Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 6 (2015), pp. 4063-4074

http://www.ripublication.com

Free Convection Effect on the Flow of A Dusty Viscous Fluid Past A Hot Vertical Porous Plate with Periodic Temperature

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Abstract:

An analysis of the effect of free convection on the flow of a dusty viscous fluid is presented under the assumption that the suction velocity is constant and normal to the wall and the wall temperature is spanwise cosinusoidal. The governing equations are the Navier Stoke's equation for the momentum, Energy Equations and Continuity equations for both fluid phase and particle phase. These equations are converted to coupled ordinary differential equations by multi parameter perturbation techniques. On solving these equations the velocity and temperature for both fluid phase and particle phase are obtained. They are discussed and shown through graphs. The skin friction coefficient and the rate of heat transfer are presented numerically.

Key Words Free Convection, Dusty Fluid, Porous Plate.

Nomenclature

(u^*, v^*, w^*)–	Velocity components at a point
v_0 -	Suction velocity
l -	wavelength
υ -	Kinematic coefficient of viscosity
g-	Acceleration due to gravity
ρ-	density
β-	coefficient of volume expansion
μ-	coefficient of viscosity
κ-	Thermal conductivity
C_p-	Specific heat at constant pressure

U_∞ -	Free stream velocity
θ_0 -	constant temperature
θ_w^* -	wall temperature
θ^* -	Temperature at any point
Nm =	ρ_p-density of the particle
C_s-	Specific heat of the fluid

τ_T-$\frac{3}{2}\Pr\frac{C_s}{C_p}\tau$ τ-$\frac{2}{9}\frac{\rho_p a^2}{\mu}$ K=6$\pi\mu$a

1. Introduction

The concept of the flow and heat transfer of dusty fluids has important applications in the fields of fluidization, combustion, use of dust in gas cooling system, centrifugal separation of particles from fluid, petroleum industry, purification of crude oil, electrostatic precipitation and polymer technology. Also, heat transfer in porous medium in a two-phase fluid occurs in a number of technological applications such as thermal energy storage, geothermal systems, porous medium heat pipes, food drying, porous insulation moisture transport and post-accident analysis of liquid-cooled nuclear reactors.

There has been numerous work in the literature on the free convection effects on the dusty fluid. Ramamurthy [7], studied the free convection effects on the Stoke's problem for an infinite plate in dusty fluid, taking into consideration the cases of (i) the plate being started impulsively from rest and (ii) the plate being uniformly accelerated using laplace transform techniques. Venkatraman and Kannan[5], investigated the flow past an infinite vertical isothermal plate taking into account the viscous dissipative heat. Helmy[6] obtained an analytical solution for the free convection of a dusty conducting fluid. T. S. Zhao et. al [13] got numerical solution of a buoyancy induced flow and phase change heat transfer in a vertical porous channel heated symmetrically along its vertical walls.

Most of these studies are based on constant properties like constant temperature. In the present work, an analysis has been carried out to study the free convection effects on a dusty viscous fluid past a vertical porous plate assuming that the temperature is cosinusoidal

Mathematical Formulation

Let the wall be the z^*x^*-plane and the y^*-axis be normal to it and the positive direction of x^*-axis be vertically upwards. Let the wall be uniformly porous and the suction velocity normal to it be v_0. Let $\left(u^*, v^*, w^*\right)$ be the components of velocity of the liquid at any point $\left(x^*, y^*, z^*\right)$. Since $v^* = v_0$ throughout, w^* is independent of z^*

and we assume that $w^* = 0$ throughout. Let the fluid velocity parallel to the x^*-axis at infinity be U_∞ and the spanwise cosinusoidal wall temperature θ_w^* be,

$$\theta_w^* = \theta_0\left(1+\varepsilon\cos\frac{\pi z}{l}\right) \tag{1}$$

where ε is a small positive number. Applying Boussinesq approximation, the momentum, Energy Equations and continuity equations for the fluid in non dimensional form, are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2}$$

$$\frac{\partial u}{\partial y}=-\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-Gr\,\mathrm{Re}\,\theta-\alpha\left(u_p-u\right) \tag{3}$$

$$\frac{\partial \theta}{\partial y}=-\frac{1}{\mathrm{Pr}\,\mathrm{Re}}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)-\frac{Ec}{\mathrm{Re}}\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right)-\beta\left(\theta_p-\theta\right) \tag{4}$$

$$\frac{\partial u_p}{\partial x}+\frac{\partial v_p}{\partial y}+\frac{\partial w_p}{\partial z}=0 \tag{5}$$

$$\frac{\partial u_p}{\partial y}=-\gamma\left(u_p-u\right) \tag{6}$$

$$\frac{\partial \theta_p}{\partial y}=-\delta\left(\theta_p-\theta\right) \tag{7}$$

