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On Regular Generalized Weakly (rgw) Functions in Topological Spaces

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Abstract

The aim of this paper is to introduce rgw-continuos functions, rgw-compactnessand rgw-connectedness in topological spaces. This new class of functions lies between theclass of semi weakly generalized (brieyswg) functions and the class of regular weaklygeneralized (brieyrwg) functions. We study the fundamental properties of these classof functions.

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I. Introduction

The notation of closed set is fundamental in the study of topological spaces. In 1970, Levine [1] introduced the concept of generalized closed sets in the topology. Many researchers like T. Kong, R. Kopperman and P. Meyer [2], Caw, Ganster and Reilly [3] [4], S.P. Arya and T.M. Nour [5], N. PalaniInappan and K. ChandrasekharaRao [6], A.Pushpalatha [7], S.S. Benchalli and R.S. Wali [8] extend the work on generalized closedsets and their related concepts in topology. In 2011, the notion of Regular GeneralizedWeakly closed (brieyrgw-closed) sets was introduced. In this paper we introduce andcharacterize the rgw-closed map, rgw-continuous, rgw-irresolute, rgw-open map, Quasirgw-open and closed map along with rgw-compactness.

Throughout this paper space (X, τ) (or simply X) always means a topological space onwhich no separation axioms are assumed unless explicitly stated. For a subset

A of a space$X$, $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complementof A in X respectively.

II. Preliminaries

Definition 2.1 A subset A of X is called generalized (briefly g-closed) [1] set iff$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.2 A subset A of X is called regular open (briefly r-open) [9] set if $A = int(cl(A))$and regular (briefly r-closed) [9] set if $A = cl(int(A))$.

Definition 2.3 A subset A of X is called pre-open set [10] if $A \subseteq int(cl(A))$ and pre-closed [10] set if $A \subseteq cl(int(A))$.

Definition 2.4 A subset A of X is called pre-open set [1] if $A \subseteq cl(int(A))$ and pre-closed [1] set if $A \subseteq int(cl(A))$.

Definition 2.5 A subset A of X is called α-open [11] if $A \subseteq int(cl(int(A)))$ and α-closed [11] if $cl(int(cl(A))) \subseteq A$.

Definition 2.6 A subset A of X is called semi-preopen [12] if $A \subseteq cl(int(cl(A)))$ and semi-preclosed [12] if $int(cl(int(A))) \subseteq A$.

Definition 2.7 A subset A of X is called θ-closed [16] if $A = cl_\theta(A)$, where $cl_\theta(A) = \{x \in X : cl(U) \cap A \neq U \in A\}$.

Definition 2.8 A subset A of X is called δ-closed [16] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq U \in A\}$.

Definition 2.9 A subset A of a space (X, τ) is called regular semiopen [13] if there is regular open set U such that $U \subset A \subset cl(U)$. The family of all regular semiopen sets of$X$ is denoted by $RSO(X)$.

Definition 2.10 A subset A of a space (X, τ) is said to be semi-regular open [14] if it is both semiopen and semiclosed.

Theorem 2.11 Every regular semiopen set in X is semiopen but not conversely.

Theorem 2.12[15] If A is regular semiopen in X, then X/A is also regular semiopen.

Theorem 2.13 [15] In a space X, the regular closed sets, regular open sets and clopensets are regular semiopen.

III.Regular Generalized Weakly (rgw)-continuous and Regular Generalized Weakly (rgw)-irresolute Functions

In this section, we discuss the functions by involving rgw-closed sets and introduce a new class of regular generalized weakly (briefly rgw-continuous) continuous mapping, concept of Quasi rgw-open functions and discuss some of their basic properties and characterizations.

Definition 3.1 A function $f:(X,\tau)\to(Y,\sigma)$ is called rgw-closed map if for each closed set $F$of X, $f(F)$ is rgw-closed set in Y.

Theorem 3.2 Every rw-closed map is a rgw-closed map.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ is rw-closed map i.e. $f(A)$ is rw-closed set for each closed set $A$of X. By using theorem 2.13 $f(A)$ is rgw-closed set. Thus we have $f(A)$ is rgw-closed set for each closed set $A$of X. Hence f is rgw-closed. The converse of this theoremdoes not hold true as shown by following example.

