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Approximating the Parameters φ Through Bayes for Generalized Compound Rayleigh Distribution with Quadratic Loss

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Abstract

In this paper we dealt with the estimation of an unknown Location parameter φ of the generalized Compound Rayleigh distribution (GCRD) using Bayesian and Approximation Bayesian estimation techniques based on complete sampling data. Furthermore, the simulation results indicate that, depending on the minimum mean squared error, the Bayesian and Approximate Bayesian estimates under Quadratic loss function suggested in this paper, have significantly better performance and the Approximate Bayes estimator also performs better than the Bayes estimator.

Keywords: Generalized Compound Rayleigh Distribution; Bayes Estimators; Quadratic Loss Function; Approximation by Lindley.

1. INTRODUCTION

The Generalized Compound Rayleigh Distribution is taken from the three-parameter Burr type XII distribution with a special case of the three-parameter Burr type XII distribution. Mostert Roux, and Bekker (1999) used this distribution as a Gamma mixture of Rayleigh distribution and obtained the Compound Rayleigh model with unimodal hazard function. This unimodal hazard function is generalized and a flexible parametric model, which entrenches the Compound Rayleigh model, by adding shape parameter. Bain and Engelhardt (1991) studied this distribution (also known as the Compound Weibull distribution (Dubey 1968) from a Poisson perspective. The pdf of Generalized Compound Rayleigh Distribution is given by

$$f(x;\theta,\varphi,\delta)=\frac{\theta}{\delta}\varphi^{\frac{1}{\delta}}x^{(\theta-1)}\left(\varphi+x^{\delta}\right)^{-(\delta+1)}\;;\qquad x,\theta,\varphi,\delta>0 \tag{1.1}$$

The Quadratic loss function is commonly used loss function in estimation problems, given as $L(\hat{\Delta},\Delta)=k(\hat{\Delta}-\Delta)^2$ where $\hat{\Delta}$ is the estimate of Δ, the loss function is called quadratic weighed loss function if k=1, we have

$$L(\hat{\Delta},\Delta)=(\hat{\Delta}-\Delta)^2 \tag{1.2}$$

This loss function is symmetrical because it associates the equal importance to the losses due to overestimation and under estimation with equal magnitudes. [Ferguson (1985), Canfield (1970), Basu and Ebrahimi (1991), Zellner (1986)]. Soliman (2001) derived and discussed the properties of varian's (1975) discussed use of asymmetric loss function in such situation for a number of distributions.

We have studied the sensitivity of the Approximate Bayes estimators of model and presented a numerical study to illustrate the above technique on generated observations and the comparison is done by R-programming.

2. The Estimators

Let $x_1\le x_2\le \dots\dots\dots \le x_n$ be the n failures in complete sample case. The likelihood function is given by

$$L(\underline{x}\,|\theta,\varphi,\delta)=\left(\frac{\theta}{\delta}\right)^n U e^{-T/\delta} \tag{2.1}$$

where

$$T=\sum_{j=1}^{n} log\left[1+\frac{{x_j}^{\theta}}{\varphi}\right] \qquad \text{and} \qquad U=\prod_{j=1}^{n}\frac{{x_j}^{\theta-1}}{\varphi+{x_j}^{\theta}}$$

from equation(2.1), the log likelihood function is

$$Log\ L= n\log\theta+\frac{n}{\delta}\log\varphi-n\log\delta+(\theta-1)\sum_{j=1}^{n} log x_j-\left(\frac{1}{\delta}+1\right)\sum_{j=1}^{n} log\left(\varphi+{x_j}^{\theta}\right) \tag{2.2}$$

differentiating the equation(2.2) with respect to θ,φ and δ yields respectively we obtain the Maximum Likelihood Estimator of $\theta,\varphi\ and\ \delta$.

Applying the Newton-Raphson method $\hat{\theta}_{MLE}$ and $\hat{\varphi}_{MLE}$ can be derived and then from them $\hat{\delta}_{MLE}$ can be obtained.

