

A Deteriorating Inventory Model With Inflation And Trade-Credit Period Under Power Pattern Demand Rate

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Abstract

In this paper we assumed that retailer pays the purchasing cost for the items as soon as they are received. While in real life supplier provides a delay period known as trade-credit period to motivate the retailer to buy more. In the present study, we have to developed an inventory model with trade credit facility in which the unit holding cost is based on the time the item is in stock. The deterioration is assumed Weibull distributed. The power pattern of demand is assumed and the effect of inflation and time value of money is taken in to deliberation. Shortages are allowed and the partial backlogging rate is a continues inverse function of waiting time in purchasing the item during stock out period. Numerical illustrations are also provided to demonstrate the present study. The sensitivity analysis is presented for giving the behavior of considered inventory system.

Keywords: Inventory, power form demand rate, deterioration, trade-credit and inflation

1. Introduction and literature review

Trade-credit period is a common phenomenon in retailing, where supplier permits the retailer a fixed time period to settle the total amount owed. This provides an advantage to the retailer as they can earn interest on the accumulated revenue received

during the trade credit period. At the same time, trade credit period can also confer benefits to the supplier since the policy may attract new customers who consider it to be a type of price reduction. Advance sales policies are widely used by retailers today, including Maxim's Bakery in Hong Kong, Amazon.com, Eslitebooks.com, Movies Unlimited, Toys R Us and Electronics Boutique. Customers who accept advance sales must prepay the entire discounted purchase amount prior to the regular sale season. Alternatively, customers can purchase the product at the regular price during the regular sale season.

Trade credit period has been widely discussed in the literature. Chang et al. (2003) developed an economic order quantity model for deteriorating items, in which the supplier provides a trade credits period to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Ouyang et al. (2006) developed a general EOQ model with trade credit for a retailer to determine the optimal shortage interval and replenishment cycle.

Goyal et al. (2007) introduced a new concept where the supplier charges the retailer progressive interest rates, if the retailer exceeds the period of permissible delay. Ho et al. (2008) proposed an integrated inventory model with retail price sensitive demand and trade credit financing. Ramaekers et. al.(2008) developed an inventory model which containing uncertainty and needs a probability distribution of demand for reorder point.

In the literature on inventory control, several times reference is made to the Normal or Gamma distribution for describing the demand in the lead time. The Poisson distribution has been establish to provide a reasonable fit when the demand is very low. Chang et al. (2009b) formulated an integrated vendor–buyer inventory model with retail price sensitive demand, where the credit terms are linked to the order quantity. Kumar et al. (2009) presented an inventory control model with quadratic demand rate for decaying items under trade credits and inflation. Chen and Kang (2010) developed integrated models with permissible delay in payments for determining the optimal replenishment time interval and replenishment frequency. Kumar et al. (2012) developed a policy decisions for a price dependent demand rate inventory model with progressive payments scheme. Guria et. al. (2012) developed an inventory model involving permissible delay in payment with fully-backlogged shortage and without allowable shortage for an existing petrol/diesel retailing station. Kumar et al. (2012) presented a deterministic inventory control model for deteriorating items with price dependent demand and time varying holding cost under trade credit.

There are also many relevant articles related to trade credit period like, Goyal (1985), Dave (1985), Mandal and Phaujdar (1989), Aggarwal and Jaggi (1995), Teng (2005), Huang (2003), Chang and Teng (2004), Chung and Liao (2004), Ouyang et al. (2005), Teng et al. (2005) and Chang et al. (2009a). However, none of the models presented in the above literature incorporates advance sales.

Inflation and the time value of money is an important factor which affect the inventory system. Inflation effects the profit function as well as the total cost function so that determine the optimal inventory policies the effect of inflation cannot be ignored. Firstly Buzacott (1975) presented the EOQ model with inflation after

Buzacott so many researchers extended the work of Buzacott and also explain the effect of inflation on different cost etc. In the same field Mishra (1979) presented a note on optimal inventory management under inflation then Park (1986) shows inflationary effect on EOQ under tread credit financing. Goyal et al. (1991) shows the impact of inflation on economic order discount scheduled to increase vendor profit. Yang (2004) developed two ware house inventory models for deteriorating item with shortage under inflation. In this same field again Yang (2004) develops two ware house inventory models for deteriorating item with inflation. Sarkar et al. (2010) developed a finite replenishment model with increasing demand rate under inflation. Chaudhari (2010) presented an imperfect production process for time varying demand with inflation and time value of money. Chauhan and Singh (2014) developed a model to reflect the real situation of market for time varying deterioration items and varying demand with time, discounted cash flow (DCF) approach.