Example 3.1 Let$X=\{a,b,c,d\}$, $\tau=(X,\emptyset,\{a,b\},\{c,d\})and\sigma=\{X,\emptyset,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$. Here we have collections of rgw-closed sets in (X,σ) are $\{X,\emptyset,\{c\},\{d\},\{a,b\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$and $\{X,\emptyset,\{d\},\{a,b\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ and collection of (X,τ)closed sets is $\{X,\emptyset,\{a,b\},\{c,d\}\}$. Define $f:(X,\tau)\to(Y,\sigma)$by $f(a)=c$, $f(b)=c$, $f(c)=a$, $f(d)=b$. Now$f(\{c,d\})=\{a,b\}$and $f(\{a,b\})=c$, where $\{a,b\}$, $\{c\}$ both are rgw-closed set but $\{c\}$ is not rw-closed. Thus f is rgw-closed map but not rw-closed map.

Theorem 3.3 If a map $f:X\to Y$ is closed and a map $g:Y\to Z$ is rgw-closed then$gof:X\to Z$ is rgw-closed.

Proof: Let $f:X\to Y$ is closed and $g:Y\to Z$ is rgw-closed. To show $gof:X\to Z$isrgw-closed. Let H be a closed set in X. Then by definition of closed map $f(H)$ is closedand $gof(H)=g(f(H))$ is rgw-closed as g is rgw-closed. Thus gof is rgw-closed.

Definition 3.4 A function $f:(X,\tau)\to(Y,\sigma)$ is called rgw-continuous if $f^{-1}(V)$ isrgw-closed in (X,τ) for every closed set in (Y,σ).

Example 3.2 Let $X=\{a,b\}$, $\tau=\{X,\emptyset,\{a\},\{b\}\}$ and $Y=\{a,b,c\}$, $\sigma=\{\emptyset,X,\{a,b\}\}$define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=a$ and $f(b)=c$. Since every subset of (X,τ) isrgw-closed thus f is rgw-continuous.

Definition 3.5A function $f:(X,\tau)\to(Y,\sigma)$ is called rgw-irresolute if $f^{-1}(V)$ isrgw-closed in (X,τ) for every rgw-closed set Vin (Y,σ).

Example 3.3 Let $X = \{a,b,c\}, \tau = \{\emptyset, X, \{a,b\}\}$ and $\sigma = \{\emptyset, X, \{a,b\}, \{c\}\}$ define$f:(X,\tau) \to (X,\sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then inverse image of every rgw-closed set is rgw-closed under f. Hence f is rgw-irresolute.

Theorem 3.6 Every rgw-irresolute function is rgw-continuous, but not conversely.

Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be rgw-irresolute function, i.e. $f^{-1}(V)$ is rgw-closed in$(X,\tau)$for every rgw-closed set Vin (Y,σ). Now as every closed set is rgw-closed set [8].Thus $f^{-1}(V)$ is rgw-closed in (X,τ) for every closed set Vin (Y,σ). The converse ofabove theorem is proved with the help of following example.

Example 3.4 Consider $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{a,b\}\}$ define $f:(X,\tau) \to (X,\sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. We have the collection of rgw-closed sets for (X,τ) and (X,σ) are$\{\emptyset, X, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$and $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$ respectively. Here for each closed set of (X,σ)inverse function maps into collection of rgw-closed set in (X,τ) but for each rgw-closedin (X,σ)inverse function does not map into collection of rgw-closed set in (X,τ) as$f^{-1}(a) = b$ does not belong to rgw-closed set for (X,τ). Thus by definitions f is rgw-continuous but not rgw-irresolute.

Theorem 3.7 (i) Let $f:(X,\tau) \to (Y,\sigma)$ be rw-continuous then f is rgw-continuous, but not conversely.
(ii) Let $f:(X,\tau) \to (Y,\sigma)$ be swg-continuous then f is rgw-continuous, but not conversely.