3. Bayes estimators for $\boldsymbol{\delta}$ with known parameter $\boldsymbol{\theta\ and\ \varphi}$

If $\hat{\theta}$ and $\hat{\varphi}$ is known we assume $\delta(a,b)$ as conjugate prior for δ as

$$g(\delta|\underline{x})=\frac{b^a}{\Gamma a}\left(\frac{1}{\delta}\right)^{a+1}e^{-\frac{b}{\delta}};\qquad (a,b,\delta)>0 \tag{3.1}$$

combining the likelihood function equation(2.1) and prior density equation(3.1), we obtain the posterior density of δ in the form

$$h(\delta|\underline{x}) = \frac{\delta^{-(n+\theta-1)}e^{-\frac{(b+T)}{\delta}}}{\int_0^\infty \delta^{-(n+\theta-1)}e^{-\frac{(b+T)}{\delta}}d\delta} \tag{3.2}$$

assuming

$T = \sum_{j=1}^{n} \log\left(1+\frac{x_j^{\theta}}{\varphi}\right)$ and $U = \prod_{j=1}^{n} \frac{x_j^{\theta-1}}{(\varphi+x_j^{\theta})}$

$$h(\delta|\underline{x}) = \frac{\gamma^{-(n+\theta-1)}e^{-\frac{(b+T)}{\delta}}(b+T)^{(n+a)}}{\Gamma(n+a)} \tag{3.3}$$

Bayes Estimator of δ under Quadratic loss function (QLF)

Under Quadratic loss function is the posterior mean given by

$$\hat{\delta}_{QB} = \int_0^\infty \delta \frac{(b+T)^{(n+a)}}{\Gamma(n+a)}\left(\frac{1}{\delta}\right)^{(n+a+1)} e^{-(b+T)/\delta}d\delta \tag{3.4}$$

$$\hat{\delta}_{QB} = \frac{(b+T)^{(n+a)}}{\Gamma(n+a)}\int_0^\infty \left(\frac{1}{\delta}\right)^{(n+a)} exp^{-(b+T)/\delta}$$

$$substituting \quad y = \frac{b+T}{\delta}$$

On solving which gives

$$\hat{\delta}_{QB} = \frac{(b+T)}{(n+a-1)} \tag{3.5}$$

4. Approximate Bayes Estimator of the unknown Location parameter $\boldsymbol{\varphi}$.

The Joint prior density of the parameters θ, φ, δ is given by

$$G(\theta,\varphi,\delta) = \mathrm{g}_1(\theta)\mathrm{g}_2(\varphi)\mathrm{g}_3(\delta|\varphi)$$

$$= \frac{c}{\delta\Gamma\xi}\varphi^{-\xi}\delta^{\xi+1}exp\left[-\left(\frac{\delta}{\varphi}+\frac{\varphi}{\mu}\right)\right] \tag{4.1}$$

where

$$g_1(\theta) = c \tag{4.2}$$

$$g_2(\varphi) = \frac{1}{\mu}e^{-\frac{\delta}{\mu}} \tag{4.3}$$

$$g_3(\delta) = \frac{1}{\Gamma\xi}\varphi^{-\xi}\delta^{\xi+1}e^{-\frac{\delta}{\varphi}} \tag{4.4}$$

The Joint posterior combing the likelihood equation(2.2) and joint prior equation(4.1) is

$$h^*(\theta,\varphi,\delta|\underline{x}) = \frac{\varphi^{-\xi}\delta^{\xi+1}exp\left[-\left(\frac{\delta}{\varphi}+\frac{\varphi}{\mu}\right)\right].L(\underline{x}|\theta,\varphi,\delta)}{\int_\theta\int_\varphi\int_\delta \beta^{-\xi}\gamma^{\xi+1}exp\left[-\left(\frac{\gamma}{\beta}+\frac{\beta}{\delta}\right)\right].L(\underline{x}|\theta,\varphi,\delta)d\theta d\varphi d\delta} \tag{4.5}$$