Boundary Conditions

$$y=0: \quad u=0, \quad u=u_{p0} \quad \theta=\theta_0\left(1+\varepsilon\cos\pi z\right) \tag{8}$$

$$y\to\infty: \quad u=u_p=1, \quad \theta=\theta_p=0$$

Multi Parameter Perturbation method

Very often, a mathematical problem cannot be solved exactly, If the exact solution is available, it exhibits such an intricate dependency in the parameters that it is hard to use as such. It may be the case, however, that a parameter can be identified, say 'ε' such that the solution is available and reasonably simple for 'ε' = 0. By perturbation method, a reasonably accurate solution can be applied even for positive small values of ε. Here perturbation has been done with two parameters ε and the Eckert number Ec.

$$f = f_0+\varepsilon f_1+O\left(\varepsilon^2\right) \tag{9}$$

where f can be any one of $\mathrm{u}, \mathrm{u}_\mathrm{p}, \phi$ and ϕ_p and u_0 and ϕ_0 are functions of y only.

Assuming $\begin{matrix} u_1 = v_0(y)\cos\pi z & u_{p1} = v_{p0}(y)\cos\pi z \\ \phi_1 = \psi_0(y)\cos\pi z & \phi_{p1} = \psi_{p0}(y)\cos\pi z \end{matrix}$ (10)

Assume that $F_0 = F_{00} + EcF_{01} + O(Ec^2)$ (11)

where F can be any one of $u, u_p, \phi, \phi_p, v, v_p, \psi \; and \; \psi_p$

Boundary conditions

$y = 0:$	$u_{00} = 0$	$\phi_{00} = 1$	$u_{01} = 0$	$\phi_{01} = 0$
$y \to \infty:$	$u_{00} = 1$	$u_{p00} = 1$	$\phi_{00} = 0$	$u_{01} = 0$
	$\phi_{01} = 0$			

Substituting equation (9) in (7) and comparing the constant term, we get,

$$\phi_{00}''' + (\Pr\mathrm{Re} + \delta)\phi_{00}'' - \Pr\mathrm{Re}(\beta - \delta)\phi_{00}' = 0$$

$$\phi_{p00} = \phi_{00} - \frac{1}{\beta}\left(\phi_{00}' + \frac{1}{\Pr\mathrm{Re}}\phi_{00}''\right)$$

$$u_{00}''' + (\mathrm{Re} + \gamma)u_{00}'' - \mathrm{Re}(\alpha - \gamma)u_{00}' = -Gr\,\mathrm{Re}^2(\phi_{00}' + \gamma\phi_{00})$$

$$u_{p00} = u_{00} - \frac{1}{\alpha}\left(u_{00}' + \frac{1}{\mathrm{Re}}u_{00}'' + Gr\,\mathrm{Re}\,\phi_{00}\right)$$

On solving the above differential equations the following solutions are obtained.

$$\phi_{00} = e^{-m_1 y}$$

$$\phi_{p00} = A_1 e^{-m_1 y}$$

$$u_{00} = 1 + A_3 e^{-m_2 y} + A_2 e^{-m_1 y}$$

$$u_{p00} = 1 + A_4 e^{-m_2 y} + A_5 e^{-m_1 y}$$

Comparing the coefficient of Ec, we get,

$$\phi_{01}''' + (P\,\mathrm{Re} + \delta)\phi_{01}'' - P\,\mathrm{Re}(\beta - \delta)\phi_{01}' = -2\Pr u_{00}' u_{00}'' - \delta\Pr u_{00}'^2$$

$$\phi_{p01} = \phi_{01} - \frac{1}{\beta}\left(\phi_{01}' + \frac{1}{\Pr\mathrm{Re}}\phi_{01}'' + \frac{u_{00}'^2}{\mathrm{Re}}\right)$$

$$u_{01}''' + (\mathrm{Re} + \gamma)u_{01}'' - \mathrm{Re}(\alpha - \gamma)u_{01}' = -Gr\,\mathrm{Re}^2(\phi_{01}' + \gamma\phi_{01})$$

$$u_{p01} = u_{01} - \frac{1}{\alpha}\left(u_{01}' + \frac{1}{\mathrm{Re}}u_{01}'' + Gr\,\mathrm{Re}\,\phi_{01}\right)$$

