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Analytic Hierarchy Process Decision Making Algorithm

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Abstract

Decision making in today's world certainly incorporates the consideration of assessment in view of a number of criteria, instead of a favored single criteria. Solving a multi-criteria decision issue offers decision makers suggestions, regarding the best decision choices (Alternatives). At the point when discovering the best decision of alternatives, subject to various distinctive criteria is almost impossible, the Analytic Hierarchy Process (AHP) has been very instrumental, effective, extraordinary and much of the time utilized strategy in solving problems in much complex decision making processes. This paper briefly discusses Multi-Criteria Decision Making (MCDM) and AHP as one of the most popular MCDM methods for group decision making. Also, steps, techniques and formulae used in AHP have been discussed to help handle the problems arising from choosing alternative(s). Finally, the paper offers recommendations to researchers and professionals to apply AHP methodology techniques when analyzing multiple, complicated and conflicting decision making problems.

Keywords: Decision Making, Multi-Criteria Decision Making (MCDM), Analytic Hierarchy Process (AHP), Criteria and Alternative

1 Introduction

Decision Making involves the process of choosing from available alternatives the possible best course of action. In such situations, a variety of criteria for judging the available alternatives is pervasive. That is, for such situations, the decision maker wants to achieve his/her objective(s) in the course of his/her action while fulfilling the constraints placed by the available resources, environment and processes [1]. However, making decision involves the utilization of knowledge, insight and innovativeness with the purpose of fulfilling fundamental needs or addressing some issues. In the presence of many alternatives to choose from, making a decision requires an evaluation of a number of considerations in order to make the right choice. For example, the advantages derived from settling on the right choice, the expenses, the dangers, and misfortunes that may result from making a wrong decision and some among others have to be assessed before a final decision is made [2]. Multi Criteria Decision Making (MCDM) provides the foundation for making such evaluations in decision-making. MCDM is a branch of decision-making approach that usually deals with multiple, complicated and conflicting criteria. It involves a general class of operations research models that consider problems in decision making in the presence of many decision criteria [3]. Models under MCDM are suitable for evaluating and making decision for the best alternatives (options) in order to choose the perfect criteria.

MCDM is further classified into two main operations research models and these are; Multi Objective Decision Making (MODM) and Multi Attribute Decision Making (MADM). There are several methods in each category and each method has its own characteristics.

Multi-Objective Decision Making, also known by many names including; Multi-Objective Optimization, Multi-Objective Programming, Multi-Criteria Optimization, Pareto Optimization or Vector Optimization, is a method that focuses on problems involving mathematical optimization with two or more objectives functions that need to be optimized at the same time [3]. It utilizes mathematical optimization techniques and generally involves a procedure interrelated with computation design process. In many areas of science, including economics, logistics, engineering and mathematics where ideal or optimal choices need to be made in the presence of multiple conflicting objectives, the application of MODM has been very significant and helpful. For instance, minimizing expenses whilst maximizing comfort while purchasing a car, and maximizing performance whilst minimizing consumption of fuel and pollutants emission of a vehicle are some of the examples of MODM problems involving more objective functions.

Nevertheless, according to [4], MADM method that is one of the decision-making support methods has been found to be the base for decision-making model. This model focuses on a list of chosen criteria, its parameters and variables which one wishes to examine in decision making process. According to [5], the problems in MADM are of significance in many different fields, including economics, engineering and management. It is however obvious that much knowledge in this present world is fuzzy as opposed to precision. Imprecision originates from different sources, such as, unquantifiable data because decision makers are faced with vague or imprecise information about options in connection to attributes. It is quite remarkable that the utilization of statistics and probabilities for conventional correlation analysis has been considered inadequate in handling uncertainties associated with failures in data and modeling. MADM problems have been seen to be far reaching in real life decision-making circumstances and focus on discovery of desirable solution from a limited number of feasible alternatives evaluated on multiple properties, both quantitative and subjective. The MADM has other several classifications including; Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS); Analytical Hierarchy

Process (AHP); Simple Additive Weighting (SAW); Ordered Weighted Averages (OWA); The Simple Multi Attribute Rating Technique (SMART); Elimination et Choice Translating Reality (ELECTRE); among others. In spite of all these methods, AHP method has been considered as the most popular MCDM tool for making decision involving complex and conflicting variables. The Analytic Hierarchy Process can be used for data design, data collection, model development and data analysis tools and techniques [6].