The Approximate Bayes Estimator is given by

$$Y(\Theta) = Y(\theta, \varphi, \delta) \tag{4.6}$$

$$\hat{Y}_{APQB} = E\left(Y|\underline{x}\right) = \frac{\int_\theta \int_\varphi \int_\delta Y(\alpha,\beta,\gamma) G^*(\theta,\varphi,\delta) d\theta d\varphi d\delta}{\int_\theta \int_\varphi \int_\delta G^*(\theta,\varphi,\delta) d\theta d\varphi d\delta} \tag{4.7}$$

Lindley Approximation Procedure

The Bayes estimators of a function $\nu = \nu(\vartheta, \rho)$ of the unknown parameter ϑ and ρ under quadratic loss is the posterior mean

$$\hat{\nu}_{BQ} = E\left(\nu|\underline{x}\right) = \frac{\iint \nu(\vartheta,\rho) h^*(\theta, p|\underline{x}) d\vartheta\, d\rho}{\iint h^*(\theta, p|\underline{x}) d\vartheta\, d\rho} \tag{4.7a}$$

The ratio of integrals in equation (4.7a) does not seem to take a closed form so we must consider the Lindley approximation procedure as

$$E\left(\nu(\vartheta, \rho)|\underline{x}\right) = \frac{\int \nu(\vartheta).e^{(l(\vartheta)+\rho(\vartheta))} d\vartheta}{\int e^{(l(\vartheta)+\rho(\vartheta))}.d\vartheta} \tag{4.7b}$$

Lindley developed approximate procedure for evaluation of posterior expectation of $\nu(\vartheta)$. Several other authors have used this technique to obtain Bayes estimators (see Sinha(1986), Sinha and sloan(1988),Soliman(2001)).The posterior expectation of Lindley approximation procedure to evaluate of $\nu(\vartheta)$in equation (4.7a and 4.7b) under SELF, where where $\rho(\vartheta) = \log g(\vartheta)$, and $g(\vartheta)$ is an arbitrary function of ϑ and $l(\vartheta)$ is the logarithm likelihood function (Lindley (1980)).

The modified form of equation (4.7) is given by

$$E\left(Y(\alpha, \beta, \gamma|\underline{x}\right) = Y(\Theta) + \frac{1}{2}[A(Y_1\sigma_{11} + Y_2\sigma_{12} + Y_3\sigma_{13}) + B(Y_1\sigma_{21} + Y_2\sigma_{22} + Y_3\sigma_{23}) + P\ (Y_1\sigma_{31} + Y_2\sigma_{32} + Y_3\sigma_{33})] + (Y_1 a_1 + Y_2 a_2 + Y_3 a_3 + a_4 + a_5) + 0\left(\frac{1}{n^2}\right) \tag{4.8}$$

Above equation is evaluated at MLE = $(\hat{\theta}, \hat{\varphi}, \hat{\delta})$

where

$$a_1 = \rho_1\sigma_{11} + \rho_2\sigma_{12} + \rho_3\sigma_{13} \tag{4.9}$$

$$a_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + \rho_3\sigma_{23} \tag{4.10}$$

$$a_3 = \rho_1\sigma_{31} + \rho_2\sigma_{32} + \rho_3\sigma_{33} \tag{4.11}$$

$$a_4 = Y_{12}\sigma_{12} + Y_{13}\sigma_{13} + Y_{23}\sigma_{23} \tag{4.12}$$

$$a_5 = \frac{1}{2}(Y_{11}\sigma_{11} + Y_{22}\sigma_{22} + Y_{33}\sigma_{33})\ ; \tag{4.13}$$