Solving the above differential equations, the following solutions are obtained.

$$\phi_{01} = A_6 e^{-m_1 y} + A_7 e^{-2m_2 y} + A_8 e^{-2m_1 y} + A_9 e^{-(m_1+m_2)y}$$
$$\phi_{p01} = A_{10} e^{-m_1 y} + A_{11} e^{-2m_2 y} + A_{12} e^{-2m_1 y} + A_{13} e^{-(m_1+m_2)y}$$
$$u_{01} = A_{14} e^{-m_2 y} + A_{15} e^{-m_1 y} + A_{16} e^{-2m_2 y} + A_{17} e^{-2m_1 y} + A_{18} e^{-(m_1+m_2)y}$$
$$u_{p01} = A_{19} e^{-m_2 y} + A_{20} e^{-m_1 y} + A_{21} e^{-2m_2 y} + A_{22} e^{-2m_1 y} + A_{23} e^{-(m_1+m_2)y}$$

Comparing the coefficient of ε, we get,
$$\psi'''_{00} + (P\,\mathrm{Re} + \delta)\psi''_{00} - \left(P\,\mathrm{Re}(\beta-\delta) + \pi^2\right)\psi'_{00} - \delta\pi^2\psi_{00} = 0$$
$$\psi_{p00} = \psi_{00} - \frac{1}{\beta}\left(\psi'_{00} + \frac{1}{P\,\mathrm{Re}}\psi''_{00} - \frac{\pi^2}{\Pr\mathrm{Re}}\psi_{00}\right)$$
$$v'''_{00} + (\mathrm{Re} + \gamma)v''_{00} - \left(\mathrm{Re}(\alpha-\gamma) - \pi^2\right)v'_{00} - \pi^2\gamma v_{00} = -Gr\,\mathrm{Re}^2(\psi'_{00} + \gamma\psi_{00})$$
$$v_{p00} = v_{00} - \frac{1}{\alpha}\left(v'_{00} + \frac{1}{\mathrm{Re}}v''_{00} - \frac{\pi^2}{\mathrm{Re}}v_{00} + Gr\,\mathrm{Re}\,\psi_{00}\right)$$

Solving the above differential equations, the following solutions are obtained.
$$\psi_{00} = e^{-m_3 y}$$
$$\psi_{p00} = A_{24} e^{-m_1 y}$$
$$v_{00} = A_{25} e^{-m_4 y} + A_{26} e^{-m_3 y}$$
$$v_{p00} = A_{27} e^{-m_4 y} + A_{28} e^{-m_3 y}$$

Comparing the coefficient of ε Ec, we get,
$$\psi'''_{01} + (P\,\mathrm{Re} + \delta)\psi''_{01} - \left(P\,\mathrm{Re}(\beta-\delta) + \pi^2\right)\psi'_{01} - \delta\pi^2\psi_{01}$$
$$= -2\Pr(u''_{00}v'_{00} + u'_{00}v''_{00}) - 2\Pr\delta u'_{00}v'_{00}$$
$$\psi_{p01} = \psi_{01} - \frac{1}{\beta}\left(\psi'_{01} + \frac{1}{P\,\mathrm{Re}}\psi''_{01} - \frac{\pi^2}{\Pr\mathrm{Re}}\psi_{01} + \frac{2}{\mathrm{Re}}u'_{00}v'_{00}\right)$$
$$v'''_{01} + (\mathrm{Re} + \gamma)v''_{01} - \left(\mathrm{Re}(\alpha-\gamma) - \pi^2\right)v'_{01} - \pi^2\gamma v_{01} = -Gr\,\mathrm{Re}^2(\psi'_{01} + \gamma\psi_{01})$$
$$v_{p01} = v_{01} - \frac{1}{\alpha}\left(v'_{01} + \frac{1}{\mathrm{Re}}v''_{01} - \frac{\pi^2}{\mathrm{Re}}v_{01} + Gr\,\mathrm{Re}\,\psi_{01}\right)$$

