldots, R_k$ may be different for different choices of k-th objective function. $\eta_k = \Delta_{kL}^{(l)}$ and $\rho_k = \Delta_{kR}^{(l)}, l = 1, 2, 3, \ldots, R_k$ are the maximum allowable negative and positive deviations from the l-th aspiration level in the k-th goal, respectively. Now, we present the equivalent model as follows:

Model VI(a):

$$\begin{aligned} \max: \quad & \lambda \\ \text{subject to} \quad & \lambda \leq 1 - \frac{Z_k - g_k}{\Delta_{kR}}, \frac{Z_k - g_k}{\Delta_{kR}} \in \left\{ \frac{Z_k - g_k^{(1)}}{\Delta_{kR}^{(1)}}, \frac{Z_k - g_k^{(2)}}{\Delta_{kR}^{(2)}}, \ldots, \frac{Z_k - g_k^{(R_k)}}{\Delta_{kR}^{(R_k)}} \right\} \\ & X \in F \\ & \lambda \geq 0 \end{aligned}$$

Model VI(b):

$$\begin{aligned} \max: \quad & \lambda \\ \text{subject to} \quad & \lambda \leq 1 - \frac{g_k - Z_k}{\Delta_{kL}}, \frac{g_k - Z_k}{\Delta_{kL}} \in \left\{ \frac{g_k^{(1)} - Z_k}{\Delta_{kL}^{(1)}}, \frac{g_k^{(2)} - Z_k}{\Delta_{kL}^{(2)}}, \ldots, \frac{g_k^{(R_k)} - Z_k}{\Delta_{kL}^{(R_k)}} \right\} \\ & X \in F \\ & \lambda \geq 0 \end{aligned}$$

Model VI(c):

$$\begin{aligned} \max: \quad & \lambda \\ \text{subject to} \quad & \lambda \leq 1 - \frac{Z_k - g_k}{\Delta_{kR}}, \frac{Z_k - g_k}{\Delta_{kR}} \in \left\{ \frac{Z_k - g_k^{(1)}}{\Delta_{kR}^{(1)}}, \frac{Z_k - g_k^{(2)}}{\Delta_{kR}^{(2)}}, \ldots, \frac{Z_k - g_k^{(R_k)}}{\Delta_{kR}^{(R_k)}} \right\} \\ & \lambda \leq 1 - \frac{g_k - Z_k}{\Delta_{kL}}, \frac{g_k - Z_k}{\Delta_{kL}} \in \left\{ \frac{g_k^{(1)} - Z_k}{\Delta_{kL}^{(1)}}, \frac{g_k^{(2)} - Z_k}{\Delta_{kL}^{(2)}}, \ldots, \frac{g_k^{(R_k)} - Z_k}{\Delta_{kL}^{(R_k)}} \right\} \\ & X \in F \\ & \lambda \geq 0 \end{aligned}$$

From the above models it is observed that $\lambda \leq 1$ is the default bound for λ. Let's replace λ by ν i.e. $\nu = 1 - \lambda$. Since $\lambda \leq 1$ and 1-$\lambda \geq 0$, thus $\nu \geq 0$ should be added to the models. Therefore minimizing $1 - \lambda$ is same as maximizing ν. Now the mathematical models for Type A, Type B and Type C are represented as:

Model VII(a):

$$\begin{aligned} \min: \quad & \nu \\ \text{subject to} \quad & \nu \leq \frac{Z_k - g_k}{\Delta_{kR}}, \frac{Z_k - g_k}{\Delta_{kR}} \in \left\{ \frac{Z_k - g_k^{(1)}}{\Delta_{kR}^{(1)}}, \frac{Z_k - g_k^{(2)}}{\Delta_{kR}^{(2)}}, \ldots, \frac{Z_k - g_k^{(R_k)}}{\Delta_{kR}^{(R_k)}} \right\} \\ & X \in F \\ & \nu \geq 0 \end{aligned}$$