And

$$A = [\sigma_{11}l_{111} + 2\sigma_{12}l_{121} + 2\sigma_{13}l_{131} + 2\sigma_{23}l_{231} + \sigma_{22}l_{221} + \sigma_{33}l_{331}] \tag{4.14}$$

$$B = [\sigma_{11}l_{112} + 2\sigma_{12}l_{122} + 2\sigma_{13}l_{132} + 2\sigma_{23}l_{232} + \sigma_{22}l_{222} + \sigma_{33}l_{332}] \tag{4.15}$$

$$P = [\sigma_{11}l_{113} + 2\sigma_{13}l_{133} + 2\sigma_{12}l_{123} + 2\sigma_{23}l_{233} + \sigma_{22}l_{223} + \sigma_{33}l_{333}] \quad (4.16)$$

To apply Lindley approximation on equation (4.8) , we first obtain

$$\sigma_{ij} = [-l_{ijk}]^{-1} i,j,k = 1,2,3$$

Likelihood function from equation (2.2) is

$$Log\ L = n\log\theta + \frac{n}{\delta}\log\varphi - n\log\delta + (\theta-1)\sum_{j=1}^{n} logx_j - \left(\frac{1}{\delta} + 1\right)\sum_{j=1}^{n} log(\varphi + x_j{}^{\theta}) \quad (4.17)$$

Now differentiating log likelihood function with respect to $\theta, \varphi\ and\ \delta$,we get

$$l_{111} = \frac{2n}{\theta^3} - \left(\frac{1}{\gamma}+1\right)\omega_{133} \quad \text{where} \quad \omega_{133} = \sum \frac{x_j{}^{\theta}(\varphi - x_j{}^{\theta})(\log x_j)^3}{(\varphi+x_j{}^{\theta})^3} \quad (4.18)$$

$$l_{222} = \frac{2n}{\delta\varphi^3} - 2\left(\frac{1}{\gamma}+1\right)\delta_{13} \quad \text{where} \quad \delta_{13} = \sum_{j=1}^{n}\frac{1}{(\varphi+x_j)^3} \quad (4.19)$$

$$l_{333} = -\frac{2n}{\delta^3} - \frac{6n\ log\varphi}{\delta^4} + \frac{6}{\delta^4}\delta_{10} \quad \text{where} \quad \delta_{10} = \sum_{j=1}^{n} log\left(\varphi + x_j{}^{\theta}\right) \quad (4.20)$$

$$l_{112} = \left(\frac{1}{\delta}+1\right)\omega_{123} \quad \text{where} \quad \omega_{123} = \sum_{j=1}^{n}\frac{x_j{}^{\theta}(\varphi - x_j{}^{\theta})(\log x_j)^2}{(\varphi+x_j{}^{\theta})^3} \quad (4.21)$$

and $l_{112} = l_{121}$

$$l_{113} = \frac{\varphi}{\delta^2}\omega_{122} \quad \text{where} \quad \omega_{122} = \sum_{j=1}^{n}\frac{x_j{}^{\theta}(logx_j)^2}{(\varphi+x_j{}^{\theta})^2} \quad (4.22)$$

$$l_{221} == -2\left(\frac{1}{\delta}+1\right)\omega_{113} \quad \text{where} \quad \omega_{113} = \sum_{j=1}^{n}\frac{x_j{}^{\theta}logx_j}{(\varphi+x_j{}^{\theta})^3} \quad (4.23)$$

$$l_{221} = l_{212}$$

$$l_{223} = \frac{n}{(\delta\varphi)^2} - \frac{1}{(\delta)^2}\delta_{12} \quad \text{where} \quad \delta_{12} = \sum_{j=1}^{n}\frac{1}{(\varphi+x_j{}^{\theta})^2} \quad (4.24)$$

$$l_{331} = -\frac{2}{\delta^3}\omega_{11} \quad \text{where} \quad \omega_{11} = \sum_{j=1}^{n}\frac{x_j{}^{\delta}logx_j}{(\varphi+x_j{}^{\theta})} \quad (4.25)$$

$$l_{331} = l_{313}$$

$$l_{332} = \frac{\partial}{\partial\delta}\left(\frac{\partial^2 L}{\partial\delta\partial\varphi}\right) = \frac{2}{\delta^3}\left(\frac{n}{\varphi} - \delta_{11}\right) \quad (4.26)$$