Solving the above differential equations, the following solutions are obtained.
$$\psi_{01} = A_{29} e^{-m_3 y} + A_{30} e^{-(m_2+m_4)y} + A_{31} e^{-(m_2+m_3)y} + A_{32} e^{-(m_1+m_4)y} + A_{33} e^{-(m_1+m_3)y}$$
$$\psi_{p01} = A_{34} e^{-m_3 y} + A_{35} e^{-(m_2+m_4)y} + A_{36} e^{-(m_2+m_3)y} + A_{37} e^{-(m_1+m_4)y} + A_{38} e^{-(m_1+m_3)y}$$
$$v_{01} = A_{39} e^{-m_4 y} + A_{40} e^{-m_3 y} + A_{41} e^{-(m_2+m_4)y} + A_{42} e^{-(m_2+m_3)y} + A_{43} e^{-(m_1+m_4)y} + A_{44} e^{-(m_1+m_3)y}$$
$$v_{p01} = A_{45} e^{-m_4 y} + A_{46} e^{-m_3 y} + A_{47} e^{-(m_2+m_4)y} + A_{48} e^{-(m_2+m_3)y} + A_{49} e^{-(m_1+m_4)y} + A_{50} e^{-(m_1+m_3)y}$$

FIG-1 depicts the corrective factor of the temperature of the fluid. When the Reynolds number and the Prandtl number increase, the temperature decreases. But at the same time it increases with increase in the Grashoff number. It is obvious that dust parameters has no effect on the corrective factor of the temperature.. It is obvious from FIG-2 that the corrective factor of the temperature of the dust particles behaves in the same pattern as that of the fluid. The temperature of the dust particle decreases with increase in the Reynolds Number, Prandtl number and temperature relaxation parameter.

It is obvious from FIG-3 that the temperature of the fluid decreases with increase in the Reynolds Number, Prandtl number and temperature relaxation parameter. The temperature is 1 near the plate, but decreases to zero as it moves away from the plate.

It is clear from FIG-4 that the temperature of the dust particle is finite near the plate, but decreases to zero as y increases. It increases with increase in Reynolds Number, Grashoff number and particle mass parameter. The temperature of the dust particles decreases within increase in Prandtl number, particle concentration arameter and temperature relaxation parameter.

It is noted from FIG-5 that, as the particle concentration parameter increases the corrective factor of the velocity of the fluid decreases. But when the Prandtl number increases, the velocity increases upto certain value of y, then crossover occurs and decreases thereafter. It increases with increase in Reynolds Number, Grashoff number, particle concentration parameter, particle mass parameter and temperature relaxation parameter. For all parameters, the velocity increases upto certain value but decreases steadily after that to zero as y increases.

It is noted from FIG-6 that the corrective factor of the velocity of the dust particle is finite near the plate, but decreases to zero as y increases. As the Prandtl number increases, the velocity of the dust particle steadily decreases to zero. For the increase in other parameters, there is increase and then steady decrease.

It is seen from FIG-7 that the velocity of the fluid increases when there is increase in Reynolds number, but decreases as particle concentration parameter and time relaxation parameter increase. It starts from zero near the plate and decreases to 1 and become steady there after as y increases.

It is observed from FIG-8 that, increase in Prandtl number decreases the velocity of the fluid, whereas increase in Grashoff number and particle mass parameter increases the velocity of the fluid.

It is noted from FIG-9 that the velocity of the dust particle is finite near the plate, but decreases to 1 and become steady there after as y increases.