Model VII(b):

$$\begin{aligned} \min: \quad & \nu \\ \text{subject to} \quad & \nu \leq 1 - \frac{g_k - Z_k}{\Delta_{kL}}, \frac{g_k - Z_k}{\Delta_{kL}} \in \left\{ \frac{g_k^{(1)} - Z_k}{\Delta_{kL}^{(1)}}, \frac{g_k^{(2)} - Z_k}{\Delta_{kL}^{(2)}}, \ldots, \frac{g_k^{(R_k)} - Z_k}{\Delta_{kL}^{(R_k)}} \right\} \\ & X \in F \\ & \nu \geq 0 \end{aligned}$$

Model VII(c):

$$\begin{aligned} \min: \quad & \nu \\ \text{subject to} \quad & \nu \leq \frac{Z_k - g_k}{\Delta_{kR}}, \frac{Z_k - g_k}{\Delta_{kR}} \in \left\{ \frac{Z_k - g_k^{(1)}}{\Delta_{kR}^{(1)}}, \frac{Z_k - g_k^{(2)}}{\Delta_{kR}^{(2)}}, \ldots, \frac{Z_k - g_k^{(R_k)}}{\Delta_{kR}^{(R_k)}} \right\} \\ & \nu \leq \frac{g_k - Z_k}{\Delta_{kL}}, \frac{g_k - Z_k}{\Delta_{kL}} \in \left\{ \frac{g_k^{(1)} - Z_k}{\Delta_{kL}^{(1)}}, \frac{g_k^{(2)} - Z_k}{\Delta_{kL}^{(2)}}, \ldots, \frac{g_k^{(R_k)} - Z_k}{\Delta_{kL}^{(R_k)}} \right\} \\ & X \in F \\ & \nu \geq 0 \end{aligned}$$

The resultant mathematical models cannot be solved directly due to presence of multi-choice parameters. In order to deal with multi-choice parameters, following transformation techniques are proposed.

4.2. Interpolating polynomial approach

4.2.1 Newton's forward difference interpolating polynomial Approach

Let a function $f(x)$ be given by the following table at a discrete set of points i; $i = 0(1)n-1$.

Table 1: nodes

i	0	1	2	…	$n-1$
$f(i)$	f_0	f_1	f_2	…	f_{n-1}

The data given in the Table : nodes correspond to the sequence: 0,1,2 …, n-1. We consider the initial point as 0 and the step length as 1. Therefore it is easier to calculate the simple difference as compared to divided differences, because the forward differences do not involve division by the difference of nodes. These simple differences can be of forward differences or backward differences. Here we consider the forward differences and the interpolating polynomial based on forward differences.

From the Table 1, the n-th order forward difference $\Delta^n f_i$ is calculated using the given formula as:

$$\Delta f_i = f_{i+1} - f_i \tag{4.1}$$
$$\Delta^2 f_i = \Delta f_{i+1} - \Delta f_i \tag{4.2}$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$\Delta^n f_i = \Delta^{n-1} f_{i+1} - \Delta^{n-1} f_i \tag{4.3}$$

To find the forward difference and forward difference interpolating polynomial, Table 2 is used.

We formulate an interpolating polynomial using forward difference, we get

$$P_{n-1}(z) = f_0 + z\Delta f_0 + \frac{z(z-1)}{2!}\Delta^2 f_0 + \frac{z(z-1)(z-2)}{3!}\Delta^3 f_0 + \cdots + \frac{z(z-1)(z-2)(z-3)\dots(z-n+2)}{(n-1)!}\Delta^{n-1} f_0$$

Table 2: Newton's Forward Difference Table

i	$f(i)$	1-st order forward difference	2-nd order forward difference	…	$(n-1)$-th order forward difference
0	f_0				
		Δf_0			
1	f_1		$\Delta^2 f_0$		
		Δf_1			
2	f_2		$\Delta^2 f_1$	…	$\Delta^{(n-1)} f_0$
		Δf_2			
3	f_3		$\Delta^2 f_2$		
⋮	⋮	⋮			
n-1	f_{n-1}				

