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On Almost Semi-*I*-Continuous Functions

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Abstract

The aim of this paper is to introduce and characterize a new class of functions called almost semi-I-continuous functions in ideal topological spaces by using semi-I -open sets.[1]

1. INTRODUCTION

The concept of ideals in topological spaces has been introduced and studied by Kuratowski [9] and Vaidyanathaswamy, [21]. An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^*: P(X) \to P(X)$, called the local function [21] of A with respect to τ and I, is defined as follows: for $A \subset X$, $A^*(\tau, I)=\{x \in X | U \cap A \notin I$ for every $U \in \tau(x)\}$, where $\tau(x)=\{U \in \tau | x \in U\}$. A Kuratowski closure operator $\mathrm{Cl}^*(.)$ for a topology $\tau^*(\tau, I)$ called the *-topology, finer than τ is defined by $\mathrm{Cl}^*(A)=A \cup A^*(\tau, I)$ when there is no chance of confusion, $A^*(I)$ is denoted by A^*. If I is an ideal on X, then (X, τ, I) is called an ideal topological space. The aim of this paper is to introduce and characterize a new class of functions called almost semi-I-continuous functions in ideal topological spaces by using semi-I-open sets.

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2. PRELIMINARIES

Let *A* be a subset of a topological space (X, τ). We denote the closure of *A* and the interior of *A* by Cl(*A*) and Int(*A*), respectively. A subset *A* of a topological space (X, τ) is said to be regular open [20] if *A*=Int(Cl(*A*)). A set $A \subset X$ is said to be δ-open [22] if it is the union of regular open sets of *X*. The complement of a regular open (resp. δ-open) set is called regular closed (resp. δ-closed). The intersection of all δ-closed sets of (X, τ) containing *A* is called the δ-closure [22] of *A* and is denoted by $Cl\delta$ (*A*). A point $x \in X$ is called a θ-cluster point of *A* if $Cl(A) \cap A \neq \emptyset$ for every open set *V* of *X* containing *x*. The set of all θ-cluster points of *A* is called the θ-closure of *A* [22] and is denoted by $Cl_\theta (A)$. If $A=Cl_\theta (A)$, then *A* is said to be θ-closed [22]. The complement of θ-closed set is said to be θ-open [22]. A subset A of a topological space (X, τ) is said to be semiopen [10] (resp. preopen [11], β-open [1]) if A ⊂ Cl(Int(A)) (resp. A ⊂ Int(Cl(A)), A ⊂ Cl(Int(Cl(A)))). The set of all regular open (resp. regular closed, δ-open, δ-closed, semiopen, preopen) sets of (X, τ) is denoted by RO(X) (resp. RC(X), δO(X), δC(X), SO(X), PO(X)). A subset S of an ideal topological space (X, τ,I) is called semi-I-open[2] if S ⊂ Cl*(Int(S)).The complement of a semi –I-open set is called a semi-I-closed set [2]. The intersection of all semi-I-closed sets containing S is called the semi-I-closure of S and is denoted by sICl(S). The semi-I-interior of S is defined by the union of all semi- I-open sets contained in S and is denoted by sI Int(S). The set of all semi-I-open sets of (X, τ, I) is denoted by SIO(X). The set of all semi-I-open sets of (X, τ, I) containing a point x ∈ X is denoted by SIO(X, x).

Definition 2.1. A function f: (X, τ) → (Y, σ) is said to be:

(1) semicontinuous [10] if f^{-1} 1(V) is semiopen in X for every open set V of Y;

(2) almost continuous [19] I f^{-1} (V) is open in X for every regular open set V of X;

(3) R-map [5] if f^{-1} (V) is regular open in X for every regular open set V of X;

(4) almost semicontinuous [12] if f^{-1} (V) is semiopen in X for every regular open set V of Y.

Definition 2.2. [3] A function f: (X, τ, I) → (Y, σ, I) is said to be I- irresolute if f^{-1} (V) is semi-I-open in X for every semi-I-open subset V of Y.

Definition 2.3. A function f: (X, τ, I) → (Y, σ) is said to be:

(1) semi-I-continuous [2] if f^{-1} (V) is semi-I-open in X for every open set V of Y,

(2) weakly semi-I-continuous [16] if for each x ∈ X if for each open subset V in Y containing f(x), there exists U ∈ SIO(X, x) such that f(U) ⊂Cl(V).