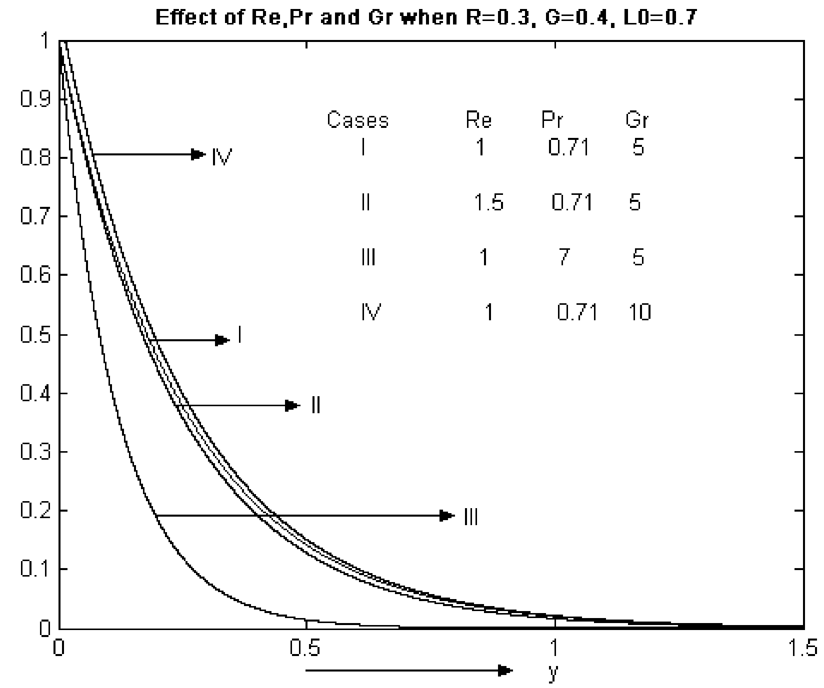

FIG-1 $\psi(y)$ *versus* y

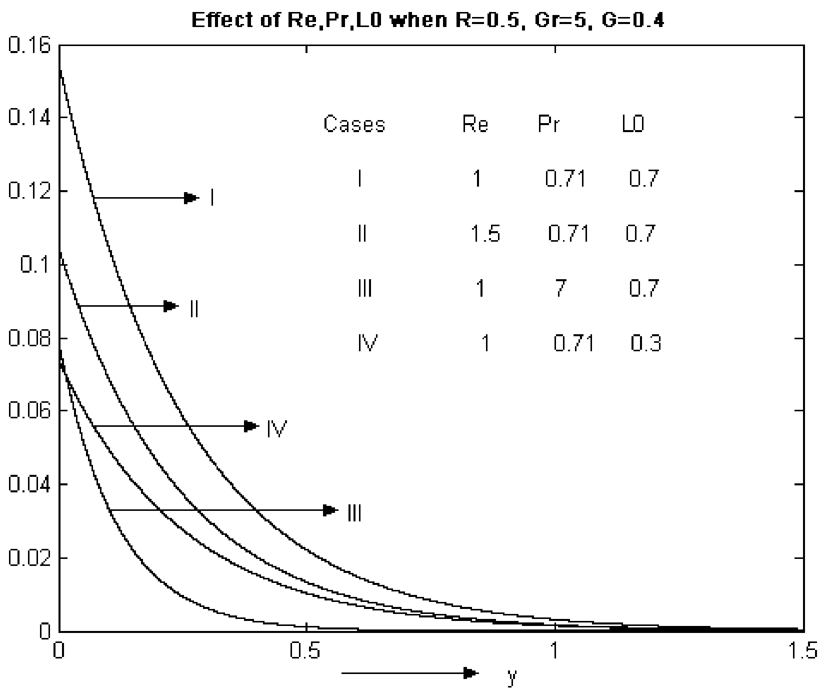

FIG-2 $\psi_p(y)$ *versus* y

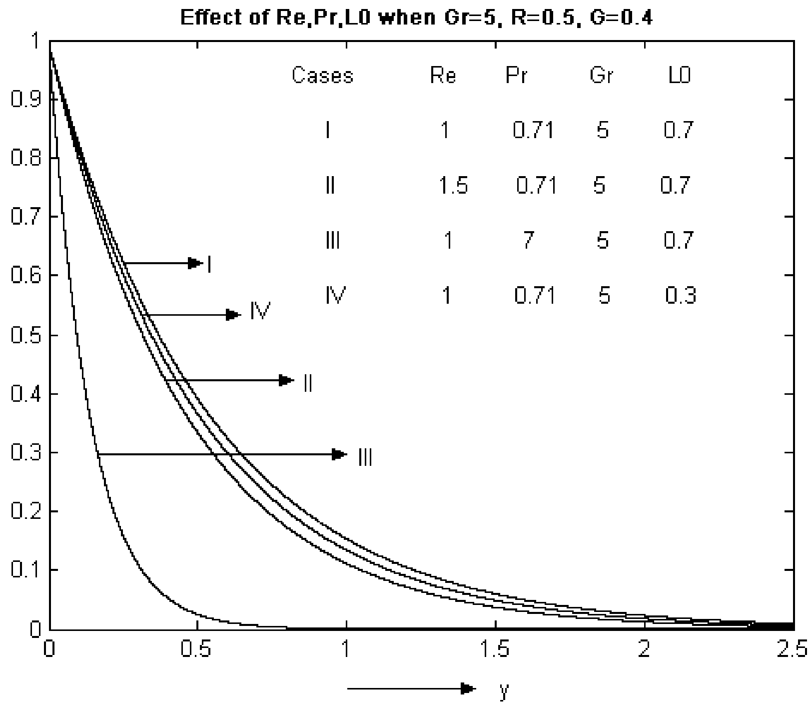

