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# Interval Valued Anti Fuzzy Subnearrings Of A Nearring Under Homomorphism

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## ABSTRACT

In this paper, we study some of the properties of interval valued anti fuzzy subnearring of a nearring under homomorphism and anti-homomorphism and prove some results on these.

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**KEY WORDS:** Interval valued fuzzy subset, interval valued fuzzy subnearring, interval valued anti fuzzy subnearring, pseudo interval valued fuzzy coset, interval valued fuzzy coset.

## INTRODUCTION:

Interval valued fuzzy sets were introduced independently by Zadeh [12], Grattan-Guiness [5], Jahn [7], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval valued membership function. Jun.Y.B and Kin.K.H [8] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju.A and Nagarajan.R[10] defined the charactarization of interval valued Anti fuzzy Left h-ideals over Hemirings. Azriel Rosenfeld [3] defined a fuzzy group. Asok Kumer Ray [2] defined a product of fuzzy subgroups.We introduce the concept of interval valued anti fuzzy subnearring of a nearring and established some results.

## 1.PRELIMINARIES:

**1.1 Definition:** Let X be any nonempty set. A mapping [M]: X $\rightarrow$ D[0, 1] is called an interval valued fuzzy subset (briefly, IVFS) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where $M^-$ and $M^+$ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X. Thus $M^-(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset. Note that [0] = [0,0] and [1] = [1, 1].

**1.2 Remark:** Let $D^X$ be the set of all interval valued fuzzy subset of X, where D means D[0, 1].

**1.3 Definition:** Let $[M] = \{\langle x, [M^-(x), M^+(x)] \rangle / x \in X \}$, $[N] = \{\langle x, [N^-(x), N^+(x)] \rangle / x \in X\}$ be any two interval valued fuzzy subsets of X. We define the following relations and operations:

(i) $[M] \subseteq [N]$ if and only if $M^-(x) \leq N^-(x)$ and $M^+(x) \leq N^+(x)$, for all x in X.

(ii) $[M] = [N]$ if and only if $M^-(x) = N^-(x)$ and $M^+(x) = N^+(x)$, for all x in X.

(iii) $[M] \cap [N] = \{ \langle x, [\min \{ M^-(x), N^-(x)\}, \min \{ M^+(x), N^+(x)\} ] \rangle / x \in X \}$.

(iv) $[M] \cup [N] = \{ \langle x, [\max \{ M^-(x), N^-(x)\}, \max \{ M^+(x), N^+(x)\} ] \rangle / x \in X \}$.

(v) $[M]^C = [1] - [M] = \{ \langle x, [1 - M^+(x), 1 - M^-(x)] \rangle / x \in X \}$.

**1.4 Definition:** Let $(R, +, \cdot)$ be a nearring. An interval valued fuzzy subset [M] of R is said to be an **interval valued fuzzy subnearring** of R if the following conditions are satisfied:

(i) $[M](x+y) \geq \text{rmin} \{ [M](x), [M](y) \}$,

(ii) $[M](-x) \geq [M](x)$,

(iii) $[M](xy) \geq \text{rmin} \{ [M](x), [M](y) \}$, for all x and y in R.

**1.5 Definition:** Let $(R, +, \cdot)$ be a nearring. An interval valued fuzzy subset [M] of R is said to be an **interval valued anti fuzzy subnearring** of R if the following conditions are satisfied:

(i) $[M](x+y) \leq \text{rmax} \{ [M](x), [M](y) \}$,

(ii) $[M](-x) \leq [M](x)$,

(iii) $[M](xy) \leq \text{rmax} \{ [M](x), [M](y) \}$, for all x and y in R.

**1.6 Definition:** Let $(R, +, \cdot)$ and $(R^l, +, \cdot)$ be any two nearrings. Let $f: R \to R^l$ be any function and [M] be an interval valued anti fuzzy subnearring in R, [V] be an interval valued anti fuzzy subnearring in $f(R) = R^l$, defined by $[V](y) = \inf_{x \in f^{-1}(y)} [M](x)$, for all x in R and y in $R^l$. Then [M] is called a pre-image of [V] under f and is denoted by $f^{-1}([V])$.