$$l_{332} = l_{323}$$

$$l_{231} = -\frac{\omega_{14}}{\delta^2} \quad (4.27)$$

$$l_{231} = l_{213}$$

$$l_{123} = -\frac{\omega_{14}}{\delta^2} \quad (4.28)$$

$$l_{123} = l_{132}$$

$$l_{133} = \frac{-2}{\delta^2}\sum_{j=1}^{n} \frac{x_j{}^{\theta}\log x_j}{(\varphi + x_j{}^{\theta})} = -\frac{2}{\delta^2}\omega_{11} \tag{4.29}$$

$$l_{122} = -2\left(\frac{1}{\delta} + 1\right)\omega_{113} \quad \text{where} \quad \omega_{113} = \sum_{j=1}^{n} \frac{x_j{}^{\theta}\log x_j}{(\varphi + x_j{}^{\theta})^3} \tag{4.30}$$

$$l_{233} = \frac{2n}{\varphi\delta^3} - \frac{2}{\delta^3}\sum_{j=1}^{n} \frac{1}{\varphi + x_j{}^{\theta}} = \frac{2}{\delta^3}\left(\frac{n}{\varphi} - \delta_{11}\right) \tag{4.31}$$

The matrix of derivatives is given as

$$[-l_{ijk}] = -\begin{bmatrix} l_{111} & l_{112} & l_{113} \\ l_{221} & l_{222} & l_{223} \\ l_{331} & l_{332} & l_{333} \end{bmatrix} \tag{4.32}$$

$$= \begin{bmatrix} \frac{2n}{\theta^3} - \left(\frac{1}{\delta} + 1\right)\omega_{133} & ,\left(\frac{1}{\delta} + 1\right)\omega_{123} & -\frac{\varphi}{\delta^2}\omega_{122} \\ -2\left(\frac{1}{\delta} + 1\right)\omega_{113} & ,\frac{2n\delta}{\delta\varphi^3} - 2\left(\frac{1}{\delta} + 1\right)\delta_{13} & \frac{n}{(\delta\beta)^2} - \frac{1}{\delta^2}\delta_{12} \\ \frac{-2}{\delta^3}\omega_{11} & ,\frac{-2}{\delta^3}\left(\frac{n}{\delta} - \delta_{11}\right) & ,-\frac{2n}{\delta^3} - \frac{6n\ log\varphi}{\delta^4} + \frac{6}{\delta^4}\delta_{10} \end{bmatrix}$$

$$[-l_{ijk}] = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix}$$

$Determinant\ of\quad [-l_{ijk}]$

$$D = \{Q_{11}[Q_{22}Q_{33} - Q_{23}Q_{32}] - Q_{12}[Q_{21}Q_{33} - Q_{31}Q_{23}] + Q_{13}[Q_{21}Q_{32} - Q_{22}Q_{33}]\} \tag{4.45}$$

$$[-l_{ijk}]^{-1} = \frac{(Adjoint\ of\ [-l_{ijk}])'}{D}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \frac{U_{11}}{D} & \frac{U_{12}}{D} & \frac{U_{13}}{D} \\ \frac{U_{21}}{D} & \frac{U_{22}}{D} & \frac{U_{23}}{D} \\ \frac{U_{31}}{D} & \frac{U_{32}}{D} & \frac{U_{33}}{D} \end{bmatrix}$$

$$[-l_{ijk}]^{-1} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}; \tag{4.33}$$