In this study, we develop an inventory model for deteriorating item with variable holding cost and allowable delay in payment is developed. The deterioration rate is assumed as Weibull distributed and the demand rate of item is assumed in power pattern form. Shortages are allowed and partially backlogged at next replenishment. The effect of inflation rate is also taken into reflection. Finally, numerical examples are separately presented to demonstrate the developed model for each scenario of permissible delay period and sensitivity analysis is also performed.

2. Notations and Assumptions:

2.1: The following notations have been used for the development of mathematical inventory model:

$I(t)$	The inventory level at any time t , $t \geq 0$;
T	Constant scheduling period or cycle length (time units);
I_{\max}	Maximum level of inventory at the start of a cycle (units);
S	Maximum quantity of demand backlogged per cycle (units);
t_1	Time of inventory cycle when there is positive inventory;
Q	Order quantity (units/cycle);
c_p	Per unit purchasing cost (\$);
r	Inflation rate;
I_e	Interest which can be earned per \$ per year;
I_c	Interest charges per \$ investment in inventory per year;
c_1	Deterioration cost per unit/ unit time (\$);
c_2	Ordering cost per order (\$);
c_3	Shortage cost per unit back-ordered/ unit time (\$/unit/unit time);
c_4	Opportunity cost due to lost sales (\$/unit);
p	Per unit selling price $p > c_p$ (\$);
M	Delay period provided to the retailer;
$ATC_i(t_1^*)$	Average total cost per unit time in for i -th case, where $i = 1, 2$.

2.2 The following assumptions have been used for the development of mathematical inventory model:

- i. The inventory system involves only one item and the cycle length is given and finite.
- ii. The replenishment occurs instantaneously at an infinite rate.
- iii. Lead time is negligible.
- iv. The distribution of time until deterioration of the item follows a two-parameter Weibull distribution.
- v. Deterioration occurs as soon as items are received in to inventory.
- vi. There is no replacement or repair of deteriorating items during the period under consideration.
- vii. The demand up to time t is assumed to be $D(t) = \frac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}$. Where the demand size d during the fixed cycle time T and $n \in (1, \infty)$ is the pattern index. Such pattern in the demand rate is called power demand pattern.
- viii. During the shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. We have defined the backlogging rate to be $\frac{1}{1 + \delta(T-t)}$ where inventory is negative. The backlogging parameter δ is a positive constant, $t_1 \leq t \leq T$.

A brief introduction to the rate of deterioration is given as follows:

t is the product life (time to deterioration), $t > 0$;

$f(t)$ is the probability density functions of product life (p.d.f.);

$F(t)$ is the cumulative distribution functions of product life (c.d.f.);

$R(t)$ is the reliability (probability of survivorship by time t);

$Z(t)$ is the instantaneous deterioration rate.

If the product life t is assumed to follow a two-parameter Weibul distribution, then p.d.f.

$$f(t) \text{ is given as } f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^\beta}, \quad (1)$$

where $\alpha > 0$ is the scale parameter and $\beta > 0$ is the shape parameter using the former definition, we have $R(t) = 1 - F(t) = e^{-\alpha t^\beta}$. (2)

$$Z(t) = \frac{\alpha\beta t^{\beta-1} e^{-\alpha t^\beta}}{e^{-\alpha t^\beta}} = \alpha\beta t^{\beta-1}, t > 0. \quad (3)$$

Equation (3) will be used in the model development. When $\beta > 1$, deteriorating rate increase with time and when $\beta < 1$ deteriorating rate decreases with time but when $\beta = 1$ deteriorating rate is constant.

3. Mathematical Modelling and Analysis:

The inventory system during a given cycle is depicted in Fig. 2. At $t=0$, an initial replenishment of Q units is made, of which S units are delivered towards backorders, leaving a balance of I_{max} units in the initial inventory. It is shown in figure above that from $t=0$ to $t_1=0$ time units, the inventory level decreases due to both demand and deterioration. At t_1 , the inventory level is zero. During the time $(T-t_1)$ part of the shortage is backlogged and part of it is lost sales. Only the backlogging items are replaced by the next replenishment.