**1.7 Definition:** Let [M] be an interval valued anti fuzzy subnearring of a nearring R. Then $[M]^0$ is defined as $[M]^0(x) = [M](x)M^+(0)$, for all x in R, where 0 is the identity element of R.

**1.8 Definition:** Let [M] be an interval valued anti fuzzy subnearring of a nearring (R, +, ·) and a in R. Then the **pseudo interval valued fuzzy coset** $(a[M])^p$ is defined by $((a[M])^p)(x) = p(a)[M](x)$, for every x in R and for some p in P.

**1.9 Definition:** Let [M] be an interval valued anti fuzzy subring of a ring (R, +, ·). For any a in R, a+[M] defined by (a+[M])(x) = [M](x−a) for every x in R, is called an **interval valued fuzzy coset** of R.

## 2. SOME PROPERTIES:

**2.1 Theorem:** Let (R, +, ·) and ($R^l$, +, ·) be any two nearrings. The homomorphic image of an interval valued anti fuzzy subnearring of R is an interval valued anti fuzzy subnearring of $R^l$.

**Proof:** Let (R, +, ·) and ($R^l$, +, ·) be any two nearrings and f: $R \rightarrow R^l$ be a homomorphism. Let [M] be an interval valued anti fuzzy subnearring of R. Let [V] be the homomorphic image of [M] under f. We have to prove that [V] is an interval valued anti fuzzy subnearring of f(R) = $R^l$. Now for f(x) and f(y) in $R^l$, we have [V](f(x) −f(y)) = [V](f(x−y)) ≤ [M](x−y) ≤ rmax { [M](x), [M](y) } which implies that [V](f(x)−f(y)) ≤ rmax {[V](f(x)), [V](f(y))}. And [V](f(x)f(y)) = [V](f(xy)) ≤ [M](xy) ≤ rmax { [M](x), [M](y) } which implies that [V](f(x)f(y)) ≤ rmax {[V](f(x)), [V](f(y)) }. Hence [V] is an interval valued anti fuzzy subnearring of a nearring $R^l$.

**2.2 Theorem:** Let (R, +, ·) and ($R^l$, +, ·) be any two nearrings. The homomorphic pre-image of an interval valued anti fuzzy subnearring of $R^l$ is an interval valued anti fuzzy subnearring of R.

**Proof:** Let (R, +, ·) and ($R^l$, +, ·) be any two nearrings and f: $R \rightarrow R^l$ be a homomorphism. Let [V] be an interval valued anti fuzzy subnearring of f(R) = $R^l$. Let [M] be the pre-image of [V] under f. We have to prove that [M] is an interval valued anti fuzzy subnearring of R. Let x and y in R. Then [M](x−y) = [V](f(x−y)) = [V](f(x) −f(y)) ≤ rmax { [V](f(x)), [V](f(y)) } = rmax {[M](x), [M](y) } which implies that [M](x−y) ≤ rmax { [M](x), [M](y) } for x and y in R. And [M](xy) = [V](f(xy)) = [V](f(x)f(y)) ≤ rmax { [V](f(x)), [V](f(y)) } = rmax {[M](x), [M](y) } which implies that [M](xy) ≤ rmax { [M](x), [M](y) } for x and y in R. Hence [M] is an interval valued anti fuzzy subnearring of the nearring R.

**2.3 Theorem:** Let (R, +, ·) and ($R^l$, +, ·) be any two nearrings. The anti-homomorphic image of an interval valued anti fuzzy subnearring of R is an interval valued anti fuzzy subnearring of $R^l$.

**Proof:** Let (R, +, ·) and ($R^l$, +, ·) be any two nearrings and f: $R \rightarrow R^l$ be an anti-homomorphism. Let [M] be an interval valued anti fuzzy subnearring of R. Let [V] be the homomorphic image of [M] under f. We have to prove that [V] is an interval valued anti fuzzy subnearring of f(R) = $R^l$. Now let f(x) and f(y) in $R^l$, we have

[V](f(x)–f(y)) = [V](f(y–x)) ≤ [M](y–x) ≤ rmax {[M](x), [M](y)} which implies that [V](f(x)–f(y)) ≤ rmax { [V](f(x)), [V](f(y)) }. And [V](f(x)f(y)) = V(f(yx)) ≤ [M](yx) ≤ rmax { [M](x), [M](y) } which implies that [V](f(x)f(y)) ≤ rmax {[V](f(x)), [V](f(y)) }. Hence [V] is an interval valued anti fuzzy subnearring of $R^l$.