2 AHP Methodology

Decision-making techniques range from dependence on chance (for example; flipping a coin) to the utilization of more organized tools. Sound decision-making takes account of measuring all the significant factors. Decision making in the present world has been inherently perplexing especially when numerous factors must be weighed against contending priorities [2]. That is, the decision maker needs to account for the numerous but often-conflicting objectives and this normally result in a number of alternatives to choose from. One of the tools used in most recent times was developed by [7] in the 1970s to identify, assess, prioritize and evaluate choices made during decision-making is the AHP. This methodology was created to optimize processes through prioritization of variables in making complex decisions especially when one is confronted with a mix of quantitative, qualitative and in some cases problems involving differing factors. In other words, the method focuses on prioritizing selection criteria, and distinguishing the more important criteria from the less important ones. As opined by [8], AHP is simple technique with a focus on pair to pair comparisons that are suitable for assessing both qualitative and quantitative design. Its method has broad application in group decision making and is widely used around the world in different fields including; engineering, business management, government, education, telecommunication, health, and others. According to [9], Analytic Hierarchy Process is made up of suitable techniques for prioritizing critical management problems.

AHP likewise utilizes actual measures like numbers, price, or subjective opinions as inputs into a matrix. The output incorporates ratio scales and consistency indices obtain through computation of principal eigenvectors and eigenvalues. Because human judgment sometimes tends to be subjective in nature, AHP allows some measures of [7]. For instance, inconsistencies emerge when contrasting three items, Orange, Apple, and Mango. Case in point, if Orange is more favored over Apple and Apple is more favored over Mango, then by logical preference (by transitive property) is that, Orange ought to be more favored over Mango. If not, then the comparison is considered as inconsistent.

Also, AHP applies judgment to analyze data. It utilizes the judgments of decision makers to structure decision problems into hierarchies. In order to solve the problem, the problem complexities are then represented by the levels in the hierarchy. The hierarchy is utilized to obtain ratio scaled measures for decision options (alternatives) and also the relative value that the alternatives have against the objectives. That is, AHP constructs ranking of decision items utilizing comparisons or correlations between every pair of items constituted as a matrix. The matched comparisons generate weighting scores that measure the amount of significance items and criteria have with one another. Matrix algebra is then used to sort out variables to arrive at the best decision choice.

3 Steps in AHP Methodology

[6] said, there are four (4) steps of calculation that are considered in AHP and these are; 1. Construction of Structural Hierarchy; 2. Construction of Comparative Judgments (Pair-Wise Comparison Matrices); 3. Weight Determination through Normalization Procedure; and 4. Synthesis of Weight and Consistency Test.

1) Construction of Structural Hierarchy where objective are highlighted and criteria and alternatives identified. A complex decision should be decomposed into a structural hierarchy from the goal to the various criteria and sub-criteria to the very lowest level in descending order. The goal is represented at the topmost level on the hierarchy. Also, the criteria and the sub-criteria are represented at the middle of the hierarchy. Lastly, the alternatives are set down at the very last level on the hierarchy. This is illustrated in the figure 1 below;