Now we formulate a multi-choice goal programming model for the multi-objective linear programming problem by using Newton's forward difference interpolating polynomial as:

$$\text{min}: \quad \sum_{k=1}^{K} \omega_k(\rho_k + \eta_k) \tag{4.4}$$
$$\text{subject to} \quad Z_k(X) + \eta_k - \rho_k = P_{R_k - 1}(z), k = 1, 2, 3, \dots, K \tag{4.5}$$
$$X \in F \tag{4.6}$$
$$z \text{ is an integer and } 0 \leq z \leq R_k - 1 \tag{4.7}$$
$$\rho_k, \eta_k, x_j, z \geq 0, \quad j = 1, 2, \dots, n; k = 1, 2, 3, \dots, K \tag{4.8}$$

After applying the multi choice fuzzy goal programming , the resulting Multi-Choice Fuzzy Goal Programming Problem using Newton's forward difference interpolating polynomial can be expressed as:

Model VIII(a):

$$\begin{aligned}
\text{min :} \quad & \nu \\
\text{subject to} \quad & Z_k + \eta_k - \rho_k \leq P_{R_k - 1}(z),\ k = 1, 2, \ldots, K \\
& \nu + \frac{1}{\Delta_{kL}}\eta_k + \frac{1}{\Delta_{kR}}\rho_k \leq 1 \\
& \eta_k \times \rho_k = 0 \\
& z \text{ is an integer and } 0 \leq z \leq R_k - 1 \\
& X \in F \\
& \nu, \eta_k, \rho_k \geq 0
\end{aligned}$$

Integer z ensures only one aspiration level must be chosen in each goal. ρ_{ij}, η_{ij} are the maximum allowable positive and negative deviation from the j-th aspiration level in the i-th goal, respectively.

Using Model - VII(c), we can express the mathematical formulation as:

Model VIII(b):

$$\begin{aligned}
\text{min :} \quad & \nu \\
\text{subject to} \quad & \nu \leq P_{R_k - 1}(z),\ k = 1, 2, 3, \ldots, K \\
& \nu \leq P^1_{R_k - 1}(z),\ k = 1, 2, 3, \ldots, K \\
& X \in F \\
& \nu \geq 0
\end{aligned}$$

4.3. Binary variable approach

Theorem 4.1. Every natural number can be expressed as sum of 2^k number of terms and each term is a power of 2, where $k \in \mathbb{N} \cup \{0\}$.

The proof of the above theorem is obvious. Now we formulate a multi-choice goal programming model for the multi-objective linear programming problem by using binary variables as.

$$\min: \quad \sum_{k=1}^{K} \omega_k(\rho_k + \eta_k) \tag{4.9}$$

$$\text{subject to} \quad Z_k(X) + \eta_k - \rho_k = P_k(\theta_k^{(1)}, \theta_k^{(2)}),\ k = 1, 2, 3, \ldots, K \tag{4.10}$$

$$X \in F \tag{4.11}$$

$$\theta_k^{(1)}, \theta_k^{(2)} \in \{0, 1\} \tag{4.12}$$

$$\rho_k, \eta_k, x_j, z \ \geq\ 0, \quad j = 1, 2, \ldots, n;\ k = 1, 2, 3, \ldots, K \tag{4.13}$$

After applying the concept of multi choice fuzzy goal programming, the resulting multi-choice fuzzy goal programming problem using binary variable can be expressed as:

Model - IX(a):

$$\begin{aligned}
\min: \quad & \nu \\
\text{subject to} \quad & Z_k + \eta_k - \rho_k \leq P_k(\theta_k^{(1)}, \theta_k^{(2)}), k = 1, 2, \ldots, K \\
& \nu + \frac{1}{\Delta_{kL}}\eta_k + \frac{1}{\Delta_{kR}}\rho_k \leq 1 \\
& \eta_k \times \rho_k = 0 \\
& X \in F \\
& \nu, \eta_k, \rho_k \geq 0
\end{aligned}$$

Binary numbers $\theta_k^{(1)}, \theta_k^{(2)}$ ensures only one aspiration level must be chosen in each goal. ρ_{ij}, η_{ij} are the maximum allowable positive and negative deviation from the j-th aspiration level in the i-th goal, respectively. Using Model - VII(c), we can express MCFGP problem using binary variable as:

Model IX(b):

$$\begin{aligned}
\min: \quad & \nu \\
\text{subject to} \quad & \nu \leq P_k^1(\theta_k^{(1)}, \theta_k^{(2)}),\ k = 1, 2, \ldots, K \\
& \nu \leq P_k^2(\theta_k^{(1)}, \theta_k^{(2)}),\ k = 1, 2, \ldots, K \\
& X \in F \\
& \nu \geq 0
\end{aligned}$$

5. Numerical Example

A company is manufacturing three products y_1, y_2 and y_3. For the product y_1 there are three customers A, B and C with "approximate" demands 30, 50 and 70 respectively.