FIG-3 $\theta(y)$ *versus* y

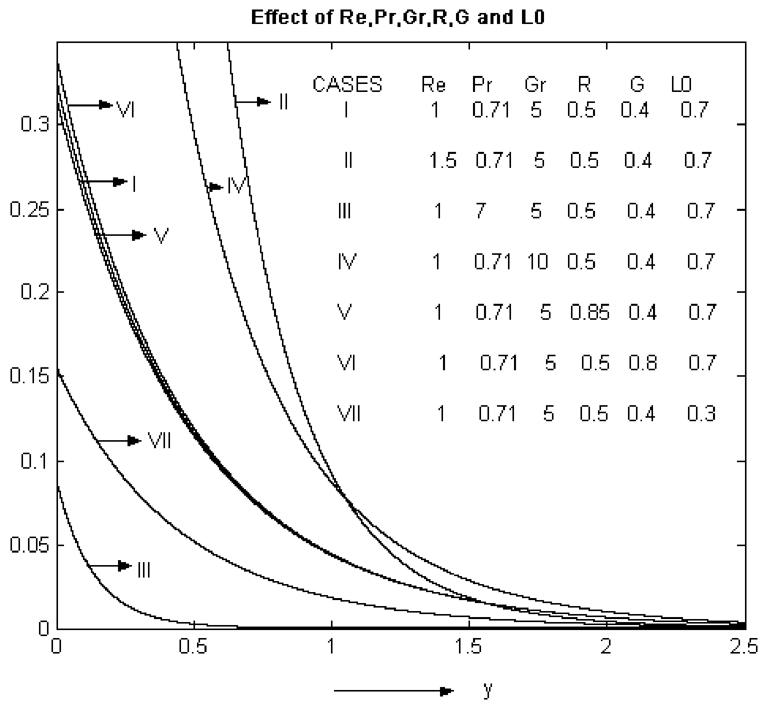

FIG-4 $\theta_p(y)$ *versus* y

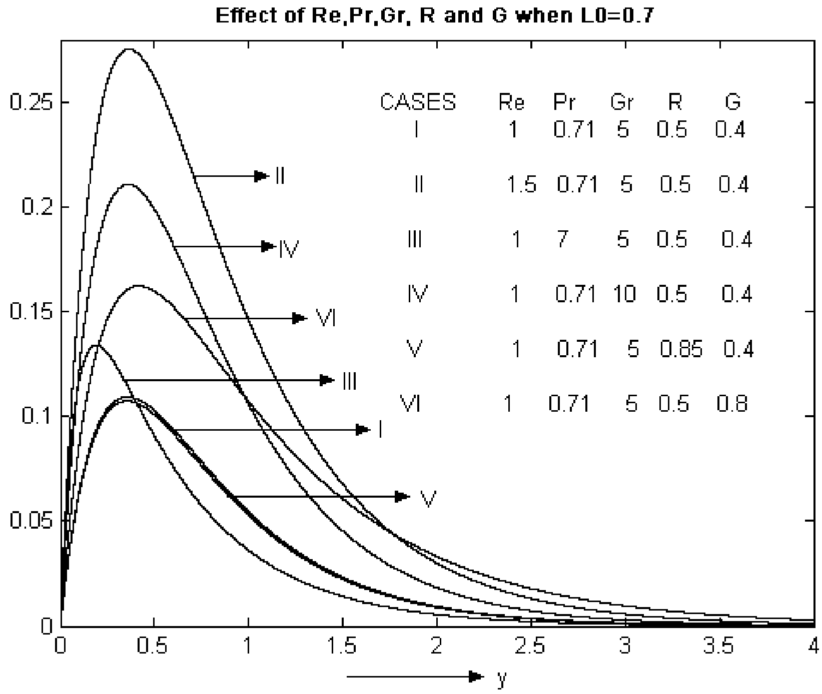