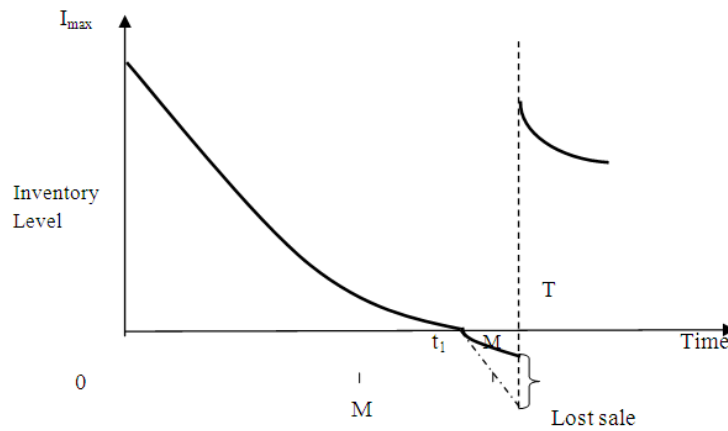


Fig. 1. Graphical representation of inventory system with delay period.

The differential equation describing $I(t)$ over the length t_1 is given as follow:

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -D(t) ; 0 \leq t \leq t_1. \tag{4}$$

The boundary conditions are $I(0) = I_{max}$ and $I(t_1) = 0$.

The approximate solution of equation (4) by neglecting higher order term of α is

$$I(t) = \frac{d}{T^{1/n}} \left[(t_1^{1/n} - t^{1/n}) + \frac{\alpha(t_1^{1+n\beta} - t^{1+n\beta})}{(1+n\beta)} \right] e^{-\alpha t^\beta} ; 0 \leq t \leq t_1. \tag{5}$$

Now, again taking the first two terms of the exponential series and neglecting the terms containing α^2 the equation (5) becomes

$$I(t) = \frac{d}{T^{1/n}} \left[(t_1^{1/n} - t^{1/n}) + \frac{\alpha(t_1^{1+n\beta} - t^{1+n\beta})}{(1+n\beta)} \right] (1 - \alpha t^\beta) ; 0 \leq t \leq t_1 \tag{6}$$

So, the maximum inventory level for each cycle can be obtained as

$$I_{\max} = I(0) = \frac{d}{T^{1/n}} \left[t_1^{1/n} + \frac{\alpha t_1^{\frac{1}{n}+\beta}}{(1+n\beta)} \right] \tag{7}$$

During the shortage interval t_1, T , the demand at time t is partially backlogged at the fraction $\frac{1}{1+\delta(T-t)}$. Thus, the differential equation governing the amount of demand backlogged is as below.

$$\frac{dI}{dt} = -\frac{dt^{1-n/n}}{nT^{1/n} (1+\delta(T-t))}; t_1 \leq t \leq T \tag{8}$$

with the boundary condition $I(t_1) = 0$. The solution of equation (8) by neglecting higher order term of δ is

$$I(t) = -\frac{d}{T^{1/n}} \left[(1-\delta T)(t^{1/n} - t_1^{1/n}) + \frac{\delta}{(1+n)} (t^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}) \right]; t_1 \leq t \leq T \tag{10}$$

Let $t = T$ in (10), we obtain the maximum amount of demand backlogged per cycle as follows.

$$S = -I(T) = \frac{d}{T^{1/n}} \left[(1-\delta T)(T^{1/n} - t_1^{1/n}) + \frac{\delta}{(1+n)} (T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}) \right] \tag{11}$$

Hence, the order quantity per cycle is given by

$$Q = I_{\max} + S = \frac{d}{T^{1/n}} \left[t_1^{1/n} + \frac{\alpha t_1^{\frac{1}{n}+\beta}}{(1+n\beta)} + (1-\delta T)(T^{1/n} - t_1^{1/n}) + \frac{\delta}{(1+n)} (T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}) \right] \tag{12}$$

