Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 4 (2015), pp. 2507-2510

http://www.ripublication.com

# Learning the parameters of the model represents the interaction between multiple binary search treesdefined on Markov chain

**H.El –zohny**

*Professor Of Mathematics At Al Azhar University*

**S. Radwan**

*Professor Of Mathematics At Al Azhar University*

**Z.Mohammed**

*Mathematics 'S Researcher*

We will introduce parameters estimation of themodel describing the interaction between multiple processes by binary search tree defined on Markov chain and hidden Markov model.

## 1. Parameters estimation of the model:

Parameter estimation of hidden Markov model that defines the probability distribution over a sequence of observations that in turns generate from a process whose state is hidden from the observer, was obtained by Baum-Welch re-estimation formulas in which the re estimated formulas for $\pi$(the initial distribution), A (the state transition probabilities), B (the output distribution) namely $(\bar{\pi}, \bar{A}, \bar{B})$ in place of $\pi, A \text{ and } B$.

We use this algorithm to estimate the parameters of the model.

Consider the simplest form of interaction between two processes by binary search tree model:

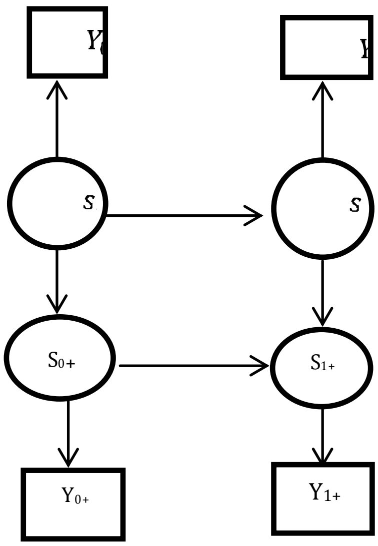


**Figure 1: A representation of the interaction between multiple processes have certain condition such that two binary search trees could interact if root of one is greater than root of the other and the same thing with left and right sub trees and then represent between them by a directed acyclic graph shown above.**

We can write joint distribution as:

$$pr(\{s_t, s_{t+}, y_t, y_{t+}\}) = \mathrm{p}(s_0)p(y_0|s_0)p(y_{0_+}|s_{0_+}) \prod_{t.t_+=1}^{T} P(s_t|s_{t-1})\, p(y_t|s_t)p(s_{t_+}|s_{t_+-1})p(s_{t_+}|s_t)p(y_{t_+}|s_{t_+}) \quad (1)$$

So define the parameters of the model:

i. Initial state distribution $=\pi_i$ defining $p(s_0)$.

ii. State transition probabilities $=a_{ij}, c_{jj}, d_{ij}$ defining $p(s_t|s_{t-1}), p(s_{t_+}|s_t)$ *and* $p(s_{t_+}|s_{t_+-1})$ *respectively.*

iii. Output distribution$=b_j, e_j$ defining $p(y_t|s_t)$ and $p(y_{t_+}|s_{t_+})$ respectively.

We will use the Baum-Welch algorithm and define some appropriate difference to re estimate all the parameters of the model.
In that algorithm we define:

$$\gamma_t(i) = pr(i_t = q_i|O,\lambda) \quad (2)$$

$$\xi_t(i,j) = pr(i_t = q_i, i_{t+1} = q_j|O,\lambda) \quad (3)$$

$$\sum_{t=1}^{T-1} \xi_t(i,j) = The\ expected\ number\ of\ transitions\ from\ state\ q_i\ to\ state\ q_j. \tag{4}$$

$$\sum_{t=1}^{T-1} \gamma_t(i) = Expected\ number\ of\ transitions\ from\ state\ q_i. \tag{5}$$

At this point we define the new formulas:

$$\sigma_{t,t_+}(i,j) = pr\left(i_t = q_i, i_{t_+} = q_j \middle| \lambda\right) \tag{6}$$

, where $\sigma_{t,t_+}(i,j)$ means the probability of being in state $q_i$ and make a transition to state $q_j$ in the next chain.

$$\gamma_{t_+}(i) = pr\left(i_{t_+} = q_i \middle| O, \lambda\right) \tag{7}$$

, where $\gamma_{t_+}(i)$ is the probability of being in state $q_i$ at time $t_+$.

$$\xi_{t_+}(i,j) = pr\left(i_{t_+} = q_i, i_{t_+ +1} = q_j \middle| O, \lambda\right) \tag{8}$$

, where $\xi_{t_+}(i,j)$ is the probability of a path being in state $q_i$ at time $t_+$ and make a transition to state $q_j$ at time $t_+ + 1$.

We can write the re estimated formulas in the following way:

i. $\overline{\pi_\iota} = \gamma_1(i), 1 \leq i \leq N.$ (9)

ii. $$\overline{a_{\iota J}} = \sum_{t=1}^{T-1} \xi_t(i,j) \Big/ \sum_{t=1}^{T-1} \gamma_t(i) \tag{10}$$

iii. $$\overline{c_{JJ}} = \sum_{t,t_+=1}^{T-1} \sigma_{t,t_+}(i,j) \Big/ \sum_{t=1}^{T-1} \gamma_t(i) \tag{11}$$

iv. $$\overline{d_{\iota J}} = \sum_{t_+}^{T-1} \xi_{t_+}(i,j) \Big/ \sum_{t_+=1}^{T-1} \gamma_{t_+}(i)$$

(12)

v. $$\overline{b_J} = {\sum_{t=1}^{T} \gamma_t(j) \atop O_t} \Big/ \sum_{t=1}^{T} \gamma_t(j)$$

(13)

vi. $$\overline{e_J} = {\sum_{t_+=1}^{T} \gamma_{t_+}(j) \atop O_t} \Big/ \sum_{t_+=1}^{T} \gamma_{t_+}(j)$$

(14)

**Reference [1]:**

1. The connection between the binary search tree and the dynamical systems trees, international journal of applied science and technology, H. El-Zohy, S. Radwan, Z.mohammed.