FIG-5 $v_0(y)$ *versus* y

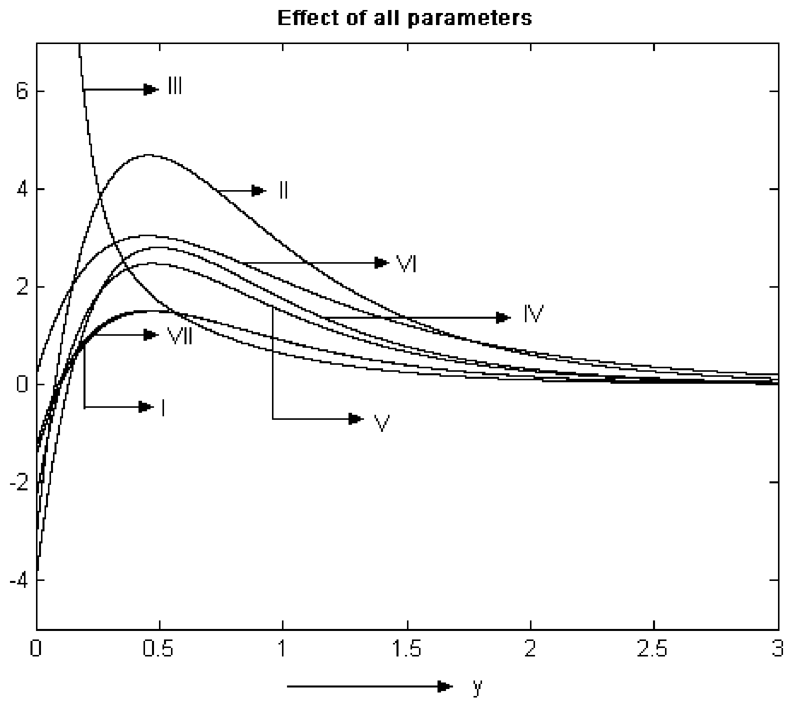

FIG-6 $v_{p0}(y)$ *versus* y

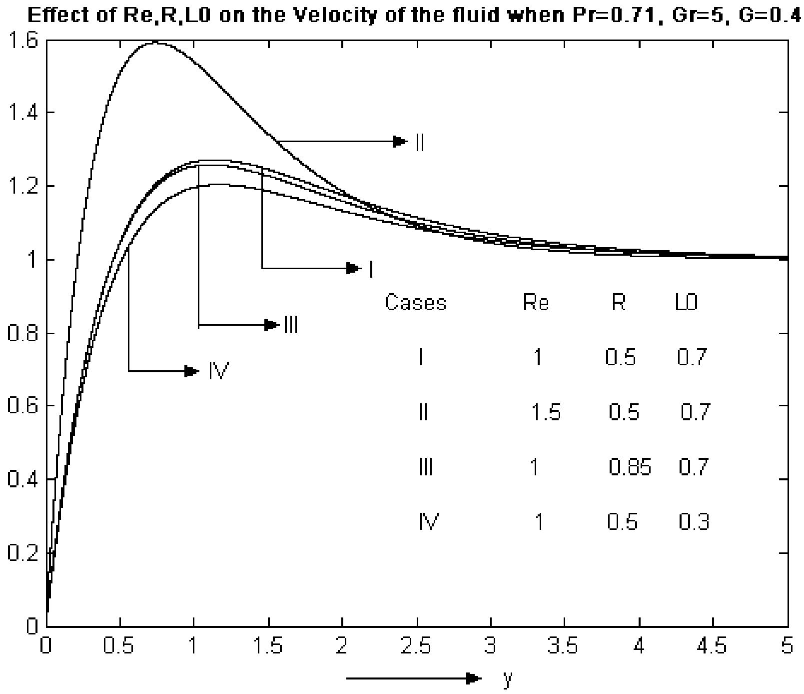

FIG-7 $u(y)$ *versus* y