Approximate Bayes Estimator

$Y(\alpha, \beta, \gamma) = Y$

$\hat{Y}_{APQB} = E(Y|\underline{x})$

evaluated from equation number and from joint prior density , we have

$G(\theta, \varphi, \delta) = g(\theta)g_2(\varphi)g_3(\delta|\varphi)$

$$= \frac{c}{\varepsilon\Gamma\xi}\beta^{-\xi}\gamma^{\xi-1}exp\left[-\left(\frac{\delta}{\varphi}+\frac{\varphi}{\varepsilon}\right)\right];$$

$$\rho = \log G = \log C - log\varepsilon - log[\xi + (\xi-1)log\delta - \xi log\varphi$$
$$-\left(\frac{\delta}{\varphi}+\frac{\varphi}{\varepsilon}\right) \quad (4.34)$$

Log G= $constant - \xi log\beta + (\xi-1)log\gamma - \frac{\delta}{\varphi} - \frac{\varphi}{\varepsilon}$

$$\rho_1 = \frac{\delta\rho}{\delta\theta} = 0 \quad (4.35)$$

$$\rho_2 = \frac{\delta\rho}{\delta\varphi} = \frac{-\xi}{\varphi} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon} \quad (4.36)$$

$$\rho_3 = \frac{\delta\rho}{\delta\delta} = \frac{\xi-1}{\delta} - \frac{1}{\varphi} \quad (4.37)$$

Using equation (4.14) to equation (4.37), we have

$$A = \frac{1}{D}\Big[U_{11}\left(\frac{2n}{\theta^3} - \left(\frac{1}{\delta}+1\right)\omega_{133}\right) + 2U_{12}\left(\frac{1}{\delta}+1\right)\omega_{123} + 2U_{13}\frac{\varphi}{\delta^3}\omega_{122} - 2U_{23}\frac{\omega_{14}}{\delta^2} - 2U_{22}\left(\frac{1}{\delta}+1\right)\omega_{113} - \frac{2}{\delta^3}U_{33}\omega_{11}\Big] \quad (4.38)$$

$$B = \frac{1}{D}\Bigg[\left(\frac{1}{\delta}+1\right)\omega_{123}U_{11} - 4U_{12}\left(\frac{1}{\delta}+1\right)\omega_{113} - 2U_{13}\left(-\frac{\omega_{14}}{\delta^2}\right) + (U_{22}+2U_{23})\left(\frac{n}{(\delta\varphi)^2} - \frac{1}{\delta^2}\delta_{12}\right) + U_{33}\left(-\frac{2}{\delta^3}\left(\frac{n}{\varphi} - \delta_{11}\right)\right)\Bigg] \quad (4.39)$$

$$P = \frac{1}{D}\Big[\frac{U_{11}\varphi}{\delta^2}\omega_{122} - \frac{2U_{12}\omega_{14}}{\delta^4} - \frac{4U_{13}\omega_{11}}{\delta^3} + \frac{4U_{23}}{\delta^3}\left(\frac{n}{\delta} - \delta_{11}\right) + U_{22}\left(\frac{n}{\delta^2\varphi^2} - \frac{1}{\delta^2}\delta_{12}\right) + U_{33}\left(-\frac{2n}{\delta^3} - \frac{6n\,log\varphi}{\delta^4} + \frac{6}{\delta^4}\delta_{10}\right)\Big] \quad (4.40)$$

Now

$$\hat{Y}_{APQB} = E(Y|\underline{x})$$

$$E(Y|\underline{x}) = u + (u_1a_1 + u_2a_2 + u_3a_3 + a_{4+}a_5) + \frac{1}{2}[A(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13}) + B(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23}) + P(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})] + 0\left(\frac{1}{n^2}\right) \quad (4.41)$$

$$E(Y|\underline{x}) = U + \psi_1 + \psi_2$$

where

$$\psi_1 = u_1a_1 + u_2a_2 + u_3a_3 + a_4 + a_5 \quad (4.42)$$

$$\psi_2 = \frac{1}{2}[(A\sigma_{11} + B\sigma_{21} + P\sigma_{31}).Y_1 + (A\sigma_{12} + B\sigma_{22} + P\sigma_{32}).Y_2 + (A\sigma_{13} + B\sigma_{23} + P\sigma_{33})Y_3] \quad (4.43)$$