The present value of the ordering cost per cycle is

$$OC = A \tag{13}$$

The present value of the purchasing cost per cycle is

$$PC = c_p \left[\frac{d}{T^{1/n}} \left(t_1^{1/n} + \frac{\alpha t_1^{\frac{1}{n}+\beta}}{(1+n\beta)} \right) + 1 - rT (1-\delta T)(T^{1/n} - t_1^{1/n}) + \frac{\delta}{(1+n)} \left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1} \right) \right] \tag{14}$$

The present value of the deterioration cost per cycle is

$$DC = \int_0^{t_1} c_1 \alpha \beta t^{\beta-1} I(t) e^{-rt} dt, \\ = \frac{c_1 \alpha \beta d}{T^{1/n}} \left[\frac{t_1^{\frac{1}{n}+\beta}}{\beta(1+n\beta)} + \frac{\alpha t_1^{\frac{1}{n}+2\beta} (1+n\beta - 2n^2\beta + 2n^2\beta^2)}{2\beta(1+n\beta)(1+2n\beta)} - \frac{nrt_1^{\frac{1}{n}+\beta+1}}{(1+n+n\beta)} + \frac{\alpha rt_1^{\frac{1}{n}+2\beta+1} (2n^2\beta - 2n^2\beta^2 - n\beta^2 - \beta)}{(1+\beta)(1+2\beta)(1+n\beta)(1+n+2n\beta)} \right] \tag{14}$$

The present value of the shortage cost per cycle is

$$\begin{aligned}
 SC &= \int_{t_1}^T c_3 (-I(t)) e^{-rt} dt, \\
 &= \frac{c_3 d}{T^{1/n}} \left[1 - \delta T \left\{ \frac{n}{(1+n)} \left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1} \right) - \frac{nr}{(1+2n)} \left(T^{\frac{1}{n}+2} - t_1^{\frac{1}{n}+2} \right) \right. \right. \\
 &\quad \left. \left. + \frac{t_1^{\frac{1}{n}+1}}{(1+n)} - t_1^{\frac{1}{n}} T - t_1 + \frac{rt_1^{\frac{1}{n}}}{2} T^2 - t_1^2 \right\} + \frac{\delta}{(1+n)} \left\{ \frac{n}{(1+2n)} \left(T^{\frac{1}{n}+2} \right. \right. \right. \\
 &\quad \left. \left. \left. - t_1^{\frac{1}{n}+2} \right) - t_1^{\frac{1}{n}+1} T - t_1 - \frac{nr}{(1+2n)} \left(T^{\frac{1}{n}+3} - t_1^{\frac{1}{n}+3} \right) + \frac{rt_1^{\frac{1}{n}+1}}{2} \left(T^2 - t_1^2 \right) \right\} \right] \tag{15}
 \end{aligned}$$

The present value of the opportunity cost per cycle is

$$\begin{aligned}
 OPC &= \int_{t_1}^T c_4 \left[1 - \frac{1}{\{1 + \delta(T-t)\}} \right] D(t) e^{-rt} dt, \\
 &= \frac{c_4 d \delta}{T^{1/n}} \left[\frac{n T^{\frac{1}{n}+1}}{(1+n)} - \frac{nr T^{\frac{1}{n}+2}}{(1+n)(1+2n)} - t_1^{\frac{1}{n}} T + \frac{1+rT}{(1+n)} t_1^{\frac{1}{n}+1} - \frac{rt_1^{\frac{1}{n}+2}}{(1+2n)} \right] \tag{16}
 \end{aligned}$$

In time dependent demand phenomena of inventory control, demand takes time to come at every unit of item held in stock, that is, every unit have different time of storage. This fact gives us a proper motivation to lay down a time dependent holding cost. Here, we have taken Weibull distributed holding cost. The distribution of the time that an item is to hold has a probability density function $g(t)$ and a cumulative distribution function $G(t)$. The two-parameter Weibull density function is $g(t) = \eta \gamma t^{\gamma-1} \exp(-\eta t^\gamma)$ where η is the scale parameter, $\eta > 0, \gamma$ the shape parameter, $\gamma > 0, t =$ time to storage, $t > 0$.

The holding cost of a unit per unit of time is $\frac{g(t)}{1-G(t)} = \eta \gamma t^{\gamma-1}$.

When $\gamma > 1$, it has an increasing of holding cost of a unit per unit of time; when $\gamma < 1$, it has decreasing rate of holding cost of a unit per unit of time.