Proof: (i) Let $f:(X,\tau) \to (Y,\sigma)$ be rw-continuous i.e. for each V closed in (Y,σ) wehave $f^{-1}(V)$ is regular weakly closed in (X,τ). Thus by theorem 2.13 [As every rw-closedset is rgw-closed] $f^{-1}(V)$ is rgw-closed. i.e. for each V closed in (Y,σ) we have $f^{-1}(V)$is rgw-closed in (X,τ). Hence f is rgw-continuous. The converse of this does not holdtrue as shown in following example.
(ii) Let $f:(X,\tau) \to (Y,\sigma)$ be swg-continuous i.e. for each U closed in (Y,σ) we have$f^{-1}(U)$ is swg-closed in (X,τ). We have $f^{-1}(U)$ is rgw-closed [17] i.e. for each closed on Y we have $f^{-1}(U)$ is rgw-closed in X. Hence f is rgw-continuous. The converse of thisdoes not hold true as shown in following example.

Example 3.5 Consider $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$ define $f:(X,\tau) \to (X,\tau)$ by $f(a) = b$, $f(b) = c$, $f(c) = d$ and $f(d) = c$. We have the collection of closed sets, rgw-closed sets and rw-closed sets in$(X,\tau)$are $\{\emptyset, X, \{d\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$, $\{\emptyset, X, \{c\}, \{d\}, \{c,d\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$ and $\{\emptyset, X, \{d\}, \{a,b\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$ respectively. Now we see for each closed set in (X,τ) inverse image of $f$maps into collection of rgw-closed set in (X,τ)but it does not mapsinto collection of rw-closed sets, as $f^{-1}\{d\} = \{c\}$belong to rgw-closed set in (X,τ)but it does not maps into collection of rw-closed sets in (X,τ). Thus by definitions f is rgw-continuousbut not rw-continuous.

Example 3.6 Consider $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\},\{b\},\{a,b\}\}$ and $\sigma = \{\emptyset, X, \{c\}\}$, define g: $(X,\tau) \to (X,\sigma)$ by $g(a) = b$, $g(b) = a$ and $g(c) = c$. For (X,τ), we have the collection of rgw-closed sets and swg-closed sets $\{\emptyset, X, \{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and $\{\emptyset, X, \{c\},\{a,c\},\{b,c\}\}$ respectively and closed set for $\{X,\sigma\}$ is $\{\emptyset, X, \{a,b\}\}$. Now we see for each closed set in (X,σ) inverse image of f maps into collection of rgw-closed set in (X,τ) but it does not map into collection of swg-closed sets in (X,τ) as $f^{-1}(a,b) = \{b,a\}$which belongs to rgw-closed set in (X,τ) but does not belong to swg-closed set in (X,τ).

Theorem 3.8 The composition of two rgw-continuous functions need not be rgw-continuous.

Proof: For this we consider the following example

Example 3.7 Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\},\{b\},\{a,b\}\}$, $\sigma = \{\emptyset, X, \{c\},\{b,c\}\}$ and$\eta = \{\emptyset, X, \{b,c\}\}$. Define f: $(X,\tau) \to (X,\sigma)$by $f(a) = b$, $f(b) = c$ and $f(c) = a$ and g: $(X,\sigma) \to (X,\eta)$ by $g(a) = b$, $g(b) = a$, $g(c) = c$. We have collection of rgw-closed set for (X,τ) is $\{\emptyset, X, \{c\},\{a,b\},\{a,c\},\{b,c\}\}$ and for (X,σ) is $\{\emptyset, X, \{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\}\}$ respectively. We see that for each closed set in (X,σ)inverse image of f maps into thecollection of rgw-closed sets in(X,τ) and for each closed set in (X,η) inverse imageof g maps into the collection of rgw-closed sets in (X,σ). Thus both f and g are rgw-continuous maps.But for $gof : (X,\tau) \to (X,\eta)$ we see for each closed set in (X,η)inverse image of gof does not map into collection of rgw-closed in (X,τ) as$\{a\}$ is closedin $(X,\eta)$and $(gof)^{-1}(\{a\}) = f^{-1}\big(g^{-1}(\{a\})\big) = f^{-1}(b) = a$, which is not$rgw$-closed in$(X,\tau)$.

Theorem 3.9 Let f: $(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma) \to (Z,\eta)$ be any two functions then
(i) gof is rgw-continuous, if g is continuous and f is rgw-continuous.
(ii) gof is rgw-irresolute, if g is rgw-irresolute and f is rgw-irresolute.
(iii) gof is rgw-continuous, if g is rgw-continuous and f is rgw-irresolute.