**2.4 Theorem:** Let (R, +, ·) and ($R^l$, +, ·) be any two nearrings. The anti-homomorphic pre-image of an interval valued anti fuzzy subnearring of $R^l$ is an interval valued anti fuzzy subnearring of R.

**Proof:** Let (R, +, ·) and ($R^l$, +, ·) be any two nearrings and f: R → $R^l$ be an anti-homomorphism. Let [V] be an interval valued anti fuzzy subnearring of f(R) = $R^l$. Let [M] be the pre-image of [V] under f. We have to prove that [M] is an interval valued anti fuzzy subnearring of R. Let x and y in R. Now [M](x–y) = [V](f(x–y)) = [V](f(y)–f(x)) ≤ rmax { [V](f(x)), [V](f(y)) } = rmax { [M](x), [M](y) } which implies that [M](x–y) ≤ rmax { [M](x), [M](y) } for all x and y in R. Now [M](xy) = [V](f(xy)) = [V](f(y)f(x)) ≤ rmax { [V](f(x)), [V](f(y)) } = rmax {[M](x), [M](y)} which implies that [M](xy) ≤ rmax {[M](x), [M](y) } for all x and y in R. Hence [M] is an interval valued anti fuzzy subnearring of the nearring R.

**In the following Theorem ◦ is the composition operation of functions:**

**2.5 Theorem:** Let [M] be an interval valued anti fuzzy subnearring of a nearring H and f is an isomorphism from a nearring R onto H. Then [M]◦f is an interval valued anti fuzzy subnearring of R.

**Proof:** Let x and y in R and [M] be an interval valued anti fuzzy subnearring of the nearring H. Then we have ([M]◦f)(x–y) = [M](f(x–y)) = [M](f(x) – f(y)) ≤ rmax {[M](f(x)), [M](f(y))} ≤ rmax { ([M]◦f)(x), ([M]◦f)(y) } which implies that ([M]◦f)(x–y) ≤ rmax {([M]◦f)(x), ([M]◦f)(y)}. Then we have ([M]◦f)(xy) = [M](f(xy)) = [M](f(x)f(y)) ≤ rmax { [M](f(x)), [M](f(y)) } ≤ rmax {([M]◦f)(x), ([M]◦f)(y)} which implies that ([M]◦f)(xy) ≤ rmax {([M]◦f)(x), ([M]◦f)(y) }. Therefore ([M]◦f) is an interval valued anti fuzzy subnearring of the nearring R.

**2.6 Theorem:** Let [M] be an interval valued anti fuzzy subnearring of a nearring H and f is an anti-isomorphism from a nearring R onto H. Then [M]◦f is an interval valued anti fuzzy subnearring of R.

**Proof:** Let x and y in R and [M] be an interval valued anti fuzzy subnearring of the nearring H. Then we have ([M]◦f)(x–y) = [M](f(x–y)) = [M](f(y)– f(x)) ≤ rmax {[M](f(x)), [M](f(y)) }≤ rmax { ([M]◦f)(x), ([M]◦f)(y) } which implies that ([M]◦f)(x–y) ≤ rmax {([M]◦f)(x), ([M]◦f)(y)}. Then we have ([M]◦f)(xy) = [M](f(xy)) = [M](f(y)f(x)) ≤ rmax { [M](f(x)), [M](f(y)) } ≤ rmax {([M]◦f)(x), ([M]◦f)(y)} which implies that ([M]◦f)(xy) ≤ rmax { ([M]◦f)(x), ([M]◦f)(y) }. Therefore ([M]◦f) is an interval valued anti fuzzy subnearring of R.

**2.7 Theorem:** Let [M] be an interval valued anti fuzzy subnearring of a nearring R, $[M]^+$ be an interval valued fuzzy set in R defined by $[M]^+(x) = [M](x) +[1]-[M](0)$, for all x in R, where 0 is the identity element. Then $[M]^+$ is an interval valued anti fuzzy subnearring of the nearring R.