If we let $\gamma = 1$, the Weibull distribution is reduced to the constant distribution.

The present value of the holding cost per cycle is

$$\begin{aligned}
 HC &= c_2 \int_0^{t_1} \eta \gamma t^{\gamma-1} I(t) e^{-rt} dt \\
 &= \frac{c_2 \eta \gamma d}{T^{1/n}} \left[\frac{t_1^{\frac{1}{n}+\gamma}}{\gamma(1+n\gamma)} + \frac{\eta t_1^{\frac{1}{n}+2\gamma} (1+n\gamma - 2n^2\gamma + 2n^2\gamma^2)}{2\gamma(1+n\gamma)(1+2n\gamma)} \right. \\
 &\quad \left. - \frac{nrt_1^{\frac{1}{n}+\gamma+1}}{(1+n+n\gamma)} + \frac{\eta rt_1^{\frac{1}{n}+2\gamma+1} (2n^2\gamma - 2n^2\gamma^2 - n\gamma^2 - \gamma)}{(1+\gamma)(1+2\gamma)(1+n\gamma)(1+n+2n\gamma)} \right] \tag{17}
 \end{aligned}$$

Now, according to the position of delay period M there may arise two different cases such as:

Case (1) $M < t_1$ and Case (2) $t_1 \leq M < T$

Case 1: ($M < t_1$)

In this case, the present value of the interest earned per cycle is

$$IE_1 = pI_e \int_0^M D(t)te^{-rt} dt = \frac{pI_e d}{T^{1/n}} \left[\frac{M^{\frac{1}{n}+1}}{1+n} - \frac{rM^{\frac{1}{n}+2}}{1+2n} \right] \tag{18}$$

The present value of the interest paid per cycle is

$$\begin{aligned} IC_1 &= c_p I_c \int_M^{t_1} I(t)e^{-rt} dt \\ &= \frac{c_p I_c d}{T^{1/n}} \left[t_1^{\frac{1}{n}+1} - M^{\frac{1}{n}+1} - \frac{n}{1+n} \left(t_1^{\frac{1}{n}+1} - M^{\frac{1}{n}+1} \right) - \frac{\alpha t_1^{\frac{1}{n}}}{1+\beta} t_1^{\beta+1} - M^{\beta+1} \right. \\ &\quad + \frac{\alpha n}{1+n+n\beta} \left(t_1^{\frac{1}{n}+\beta+1} - M^{\frac{1}{n}+\beta+1} \right) - \frac{rt_1^{\frac{1}{n}}}{2} t_1^2 - M^2 + \frac{rn}{1+2n} \left(t_1^{\frac{1}{n}+2} - M^{\frac{1}{n}+2} \right) \\ &\quad + \frac{\alpha rt_1^{\frac{1}{n}}}{\beta+2} t_1^{\beta+2} - M^{\beta+2} - \frac{\alpha rn}{1+2n+n\beta} \left(t_1^{\frac{1}{n}+\beta+2} - M^{\frac{1}{n}+\beta+2} \right) \\ &\quad + \frac{\alpha}{1+n\beta} \left(t_1^{\frac{1}{n}+\beta} \left(-M \right) - \frac{rt_1^{\frac{1}{n}+\beta}}{2} \left(-M^2 \right) - \frac{n}{(+n+n\beta)} \left(t_1^{\frac{1}{n}+\beta+1} - M^{\frac{1}{n}+\beta+1} \right) \right. \\ &\quad \left. + \frac{rn}{(+2n+n\beta)} \left(t_1^{\frac{1}{n}+\beta+2} - M^{\frac{1}{n}+\beta+2} \right) \right) \Bigg] \end{aligned} \tag{19}$$

Case 2: $t_1 \leq M < T$

The present value of the interest earned per cycle is

$$\begin{aligned} IE_1 &= pI_e \left[\int_0^{t_1} D(t)te^{-rt} dt + M - t_1 - 1 - rt_1 \int_0^{t_1} D(t)dt \right] \\ &= \frac{pI_e d}{T^{1/n}} \left[\frac{t_1^{\frac{1}{n}+1}}{1+n} - \frac{rt_1^{\frac{1}{n}+2}}{1+2n} + M - t_1 - 1 - rt_1 t_1^{\frac{1}{n}} \right] \end{aligned} \tag{20}$$