evaluated at the MLE $\hat{Y} = (\hat{\theta}, \hat{\varphi}, \hat{\delta})$ where

$$a_1 = \sigma_{11} + \left(\frac{-\varepsilon}{\varphi} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon}\right)\frac{Y_{12}}{D} + \left(\frac{\varepsilon-1}{\delta} - \frac{1}{\varphi}\right)\frac{Y_{13}}{D} \quad (4.44)$$

$$a_2 = 0.\sigma_{21} + \left(\frac{-\xi}{\beta} + \frac{\gamma}{\beta} - \frac{1}{\delta}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\gamma} - \frac{1}{\beta}\right)\frac{Y_{23}}{D} \tag{4.45}$$

$$a_3 = \sigma_{31} + \left(\frac{-\xi}{\varphi} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon}\right)\frac{Y_{32}}{D} + \left(\frac{\varepsilon-1}{\delta} - \frac{1}{\varphi}\right)\frac{Y_{33}}{D} \tag{4.46}$$

$$a_4 = \frac{Y_{12}}{D}U_{12} + \frac{Y_{13}}{D}U_{13} + \frac{Y_{23}}{D}U_{23} \tag{4.47}$$

$$a_5 = \frac{1}{2D}(U_{11}Y_{11} + U_{22}Y_{22} + U_{33}Y_{33}) \tag{4.48}$$

Approximate Bayes Estimate Under Quadratic Loss Function

$$\hat{Y}_{APQB} = E(\vartheta) = \vartheta$$

where

$$E_Y(\vartheta|\underline{x}) = \frac{\int_\theta\int_\varphi\int_\delta \theta G^*(\theta,\psi,\delta)\partial\theta\partial\psi\partial\delta}{\int_\theta\int_\varphi\int_\delta G^*(\theta,\psi,\delta)\partial\theta\partial\varphi\partial\delta} \tag{4.49}$$

The above equation (4.49) is evaluated by method of Lindley approximation by replacing ϑ by $Y(\theta,\psi,\delta)$ in equation (4.49)

Special cases:–

$$Y(\theta,\varphi,\delta) = Y$$

1. Approximate Bayes Estimate of $\boldsymbol{\varphi}$

$$Y(\theta,\varphi,\delta) = Y$$

$$Y = \varphi$$

$$Y_1 = Y_{11} = Y_{12} = Y_{13} = 0$$

$$Y_3 = Y_{31} = Y_{32} = Y_{33} = 0$$

$$Y_2 = \frac{\partial Y}{\partial \varphi} = 1 \quad ; \qquad Y_{21} = Y_{22} = Y_{23} = 0$$

$$E_u(Y|\underline{x}) = \varphi + \phi_1 + \phi_2 \tag{4.50}$$

where

$$\phi_1 = \left(\frac{\xi}{\varphi} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon}\right)\frac{U_{22}}{D} + \left(\frac{\xi-1}{\delta} - \frac{1}{\varphi}\right)\frac{U_{23}}{D}$$

$$\phi_2 = \frac{1}{2}[(A\sigma_{12} + B\sigma_{22} + P\sigma_{32})]$$

$$E_u(Y|\underline{x}) = \varphi + \left(\frac{\xi}{\varphi} + \frac{\delta}{\varphi^2} - \frac{1}{\varepsilon}\right)\frac{Y_{22}}{D} + \left(\frac{\xi-1}{\delta} - \frac{1}{\varphi}\right)\frac{Y_{23}}{D} + \frac{1}{2D}(Y_{11}A + Y_{22}B + Y_{32}P)$$

$$\hat{\theta}_{APBQ} = [\varphi + \Delta_2], \ \ at\ (\hat{\theta}_{ML}, \hat{\varphi}_{ML,}\hat{\delta}_{ML}) \tag{4.51}$$

where;