**Proof:** Let x and y in R. We have $[M]^+(x-y) = [M](x-y)+[1]- [M](0) \leq$ rmax { [M](x), [M](y) } + [1]– [M](0) = rmax { { [M](x) + [1] –[M](0) }, { [M](y) + [1] – [M](0) } } = rmax { $[M]^+(x)$, $[M]^+(y)$ }. Therefore $[M]^+(x-y) \leq$ rmax { $[M]^+(x)$, $[M]^+(y)$ } for all x and y in R. Similarly $[M]^+(xy) = [M](xy) + [1] - [M](0) \leq$ rmax { [M](x), [M](y) } + [1] – [M](0) = rmax { { [M](x) + [1] – [M](0) }, { [M](y) + [1] – [M](0) } }= rmax { $[M]^+(x)$, $[M]^+(y)$ }. Therefore $[M]^+(xy) \leq$ rmax { $[M]^+(x)$, $[M]^+(y)$ } for all x and y in R. Hence $[M]^+$ is an interval valued anti fuzzy subnearring of the nearring R.

**2.8 Theorem:** Let [M] be an interval valued anti fuzzy subnearring of a nearring R, $[M]^+$ be an interval valued fuzzy set in R defined by $[M]^+(x) = [M](x) + [1]-[M](0)$ for all x in R, where 0 is the identity element. Then there exists 0 in R such that [M](0) = [1] if and only if $[M]^+(x) = [M](x)$.

**Proof**: It is trivial.

**2.9 Theorem:** Let [M] be an interval valued anti fuzzy subnearring of a nearring R, $[M]^+$ be an interval valued fuzzy set in R defined by $[M]^+(x) = [M](x) + [1]-[M](0)$, for all x in R, where 0 is the identity element. Then there exists x in R such that $[M]^+(x) = [1]$ if and only if x = 0.

**Proof**: It is trivial.

**2.10 Theorem:** Let [M] be an interval valued anti fuzzy subnearring of a nearring R, $[M]^+$ be an interval valued fuzzy set in R defined by $[M]^+(x) = [M](x) + [1]-[M](0)$ for all x in R, where 0 is the identity element. Then $([M]^{+)+} = [M]^+$.

**Proof**: Let x and y in R. We have $([M]^+)^+(x) = [M]^+(x) + [1]-[M]^+(0)$ = { [M](x) + [1]-[M](0) } + [1]-{ [M](0) + [1]-[M](0) } = [M](x) + [1]-[M](0) = $[M]^+(x)$. Hence $([M]^{+)+} = [M]^+$.

**2.11 Theorem:** Let [M] be an interval valued anti fuzzy subnearring of a nearring R. Then $[M]^0$ is an interval valued anti fuzzy subnearring of the nearring R.

**Proof**: For any x in R, we have $[M]^0(x-y) = [M](x-y)M^+(0) \leq M^+(0)$ rmax {[M](x), [M](y)} = rmax { $[M](x)M^+(0)$, $[M](y)M^+(0)$ } = rmax { $[M]^0(x)$, $[M]^0(y)$ }. That is $[M]^0(x-y) \leq$ rmax {$[M]^0(x)$, $[M]^0(y)$} for all x and y in R. Similarly $[M]^0(xy)$ = $[M](xy)\ M^+(0) \leq M^+(0)$ rmax {[M](x), [M](y)} = rmax { $[M](x)M^+(0)$, $[M](y)M^+(0)$ }

= rmax { $[M]^0(x)$, $[M]^0(y)$ }. That is $[M]^0(xy) \leq$ rmax { $[M]^0(x)$, $[M]^0(y)$ } for all x and y in R. Hence $[M]^0$ is an interval valued anti fuzzy subnearring of the nearring R.

**2.12 Theorem:** Let (R, +,.) be a nearring. [M] is an interval valued fuzzy subnearring of R if and only if $[M]^c$ is an interval valued anti-fuzzy subnearring of R.