Proof: (i) Let f and g as defined above be rgw-continuous and continuous respectively.Let$V$ be closed in (Z,η), then by definition of continuity $g^{-1}(V)$ is closed in (Y,σ) andrgw-continuity of f implies $f^{-1}(g^{-1}(V))$ is rgw-closed in (X,τ). That is $(gof)^{-1}(V)$ isrgw-closed in (X,τ) where V is closed in (Z,η). Hence gof is rgw-continuous.
(ii) Let f and g as defined above be rgw-irresolute. Let V be rgw-closed in (Z,η), then by definition of rgw-irresolute function $g^{-1}(V)$ is rgw-closed in (Y,σ). As f is rgw-irresolute so $f^{-1}(g^{-1}(V))$ is rgw-closed in (X,τ). That is $(gof)^{-1}(V)$ is rgw-irresolute in$(X,\tau)$where V is rgw-irresolute in (Z,η). Therefore gof is rgw-irresolute.

(iii) Let f and g as defined above be rgw-irresolute and rgw-continuous respectively. Let V be closed in (Z,η), then by definition of rgw-continuity $g^{-1}(V)$ is rgw-closed in (Y,σ). As f is rgw-irresolute so $f^{-1}(g^{-1}(V))$ is rgw-closed in (X,τ). That is $(gof)^{-1}(V)$ is rgw-continuous where V is closed in (Z,η). Therefore gof is rgw-continuous.

Definition 3.10 A map $f:(X,\tau)\to(Y,\sigma)$ is called a rgw-open map if the image $f(A)$ is rgw-open in (Y,σ) for each open set A in (X,τ).

Definition 3.11 A function $f:(X,\tau)\to(Y,\sigma)$ is said to be Quasi rgw-open if theimage of every rgw-open set in X is open in Y.

Example 3.8 Let $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$ and $\sigma=\{\emptyset,X,\{c\},\{b,c\}\}$ define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=c, f(b)=c$ and $f(c)=c$. We have the collectionof rgw-open set in (X,τ) are $\{\emptyset,X,\{a\},\{b\},\{c\},\{a,b\}\}$. Now we see the image of each rgw-open subset maps into open set of (Y,σ). Finally, by definition 3.11, f is Quasi rgw-open.

Theorem 3.12 A function $f:(X,\tau)\to(Y,\sigma)$ is said to be Quasi rgw-open if and onlyif for every subset U of X, $f(rgw-int(U))\subset int(f(U))$.

Proof. Let f be a Quasi rgw-open function and U be a subset of X. Now we have $int(U)\subset U$ and $rgw-int(U)$ is a rgw-open set. Hence we obtain that $f(rgw-int(U))\subset f(U)$. As $f(rgw-int(U))$ is open $f(rgw-int(U))\subset int(f(U))$.
Conversely, assume that U is a rgw-open set in X then $f(U) = f(rgw-int(U))\subset int(f(U))$ but $int(f(U))\subset f(U)$. Consequently $f(U)=int(f(U))$ and hence f is Quasi rgw-open.

Theorem 3.13 If a function $f:(X,\tau)\to(Y,\sigma)$ is Quasi rgw-open then $rgw-int(f^{-1}(G))\subset f^{-1}(int(G))$ for every subset G of Y.

Proof. Let G be any arbitrary subset of Y. Then $rgw-int(f^{-1}(G))$ is a rgw-open set in X and f is Quasi rgw-open, then $f(rgw-int(f^{-1}(G)))\subset int(f(f^{-1}(G)))\subset int(G)$. Thus $rgw-int(f^{-1}(G))\subset f^{-1}(int(G))$. Recall that a subset S is called a rgw-neighbourhoodof a point x of X, if there exist a rgw-open set U such that $x\in U\subset S$.

Definition 3.14 A map $f:(X,\tau)\to(Y,\sigma)$ is called a rgw-closed map if the image $f(A)$ is rgw-closed in (Y,σ) for each closed set A in (X,τ).

Definition 3.15 A function $f:X\to Y$ is said to be Quasi rgw-closed if the image ofeach rgw-closed set in X is closed in Y. i.e. every Quasi rgw-closed function is closed aswell as rgw-closed.

Remark 3.16 Every rgw-closed function need not be Quasi rgw-closed as shown by thefollowing example.