In this case, there is no interest paid. So, the present value of the interest paid per cycle is

$$IC_2 = 0 \tag{21}$$

Thus, the average total cost $ATC_i(t_1), i = 1, 2$ is given by

$$ATC_i(t_1) = \frac{1}{T} [OC + PC + DC + HC + SC + OPC + IC_i - IE_i], i = 1, 2 \tag{22}$$

Now, our objective is to determine the optimal values of shortage point t_1 in order to minimize the average total cost $ATC_i(t_1), i = 1, 2$. The optimal solutions t_1^* need to satisfy the following equation

$$\frac{dATC_i(t_1)}{dt_1} = 0 \tag{23}$$

The average total cost $ATC_i(t_1)$, $i=1,2$ will be minimum if

$$\left. \frac{d^2 ATC_i(t_1)}{dt_1^2} \right|_{t_1=t_1^*} > 0$$

4. Numerical Examples:

For illustration of the developed model, numerical examples are presented for a single product. To perform the numerical analysis, data has been taken randomly from literatures in appropriate units.

Example 1: We consider an inventory system which verifies the described assumptions above. The input data of parameters is taken randomly as $T=2, \eta=0.6, \gamma=2, \alpha=0.3, \beta=2, n=4, d=60, c_2=20, c_p=10, c_1=3, c_3=5, M=1.3$
 $c_4=14, I_e=0.14, I_c=0.16$ and $r=0.1$. By using MATHEMATICA 8.0, Average Total Cost $ATC_1(t_1)$ with the optimal value of t_1^* is calculated. The Optimal Order Quantity (Q^*) is also calculated. The optimal values are given as follows: $t_1^*=1.4784$, $Q^*=64.6321$, and $ATC_1(t_1^*)=52.2363$. the convexity of the average total cost function is shown in Fig. 3.

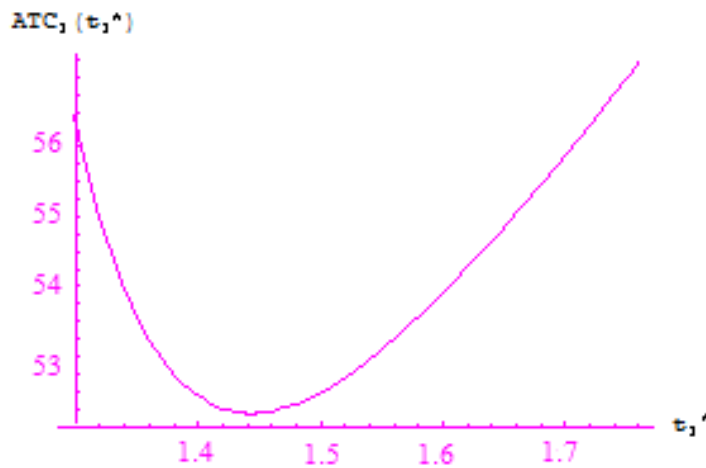


Fig. 2: Convexity of the average cost regarding t_1^* .

Example 2: The input data of parameters is same as in Example 1 except $M=1.7$ By using MATHEMATICA 8.0, Average Total Cost $ATC_1(t_1)$ with the optimal value of t_1^* is calculated. The Optimal Order Quantity (Q^*) is also calculated. The optimal

values are given as follows:

$$t_1^* = 1.6152, Q^* = 72.6421, ATC_2(t_1^*) = 69.2367.$$

5. Sensitivity Analysis

To discuss the effect of changes of model parameters $T, \eta, \gamma, \alpha, \beta, c_1, c_3$ and r on the optimal value of the average total cost, the shortage point and the value of Order Quantity per cycle for case 1, the different values of these parameter according to $\pm 5\%$ and $\pm 10\%$ change in each have taken and its effect on $ATC_1(t_1^*), t_1^*$ and Q^* are presented in the following Table 1. This analysis is based on Example 1.