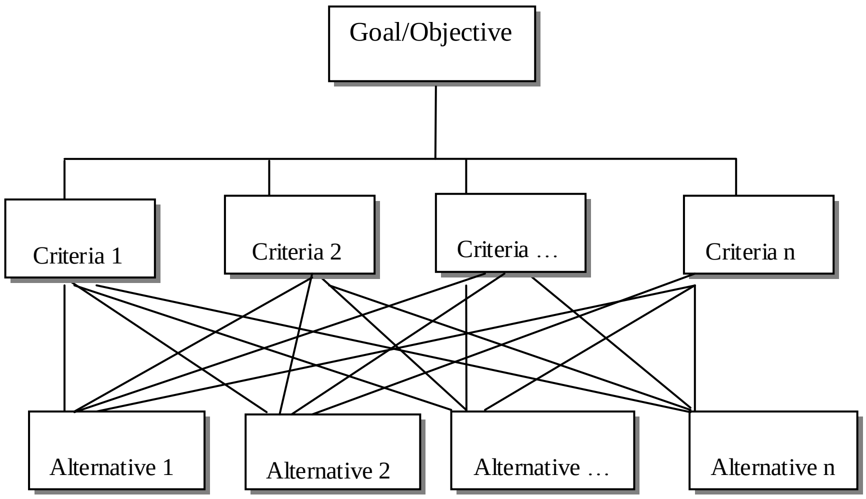

Figure 1: Construction of Structural Hierarchy

From the figure 1, the top level has to do with the goals, the second level contains criteria for ranking and the last level consists of the alternatives [6].

2) Construction of Comparative Judgments (Pair-Wise Comparison Matrices) for all the criteria and alternatives. This pair-wise comparison is adopted from the studies of [10]. Once a hierarchy is constructed, the following step has to do with determining the priorities of the variables at each level by constructing a set of comparison matrices of all the variables in relation to each other. The pair-wise comparison illustrates how much a variable ‘A’ is more favorable or important than variable ‘B’. These logical preferences are measured using an

Opinion Scaling/Pair-wise Comparison Evaluation Scaling from point one-nine scaling (1-9) as shown in table 1.1 below. The matrix is represented mathematically as;

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad (1)$$

Where
A = $\mathrm{a_{ij}}$, $\mathrm{a_{ij}} > 0$ and $\frac{1}{\mathrm{a_{ji}}} = \mathrm{a_{ij}}\ [\mathrm{a_{ij}}]$, where, i, j $= 1, 2, \ldots, n$,
If, $\mathrm{a_{ij}} = 1$ then, $i = j$,
If, $\mathrm{a_{ij}} = \frac{1}{\mathrm{a_{ji}}}$ then, $i \neq j$

If "n" number(s) are given for pair-wise comparison, then AHP performs the above process to determine the weights of criteria. A = n x n, n represents the comparison number of variables, "A" represent the alternatives and a_{11} - a_{1n} , and others in the equation represent the pair-wise comparison(s). The value of the variables that are related to the diagonal of the matrix is equal to 1, such as $a_{ij} = 1$. Based on the given equation it is possible for a preference to be assumed as reciprocal, and this is represented as $a_{ij} = \frac{1}{a_{ji}}$ for, $i \neq j$. For instance, if i-th variable is, X times more favorable than the j-th variable, then, Aij = x, at this point it is assumed that j-th variable(s) is 1/ x, as important between i-th variable $a_{ji} = \frac{1}{x}$ and/or $a_{ji} = \frac{1}{a_{ji}}$. A bi-way scale of 1-9 from extreme favored (9) to equal (1) is used for comparison in order to know the degree of importance [10].

3) Weight Determination through Normalization Procedure. To determine the weights of the criteria and local weight of the alternatives from the pair-wise comparison matrices, each value in a column 'j' is divided by the total of the values in a column 'j'. The total value of the columns in the matrix must be 1, hence, a normalization of the pair-wise comparison matrix [10].This is represented in the equation below;

$$Aw = \begin{bmatrix} \frac{a_{11}}{\sum a_{i1}} & \frac{a_{12}}{\sum a_{i2}} & \cdots & \frac{a_{1n}}{\sum a_{in}} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \frac{a_{n1}}{\sum a_{i1}} & \frac{a_{n2}}{\sum a_{i2}} & \cdots & \frac{a_{nn}}{\sum a_{in}} \end{bmatrix} \quad (2)$$

4) Synthesis of Weight and Consistency Test. Firstly, obtain a global weights of the alternatives through synthesis of the local weights. The eigenvector of matrix A will be determined by calculating C_i as the average and then the C_i as the average values in the row 'i' of Aw matrix will be calculated for the

column vector C where C_i value indicates the relative degree of importance as illustrated below. Nonetheless, Expert Choice Software [11], can also be useful software for analyzing consistency test and calculating the weighting (all the criteria and alternatives) easily.