**Proof:** Suppose [M] is an interval valued fuzzy subnearring of R. For all x and y in R, we have $[M](x\text{-}y) \geq$ min { [M](x), [M](y) }, which implies that $[1]\text{-}[M]^c(x\text{-}y) \geq$ min { { $[1]\text{-}[M]^c(x)$ }, { $[1]\text{-}[M]^c(y)$ } }, which implies that $[M]^c(x\text{-}y) \leq$ [1]-min { $[1]\text{-}[M]^c(x)$, $[1]\text{-}[M]^c(y)$ }, which implies that $[M]^c(x\text{-}y) \leq$ max { $[M]^c(x)$, $[M]^c(y)$ }. Also, $[M](xy) \geq$ min {[M](x), [M](y)}, which implies that $[1]\text{-}[M]^c(xy) \geq$ min { { $[1]\text{-}[M]^c(x)$ }, { $[1]\text{-}[M]^c(y)$ } }, which implies that $[M]^c(xy) \leq$ [1]-min { $[1]\text{-}[M]^c(x)$, $[1]\text{-}[M]^c(y)$ }, which implies that $[M]^c(xy) \leq$ max {$[M]^c(x)$, $[M]^c(y)$}. Thus $[M]^c$ is an interval valued anti-fuzzy subnearring of R. Converse also can be proved similarly.

**2.13 Theorem:** Let [M] be an interval valued anti fuzzy subnearring of a nearring R, then the pseudo interval valued fuzzy coset $(a[M])^p$ is an interval valued anti fuzzy subnearring of the nearring R, for every a in R.

**Proof:** Let [M] be an interval valued anti fuzzy subnearring of the nearring R. For every x and y in R, we have $((a[M])^p)(x-y) = p(a)[M](x-y) \leq p(a)$ rmax { [M](x), [M](y) } = rmax {p(a)[M](x), p(a)[M](y)} = rmax { $((a[M])^p)(x)$, $((a[M])^p)(y)$ }. Therefore $((a[M])^p)(x-y) \leq$ rmax { $((a[M])^p)(x)$, $((a[M])^p)(y)$ } for x and y in R. And $((a[M])^p)(xy) = p(a)[M](xy) \leq p(a)$ rmax{[M](x), [M](y)} = rmax { p(a)[M](x), p(a)[M](y) } = rmax {$((a[M])^p)(x)$, $((a[M])^p)(y)$}. Therefore $((a[M])^p)(xy) \leq$ rmax {$((a[M])^p)(x)$, $((a[M])^p)(y)$} for x and y in R. Hence $(a[M])^p$ is an interval valued anti fuzzy subnearring of the nearring R.

**2.14 Theorem:** Let (R, +,.) be a nearring. If [M] is an interval valued anti fuzzy subnearring of R, then x+[M] = y+[M] if and only if [M](x-y) = [M](0), where 0 is the identity element. In that case [M](x) = [M](y).

**Proof:** Given [M] is an interval valued anti fuzzy subnearring of R. Suppose that x+[M] = y+[M] which implies that (x+[M])(x) = (y+[M])(x) which implies that [M](x-x) = [M](x-y) which implies that [M](0) = [M](x-y). Conversely assume that [M](x-y) = [M](0) then (x+[M])(z) = [M](z-x) = [M](z-x+y-y) $\leq$ rmax { [M](z-y), [M](0) } = [M](z-y) = (y+[M])(z) which implies that (x+[M])(z) $\leq$ (y+[M])(z)-------------(1). Now (y+[M])(z) = [M](z-y) = [M](z-y+x-x) $\leq$ rmax { [M](z-x), [M](0) } = [M](z-x) = (x+[M])(z) which implies that (y+[M])(z) $\leq$ (x+[M])(z)-------------------(2). From (1) and (2) we get x+[M] = y+[M].

**2.15 Theorem:** Let (R, +,.) be a nearring. Let [M] is an interval valued anti fuzzy subnearring and x, y, u, and v be any elements in R, if x+[M] = u+[M] and y+[M] = v+[M], then (x+y)+[M] = (u+v)+[M].

**Proof:** Given [M] is an interval valued anti fuzzy subnearring of R. By Theorem 2.14, [M](x-u) = [M](y-v)= [M](0). We get [M](x+y-u-v) = [M](x-u+y-v) ≤ rmax {[M](x-u), [M](y-v)}= [M](0). Again by Theorem 2.14, (x+y)+[M] = (u+v)+[M].

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