Table 1

Parameters		t_1^*	Q^*	$ATC_1(t_1^*)$	% changed values	
					% Q^*	% $ATC_1(t_1^*)$
T	+10	1.5343	64.4450	54.0932	-0.28949	+3.55481
	+05	1.5082	64.5345	53.6813	-0.15101	+2.76628
	-05	1.4585	64.7230	51.1217	+0.14064	-2.13377
	-10	1.4221	64.8155	50.2706	+0.28376	-3.76309
η	+10	1.4571	64.2475	52.8238	-0.59506	+1.12470
	+05	1.4625	64.4390	52.5429	-0.29877	+0.58695
	-05	1.4881	64.8267	51.9188	+0.30109	-0.60782
	-10	1.4938	65.0239	51.6153	+0.60620	-1.18883
γ	+10	1.4352	64.1557	52.7680	-0.73710	+1.01787
	+05	1.4543	64.4167	52.5098	-0.33327	+0.52358
	-05	1.4961	64.8908	51.8933	+0.40026	-0.65663
	-10	1.5187	65.1232	51.6964	+0.75984	-1.03357
α	+10	1.3991	64.6840	53.4511	+0.08030	+2.32559
	+05	1.4351	64.6534	52.8425	+0.03296	+1.16050
	-05	1.5127	64.6190	51.6255	-0.02027	-1.16930
	-10	1.5545	64.5934	51.0128	-0.05988	-2.34224
β	+10	1.3669	63.9416	53.4311	-1.06835	+2.28730
	+05	1.4124	64.3483	52.8125	-0.43910	+1.10306
	-05	1.5286	64.9891	51.6055	+0.55236	-1.20759
	-10	1.5698	65.3155	50.9928	+1.05737	-2.38053
c_1	+10	1.4217	63.9649	54.4050	-1.03230	+4.15171
	+05	1.4482	64.3585	53.3903	-0.42332	+2.20919
	-05	1.4950	65.0163	51.2070	+0.59444	-1.97047

	-10	1.5282	65.3321	50.2365	+1.08305	-3.82837
c_3	+10	1.5497	65.6848	56.7483	+1.62876	+8.63767
	+05	1.5071	65.1557	54.4710	+0.81012	+4.27806
	-05	1.4469	64.1732	50.1071	-0.71002	-4.07609
	-10	1.4190	63.6785	48.4160	-1.47543	-7.31350
r	+10	1.4831	64.6581	44.3568	+0.04023	-15.0843
	+05	1.4815	64.6417	48.2874	+0.01485	-7.55969
	-05	1.4766	64.6219	56.1585	-0.01578	+7.50857
	-10	1.4742	64.6125	60.0912	-0.03033	+15.0372

Observations:

1. From Table 1 it is clear that $ATC_1(t_1^*)$ increases with increase in the values of model parameters $T, \eta, \gamma, \alpha, \beta, c_1, c_3$ and r . The obtained results show that $ATC_1(t_1^*)$ is highly sensitive to changes in $T, \alpha, \beta, c_1, c_3$ and r and less sensitive to changes in η and γ .
2. From Table 1 it is observed that $ATC_1(t_1^*)$ decreases with decrease in the values of model parameters $T, \eta, \gamma, \alpha, \beta, c_1, c_3$ and r . The obtained results show that $ATC_1(t_1^*)$ is highly sensitive to changes in $T, \alpha, \beta, c_1, c_3$ and r and less sensitive to changes in η and γ .