Example 3.9 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$and $\sigma = \{Y, \emptyset, \{a\}, \{b, c\}\}$ Define afunction $f: (X, \tau) \to (Y, \sigma) by\ f(a) = b, f(b) = c$ and $f(c) = a$. Here, we have thecollections of rgw-closed sets in (X, τ) and $(Y, \sigma)$are$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\{\emptyset, Y, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ respectively and collection of closed sets in$(X, \tau)$and $(Y, \sigma)$are $\{\emptyset, X, \{c\}\}$and $\{\emptyset, Y, \{a\}, \{b, c\}\}$respectively. Now for each closedset in (X, τ) function f maps into rgw-closed set in (Y, σ). Thus f is rgw-closed mapbut $f(\{a\}) = \{b\}$ which is not closed in (Y, σ). i.e. each rgw-closed set of (X, τ)does notmap into closed set of (Y, σ). Thus f is rgw-closed but not Quasi rgw-closed.

Definition 3.17 A function $f: (X, \tau) \to (Y, \sigma)$ is said to bergw^*-closed if for each rgw-closed set $F$of X, $f(F)$ is rgw-closed in Y.

Example 3.10 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{c\}, \{a, b\}\}$. Define a function $f: (X, \tau) \to (Y, \sigma)$by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Here, we have the collections of rgw-closed sets in (X, τ) and $(Y, \sigma)$are $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\{\emptyset, Y, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$respectively. Now for each rgw-closed set in(X, τ) function f maps into rgw-closed set in (Y, σ). Thus f is rgw^*-closed.

Theorem 3.18 Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be any two functions. Then

(i) If f is rgw-closed and g is Quasi rgw-closed, then gof is closed.

(ii) If f is Quasi rgw-closed and g is rgw-closed, thengof isrgw^*-closed.

(iii) If f is rgw^*-closed and g is Quasi rgw-closed, then gof is Quasi rgw-closed.

Proof: (i) Here f is rgw-closed and g is Quasi rgw-closed. To prove gof is closed. Let Fbe arbitrary closed in (X, τ). Since f is rgw-closed so $f(F)$ is rgw-closed in (Y, σ). Also$g$ is Quasi rgw-closed so $g(f(F))$ is closed in (Z, η). i.e. $gof(F)$ is closed in (Z, η) where$F$ is closed set in (X, τ). Thus gof is closed.

(ii) Here f is Quasi rgw-closed and g is rgw-closed. To prove gof is rgw^*-closed. Let Fbe arbitrary rgw-closed in (X, τ). Since f is Quasi rgw-closed so $f(F)$ is closed in (Y, σ).Also g is rgw-closed so $g(f(F))$ is rgw-closed in (Z, η). i.e. $gof(F)$ is rgw-closed in $(Z, \eta)$where F is rgw-closed set in (X, τ). Thus gof is rgw^*-closed.

(iii) Here f is rgw^*-closed and g is Quasi rgw-closed. To prove gof is Quasi rgw-closed.Let F be arbitrary rgw-closed in (X, τ). Since f is rgw^*-closed so $f(F)$ is rgw-closed in(Y, σ). Also g is Quasi rgw-closed so $g(f(F))$ is closed in (Z, η). i.e.

$gof(F)$ is closed in(Z,η) where F is rgw-closed set in (X,τ). Thus gof is Quasi rgw-closed.

Theorem 3.19 Let $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$ be two functions suchthat $gof : (X,\tau) \to (Z,\eta)$ is Quasi rgw-closed. Then (i) If f is rgw-irresolute surjective, then g is Quasi rgw-closed.
(ii) If g is rgw-continuous injective, then f is rgw^*-closed.