The maximum allowable negative and positive deviation of customers A, B and C from their goals are equal and set as 4, 5 and 6 respectively, for customers D and E maximum allowable negative and positive deviation are 3, 4 and 2, 3 respectively. The selling profit of these products is 10$, 12$ and 15$. However, because of some limitations such as political ones, the company has to select only one of its customers for each product. A profit of at least 850$ from products' selling is expected. Three resources S_1, S_2 and S_3 are needed to produce these products. The amounts of each resource which is needed to produce each product are presented in Table 1.

This is a case of Multi-choice Fuzzy goal Programming Problem which cannot be solved by current GP approaches. For this problem, the related goals are listed below.

$$Z_1(y_1) \cong g_1, g_1 \in \{30, 50, 70\}$$

$$Z_2(y_2) \cong g_2, g_2 \in \{15, 30\}$$

$$Z_3(y_3) \cong g_3, g_3 \in \{10, 20\}$$

Table 3: Related information about products

product	Customer	Demands	Maximum allowable neg. and pos. deviation	Profit
y_1	A	30	4	10
	B	50	5	
	C	70	6	
y_2	D	15	3	12
	E	30	4	
y_3	F	10	2	15
	G	20	3	

Table 4: The amount of consumption of resources for each product

Resource	Product		
	y_1	y_2	y_3
S_1	5	4	4
S_2	3	7	6
S_3	1	2	1

5.1. Solution by binary variable method

Based on the MCFGP method, the problem is formulated by using binary variables as follows:

$$\max: \quad \lambda \tag{5.1}$$

$$\text{subject to} \quad \lambda \leq 1 - \left[\theta_{12}\left\{\frac{y_1 - 30}{4}\theta_{11} + \frac{y_1 - 50}{5}(1-\theta_{11})\right\} + \frac{y_1 - 70}{6}(1-\theta_{12})\right] \tag{5.2}$$

$$\lambda \leq 1 - \left[\theta_{12}\left\{\frac{30 - y_1}{4}\theta_{11} + \frac{50 - y_1}{5}(1-\theta_{11})\right\} + \frac{70 - y_1}{6}(1-\theta_{12})\right] \tag{5.3}$$

$$\lambda \leq 1 - \left[\frac{y_2 - 15}{3}\theta_{21} + \frac{y_2 - 30}{4}(1-\theta_{21})\right] \tag{5.4}$$

$$\lambda \leq 1 - \left[\frac{15 - y_2}{3}\theta_{21} + \frac{30 - y_2}{4}(1-\theta_{21})\right] \tag{5.5}$$

$$\lambda \leq 1 - \left[\frac{y_3 - 10}{2}\theta_{31} + \frac{y_3 - 20}{3}(1-\theta_{31})\right] \tag{5.6}$$

$$\lambda \leq 1 - \left[\frac{10 - y_3}{2}\theta_{31} + \frac{20 - y_3}{3}(1-\theta_{31})\right] \tag{5.7}$$