$$\Delta_2 = \frac{U_{22}}{D}\left[D\left(\frac{-\xi}{\varphi}+\frac{\delta}{\varphi^2}-\frac{1}{\varepsilon}\right)+\left(\frac{1}{\theta}+1\right)\frac{\omega_{123}U_{11}}{2}-2U_{12}\left(\frac{1}{\theta}+1\right)\omega_{113}-U_{13}\frac{\omega_{14}}{\theta^2}\right.$$
$$\left.+\left(\frac{U_{22}2U_{23}}{2}\right)\left(\frac{n}{\delta^2\varphi^2}-\frac{1}{\delta^2}\varepsilon_{12}\right)+U_{33}\left(\frac{1}{\delta^3}\left(\frac{n}{\varphi}-\delta_{11}\right)\right)\right]$$
$$+\frac{U_{12}}{2D^2}\left[Y_{11}\left(\frac{2n}{\theta^3}\left(\frac{1}{\delta}+1\right)\omega_{133}+2U_{12}\left(\frac{1}{\delta}+1\right)\omega_{123}+2U_{13}\frac{\theta}{\delta^2}\omega_{122}\right.\right.$$
$$\left.\left.-2U_{23}\frac{\omega_{14}}{\delta^2}-2U_{22}\left(\frac{1}{\delta}+1\right)\omega_{113}-\frac{2}{\delta^2}U_{33}\omega_{11}\right)\right]$$
$$+\frac{U_{32}}{2D^2}\left[\frac{U_{11}}{\delta^2}\varphi\omega_{122}-2U_{12}\frac{\omega_{14}}{\delta^4}-4U_3\frac{\omega_{11}}{\delta^3}+\frac{4U_{23}}{\delta^3}\left(\frac{n}{\varphi}-\delta_{11}\right)\right.$$
$$\left.+U_{22}\left(\frac{n}{\delta^2\varphi^2}-\frac{\delta_{12}}{\delta^2}\right)+U_{33}\left(\frac{-2n}{\delta^3}-\frac{6n\,log\varphi}{\delta^4}+\frac{6}{\delta^4}\delta_{10}\right)\right]$$
$$+\frac{U_{23}}{D}\left(\frac{\xi-1}{\delta}-\frac{1}{\varphi}\right)$$

Simulations and Numerical Comparison

The simulations and numerical calculations are done by using R Language programming and results are presented in form of tables in table (1).

1. The Random variable of Generalized Compound Rayleigh Distribution is generated by R-Language programming by taking the values of the parameters $Y = \theta, \varphi, \delta$, taken as $\theta = 1$, $\varphi = 0.5$ and $\delta = 0.8$ in the equations[(4.2)-(4.4)] and equation(1.1).

2. Taking the different sizes of samples n=10(10)80 with complete sample, MLE's, the Approximate Bayes estimators, and their respective MSE's (in parenthesis) are obtained by repeating the steps 500 times, are presented in the tables from (1), and parameters of prior distribution a =2 and b =3.

3. Table (1) present the MLE of parameter of δ (for known $\theta\ and\ \varphi$) and Approximate Bayes estimators under QLF (for $\theta,\ \varphi\ and\ \delta$ unknown) and their respective MSE's. All the estimators have minimum MSE's for large sample sizes, as the sample sizes decrease, the MSE's increased.

Table (1)

Mean and MSE'S of φ

($\theta = 1$, $\varphi = 0.5$ and $\delta = 0.8$)

n	20	40	60	80	100	120	140
$\hat{\varphi}_{ML}$	0.790045	0.792261	0.878125	0.898749	0.899658	0.999001	0.998884
MSE	**[0.02441]**	**[0.02587]**	**[0.09852]**	**[0.00235]**	**[0.00457]**	**[0.00466]**	**[0.00112]**
$\hat{\varphi}_{APBQ}$	0.799878	0.798452	0.874589	0.880181	0.859235	0.979235	0.980739
MSE	**[0.00451]**	**[0.00441]**	**[0.00854]**	**[0.01571]**	**[0.01543]**	**[0.01543]**	**[0.01037]**

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