Proof: (i) Here $f:(X,\tau) \to (Y,\sigma)$ and $gof : (X,\tau) \to (Z,\eta)$ are rgw-irresolute surjectiveand Quasi rgw-closed respectively. To show g is Quasi rgw-closed. Let F be rgw-closedin (Y,σ) as f is rgw-irresolute, so $f^{-1}(F)$ is rgw-closed in (X,τ). Since gof is Quasirgw-closed and f is surjective. Thus $gof(f^{-1}(F)) = g(F)$. Which is closed in (Z,η). i.e.$g(F)$ is closed in (Z,η) where F is rgw-closed in (X,τ). Therefore g is Quasi rgw-closed.
(ii) Here $g:(Y,\sigma) \to (Z,\eta)$ and $gof : (X,\tau) \to (Z,\eta)$ are rgw-continuous injective andQuasi rgw-closed respectively. To show f is rgw^*-closed. Let F be rgw-closed in (X,τ)as gof is Quasi rgw-closed, so $gof(F)$ is closed in (Z,η). Again g is rgw-continuous andinjective function. Thus $g^{-1}(gof(F)) = f(F)$, which is rgw-closed in (Y,σ). i.e. $f(F)$ isrgw-closed in (Y,σ), where F is rgw-closed in (X,τ). Thus f is rgw^*-closed.

IV. Regular Generalized Weakly (rgw)-Compactness

In this section we extend the concept of open cover and compactness in the form of rgw-closed sets to introduce rgw-open cover and rgw-compactness and discuss their propertiesand characterization.

Definition 4.1 A collection$\{A_i : i \in \Delta\}$ of rgw-open sets in a topological space X iscalled rgw-open cover of a subset S is $S \subset \cup\{A_i : i \in \Delta\}$ holds.

Definition 4.2 A topological space (X,τ) is rgw-compact if rgw-open cover of X has a finite subcover.

Definition 4.3 A subset S of a topological space (X,τ) is said to be rgw-Compactrelative to X if for every collection$\{A_i : i \in \Delta\}$ of rgw-open subsets of X such that$S \subset \cup\{A_i : i \in \Delta\}$ there exists finite subset Δ_0 of Δ such that $S \subset \cup\{A_i : i \in \Delta_0\}$.

Definition 4.4 A subset S of a topological space X is said to be rgw-Compact if S isrgw-Compact as a subspace of X.

Theorem 4.5 Let $f:(X,\tau) \to (Y,\sigma)$ be a surjective, rgw-continuous map. If X isrgw-compact, then Y is compact.

Proof: Here, we have $f:(X,\tau)\to(Y,\sigma)$ a surjective and rgw-continuous map and X is rgw-compact. To prove Y is compact. Let $\{A_i : i\in\Delta\}$ be an open cover of Y. Then $\{f^{-1}(A_i) : i\in\Delta\}$ is a rgw-open cover of X. Since X is rgw-compact, it has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3), \dots, f^{-1}(A_n)\}$.Surjectiveness of f implies$\{A_1, A_2, A_3, \dots, A_n\}$ is an open cover of Y and hence Y is compact.

Theorem 4.6 If a map $f:(X,\tau)\to(Y,\sigma)$ is rgw-irresolute and a subset S of X isrgw-compact relative to X, then the image $f(S)$ is rgw-compact relative to Y.

Proof: Here, we have $f:(X,\tau)\to(Y,\sigma)$ is rgw-irresolute also S subset of X is rgw-compact relative to X. To prove $f(S)$ is rgw-compact relative to Y. Let$\{A_i : i\in\Delta\}$ bea collection of rgw-open sets in Y such that $(S)\subset\cup\{A_i : i\in\Delta\}$. Then $S\subset\{f^{-1}(A_i) : i\in\Delta\}$where $f^{-1}(A_i)$ is rgw-open in X for each i. Since S is rgw-compact relative toX, there exist a finite subcollection$\{A_1, A_2, \dots, A_n\}$such that $S\subset\cup\{f^{-1}(A_i) : i\in\Delta\}$ i.e.$f(S)\subset\cup\{A_i : i\in\Delta\}$. Hence, $f(S)$ is rgw-compact relative to Y.

Theorem 4.7 A rgw-closed subset of a rgw-compact space X is rgw-compact relativeto X.

Proof: Let A be a rgw-closed subset of a rgw-compact space X. Then $X\setminus A$ is rgw-openin X. Now to prove A is rgw-compact relative to X. Let Ωbe a rgw-open cover for A.Then $\{\Omega, X\setminus A\}$is a rgw-open cover for X. Since X is rgw-compact, by definition 4.2, X has a finite subcover, say$\{P_1, P_2, \dots P_n\}=\Omega_1$. If $X\setminus A$ does not belong , then $\Omega_1\setminus(X\setminus A)$ is a subcover of A. Thus by definition $4.3A$ is rgw-compact relative to X.

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