$$10y_1 + 12y_2 + 16y_3 \geq 850 \tag{5.8}$$

$$5y_1 \leq x_{11} \tag{5.9}$$

$$3y_1 \leq x_{12} \tag{5.10}$$

$$y_1 \leq x_{13} \tag{5.11}$$

$$4y_2 \leq x_{21} \tag{5.12}$$

$$7y_2 \leq x_{22} \tag{5.13}$$

$$2y_2 \leq x_{23} \tag{5.14}$$

$$4y_3 \leq x_{31} \tag{5.15}$$

$$6y_3 \leq x_{32} \tag{5.16}$$

$$y_3 \leq x_{33} \tag{5.17}$$

$$x_{11} + x_{12} + x_{13} \leq 400 \tag{5.18}$$

$$x_{21} + x_{22} + x_{23} \leq 380 \tag{5.19}$$

$$x_{31} + x_{32} + x_{33} \leq 120 \tag{5.20}$$

$$\theta_{11}, \theta_{12}, \theta_{21}, \theta_{31} \in \{0, 1\} \tag{5.21}$$

This problem is solved by using Lingo 11 [39]. The optimal solution of the above problem is obtained as $(Z_1, Z_2, Z_3 = 32.46, 29.23, 10.9)$, $(\theta_{11}, \theta_{12}, g_1 = 1,\ 1,\ 30)$, $(\theta_{21}, g_2 = 0, 30)$ and $(\theta_{31}, g_2 = 1,\ 10)$ and the value of the λ is 0.38.

5.2. Solution by Newton's forward difference interpolating polynomial method

Based on the MCFGP method, the problem is formulated by using Newton's forward difference interpolating polynomial as follows:

$$\text{max}: \quad \lambda \tag{5.22}$$

$$\text{subject to} \quad \lambda \leq \frac{(y_1 - 30)}{4} - \frac{(y_1 + 50)}{20} z_1 + \frac{(50 + y_1)}{120} z_1^2 \tag{5.23}$$

$$\lambda \leq \frac{(30 - y_1)}{4} + \frac{(y_1 + 50)}{20} z_2 - \frac{(50 + y_1)}{120} z_2^2 \tag{5.24}$$

$$\lambda \leq \frac{(y_2 - 15)}{3} - \frac{(y_2 + 60)}{12} z_3 \tag{5.25}$$

$$\lambda \leq \frac{(15 - y_2)}{3} + \frac{(y_2 + 60)}{12} z_4 \tag{5.26}$$

$$\lambda \leq \frac{(y_3 - 10)}{2} - \frac{(y_3 + 10)}{6} z_5 \tag{5.27}$$

$$\lambda \leq \frac{(10 - y_3)}{2} + \frac{(y_3 + 10)}{6} z_6 \tag{5.28}$$

$$10y_1 + 12y_2 + 16y_3 \geq 850 \tag{5.29}$$

$$5y_1 \leq x_{11} \tag{5.30}$$

$$3y_1 \leq x_{12} \tag{5.31}$$

$$y_1 \leq x_{13} \tag{5.32}$$

$$4y_2 \leq x_{21} \tag{5.33}$$

$$7y_2 \leq x_{22} \tag{5.34}$$

$$2y_2 \leq x_{23} \tag{5.35}$$

$$4y_3 \leq x_{31} \tag{5.36}$$

$$6y_3 \leq x_{32} \tag{5.37}$$

$$y_3 \leq x_{33} \tag{5.38}$$

$$x_{11} + x_{12} + x_{13} \leq 400 \tag{5.39}$$

$$x_{21} + x_{22} + x_{23} \leq 380 \tag{5.40}$$

$$x_{31} + x_{32} + x_{33} \leq 120 \tag{5.41}$$

$$z_i \text{ are integers}, \quad i = 1, 2, \ldots, 6 \tag{5.42}$$

$$z_1, z_2 \leq 2 \tag{5.43}$$

$$z_3,\ z_4,\ z_5 \text{ and } z_6 \leq 1 \tag{5.44}$$

This problem is solved by using Lingo 11 [39]. The optimal solution of the above problem is obtained as $(Z_1, Z_2, Z_3 = 32.47, 29.23, 10.90)$, $(z_1, z_2, z_3, z_4, z_5, z_6) = (0, 1, 0, 1, 0, 1)$, $(g_1,\ g_2, g_3) = (30, 15, 10)$ and the value of the λ is 0.45.

6. Conclusions

In this paper, we proposed multi-choice fuzzy goal programming problem. After assigning multiple fuzzy goals, we observe two different criteria among fuzzy target values. In first criteria we considered the case where Left spread and right spread are equal. In second criteria left spread and right spread may not be equal. Then to transform the model, we have applied two techniques, namely: Newton's forward difference interpolating polynomial approach and binary variables approach. From the results, it is observed that we obtained almost equal results using both the methods. The proposed method may be extended to fuzzy probabilistic goal programming problem.

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