$$C = \begin{bmatrix} C_1 \\ \\ C_2 \\ . \\ \vdots \\ C_n \end{bmatrix} = \begin{bmatrix} \frac{a_{11}}{\frac{\sum a_{i1}}{n}+} & \frac{a_{11}}{\frac{\sum a_{i1}}{n}+} & \cdots & \frac{a_{1n}}{+\frac{\sum a_{in}}{n}} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \frac{a_{n1}}{\frac{\sum a_{i1}}{n}+} & \frac{a_{n2}}{\frac{\sum a_{i2}}{n}+} & \cdots & \frac{a_{nn}}{+\frac{\sum a_{in}}{n}} \end{bmatrix} \quad (3)$$

Secondly, control the consistency of the weights values (C_i). To do this consistency vector will be calculated (A x C Matrix). After this, x_i will be calculated by multiplying A and C (A x C) to achieve the second, best approximation to the eigenvector. This is shown equation below [10];

$$A \times C = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \times \begin{bmatrix} C_1 \\ \\ C_2 \\ . \\ \vdots \\ C_n \end{bmatrix} = \begin{bmatrix} x_1 \\ \\ x_2 \\ . \\ \vdots \\ x_n \end{bmatrix} \quad (4)$$

Thirdly, estimate the λ_{max}. An estimation of λ_{max} will be calculated using the below formula;

$$\lambda_{max} = \sum_{i=1}^{n} \frac{x_1}{C_1} \quad (5)$$

Where λ_{max} is the eigenvalue of the pair-wise comparison matrix, then approximation to the consistency index (CI) will be calculated. Finally, the consistency judgment for appropriate value of n by CR has to be checked in order to ensure the consistency of pair-wise comparison matrix, as indicated in the representation below;

$$CI = \frac{\lambda_{max} - n}{n-1} \quad (6) \quad \text{and} \quad CR = \frac{CI}{RI} \quad (7)$$

Range of RI is a length of the sequence of (0.00, 0.00, 0.58, 0.09, 1.12, 1.24, 1.32, 1.41, 1.45, 1.49); where RI represent the random consistency index and RI values for different numbers of n from 1 to 10. If CR ≤ 0.10 (10%) then the degree of consistency is satisfactory; but if CR is > 0.10 then, there is indication of serious inconsistencies [10].

4 The Arguments for AHP

Even though AHP has received some criticism for not giving sufficient guidance on structuring a problem to be solved, constructing the levels of the criteria and the alternatives and in cases when members in a team are geographically scattered or are limited by time constraints. Also, AHP has been criticized for “rank reversal” problems (changes in the important ratings that occur at any time criteria and alternatives are added to or deleted from the original set of criteria and alternatives compared). Notwithstanding these critiques, AHP has been most instrumental and very effective method; as AHP technique is able solicit consistent subjective and objective experts' judgment through the consistency test [2]. Next, AHP is regarded as being popular because of its wide use. The nice mathematical property of AHP has attracted many researchers interest and also input data are easy to obtain [12,13]. Based on the survey conducted by [12], it was observed that AHP is the most popular method used for group decision making followed by PROMETHEE and ELECTRE. Moreover, AHP is a time-tried and tested method that has been applied in a number of decisions. The method has been exceptionally powerful in making confused, frequently irreversible decisions [2]. Also, AHP is made up of suitable techniques for prioritizing critical management problems [9]. Furthermore, AHP is intuitive appealing and flexible and many governments and corporations regularly apply the techniques for major policy decisions [14].

5 Conclusion

In this paper, a framework (MCDM-AHP) was proposed to researchers and professionals as the most suitable method when analyzing complex decision making problems. The authors started with a brief discussion on MCDM and AHP as one of the most popular MCDM methods for group decision making. Also, steps, techniques and formulae used in AHP have been discussed to help handle the problems arising from choosing alternative(s). Even though AHP have received some critiques, it was observed that, the Analytic Hierarchy Process (AHP), has been very instrumental, most effective approach, extraordinary and much of the time utilized strategy in solving problems in much complex decision making processes involving a number of criteria and alternatives. This was further demonstrated with evidences from arguments for AHP. In the presence of these evidences based on the methodology processes, along with thorough literature survey by the authors, AHP is strongly recommended to future researchers as the most suitable MCDM method when analyzing complex decision making problems.

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