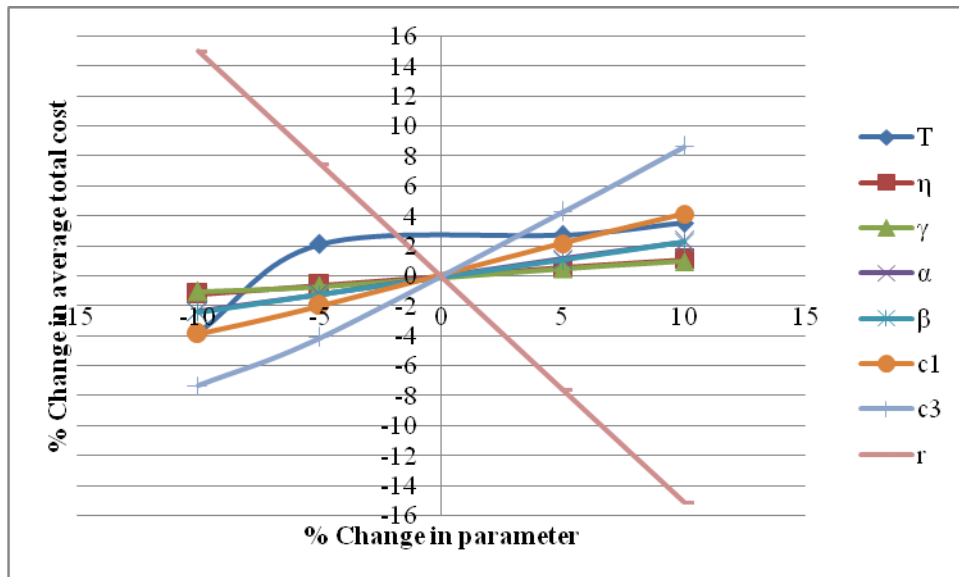


Fig. 3: Behavior of optimal average total cost w.r.t change in parameters

3. From Table 1 it is clear that Q^* increases with increase in the values of model parameters α, c_3 and r while Q^* decreases with increase in the value of T, η, γ, β and c_1 . The obtained results show that Q^* is slightly high sensitive to changes in β, c_1 and c_3 and less sensitive to changes in T, η, γ and α .
4. From Table 1 it is clear that Q^* decreases with decrease in the values of model parameters α, c_3 and r while Q^* increases with decrease in the value of $T, \eta, \gamma, \alpha, \beta$ and c_1 . The obtained results show that Q^* is slightly high sensitive to changes in β, c_1 and c_3 and less sensitive to changes in T, η, γ and α .

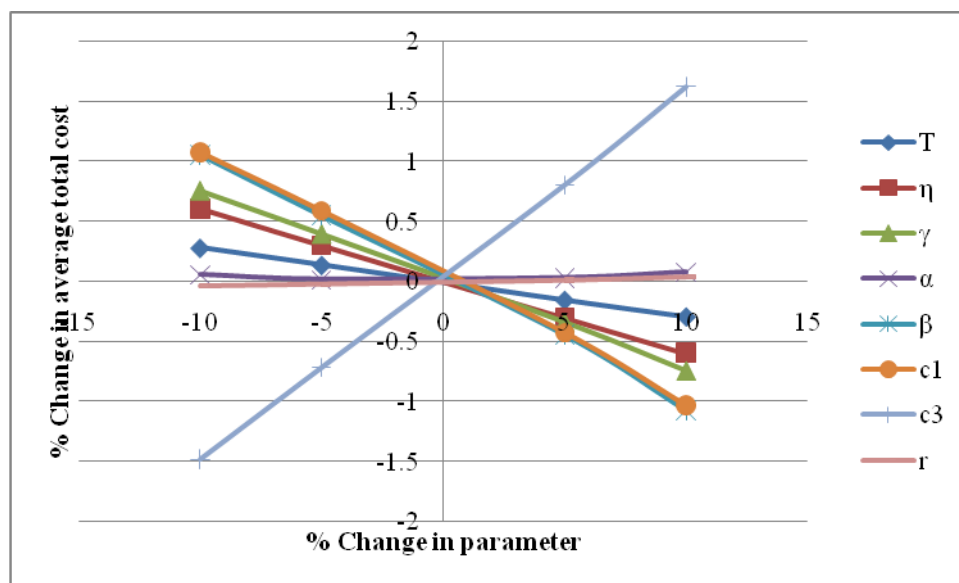


Fig. 4: Behavior of optimal order quantity w.r.t. change in parameters

6. Summary and Conclusions

In this article, an inventory model for deteriorating items with power demand pattern is developed. Shortages are allowed and the partial backlogging rate is a continuous inverse function of waiting time in purchasing the item during stock out period. The whole study is done in inflationary environment and the trade-credit financing facility is taken into consideration. According to the position of delay period there arise two cases: case (1) delay period is lesser than the shortage point and case (2) shortage point is lesser or equal to delay period. In case (1) during the delay period retailer can earn interest on revenue generated by selling his/her products and after delay period the retailer has to interest on remaining inventory. In case (2) since the delay period is greater or equal to the shortage point so the retailer does not has to pay interest and he/she can earn interest during the delay period. Since, this study considers a more practical deterioration rate; variable holding cost, inflation and permissible delay period consequently with these realistic factors the developed model is more practical

and helpful for the inventory decision makers. The further research can be done by considering probabilistic demand and progressive permissible delay period.

7. References

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