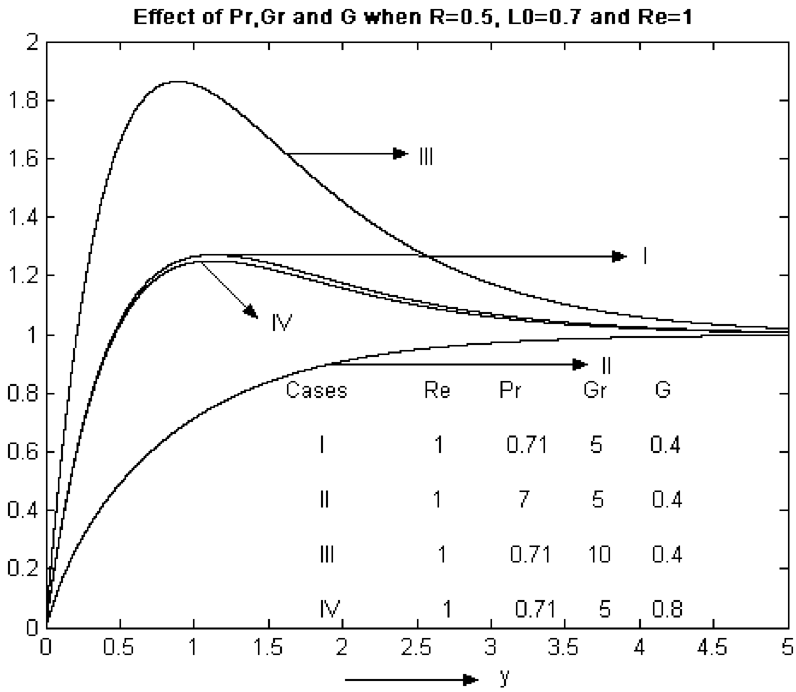

FIG-8 $u(y)$ *versus* y

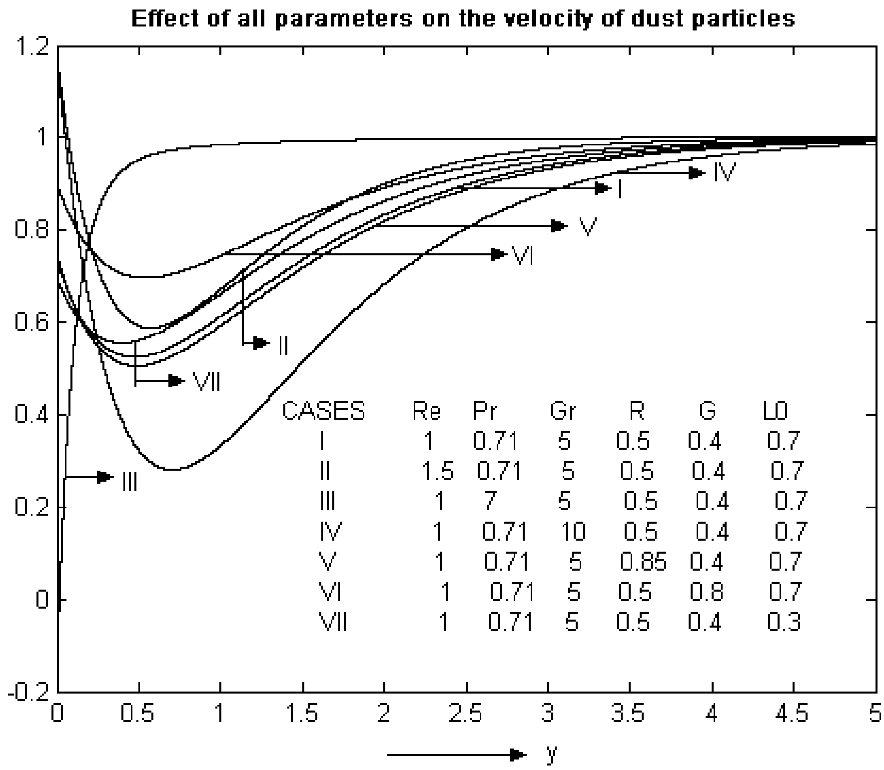

FIG-9 $u_p(y)$ *versus* y

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