Definition 2.4. An ideal topological space (X, τ, I) is said to be:

(1) semi-I-T_1 [17] (resp. r-T1 [7]) if for each pair of distinct points x and y of X, there exist semi-I-open (resp. regular open) sets and U and V such that $x \in U$, $y \in U$ and $x \in V$, $y \in V$.

(2) semi-I-T_2 [17] (resp. r-T_2 [7]) if for each pair of distinct points x and y of X, there exist semi-I-open (resp. regular open) sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Lemma 2.5. The following statements are true:

(1) Let A be a subset of a space (X, τ). Then A ∈ PO(X) if and only if s Cl(A)=Int(Cl(A)) [8].

(2) A subset A of a space (X, τ) is β-open if and only if Cl(A) is regular closed [4].

Definition 3.1. *A function f: (X, τ, I) → (Y, σ) is said to be:*

(1) almost semi-I-continuous at a point $x \in X$ if for each open subset V of Y containing f (x), there exists $U \in SIO(X, x)$ such that $f(U) \subset$ Int(Cl(*V(;*

(2) *almost semi-I-continuous if it has this property at each point of X.*

Remark 3.2. Almost semi-I-continuity implies weak semi-I-continuity and it is obvious that almost semi-I-continuity implied by semi-I- continuity. However, the converses of these implications is not true in general as the following examples show.

Example 3.3. *Let X={a, b, c}, τ={∅, {a}, {b}, {a, b}, X}, σ=*{∅, {a}, {a, b}, X} and I={∅, {a}}. Define a function f: (X, τ, I) → (Y, σ) by f (a)=b, f (b)=c and f (c)=a. Then f is almost semi-I-continuous but not semi-I-continuous.

Example 3.4. *Let X={a, b, c}, τ={∅, {a}, {b, c}}, σ={∅, {a},* {b}, {a, b}, X} and I={∅, {a}}. Then the identity function f: (X, τ, I) → (X, σ) is weakly semi-I-continuous but not almost semi-I- continuous.

Theorem 3.5. *For a function f: (X, τ, I) → (Y, σ), the following statements are equivalent:*

(1) *f is almost semi-I-continuous at $x \in X$;*

(2) $x \in \mathrm{Cl}^{\star}(\mathrm{Int}(f^{-1}(s\,\mathrm{Cl}(V()))$ *for every open set V of Y containing f (x);*

(3) $f^{-1}(V) \subset sI\,\mathrm{Int}(f^{-1}(s\,\mathrm{Cl}(V()$ *for every open set V of Y;*

(4) $sI\,\mathrm{Cl}(f^{-1}(s\,\mathrm{Int}(F() \subset f^{-1}(F)$ *for every closed set F of Y.*

Proof.

(1) $\Rightarrow$(2): Let V be an open set of Y containing $f(x)$. Then there exists $U \in SIO(X, x)$ such that $f(U) \subset \mathrm{Int}(\mathrm{Cl}(V(=s\,\mathrm{Cl}(V)$. Then $U \subset f^{-1}(s\,\mathrm{Cl}(V($. Since $U \in SIO(X, x)$, $x \in U \subset \mathrm{Cl}^{\star}(\mathrm{Int}(f^{-1}(U()) \subset \mathrm{Cl}^{\star}(\mathrm{Int}(f^{-1}(s\,\mathrm{Cl}(V()).

(2) $\Rightarrow$(3): Let V be an open set of Y containing $f(x)$ and U an open set of X containing x. Since $x \in \mathrm{Cl}^{\star}(\mathrm{Int}(f^{-1}(s\,\mathrm{Cl}(V())$, we have $x \in f^{-1}(s\,\mathrm{Cl}(V(\cap \mathrm{Cl}^{\star}(\mathrm{Int}(f^{-1}(s\,\mathrm{Cl}(V())=sI\,\mathrm{Int}(f^{-1}(s\,\mathrm{Cl}(V()$ by [[15], Lemma 2.2(a)]. Hence $f^{-1}(V) \subset sI\,\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V)))$.

(3) $\Rightarrow$(1): Let V be an open set of Y containing $f(x)$, then $x \in f^{-1}(V) \subset sI\,\mathrm{Int}(f^{-1}(s\,\mathrm{Cl}(V()$. Set $U=sI\,\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V()$, then $U \in SIO(X, x)$ such that $f(U) \subset s\mathrm{Cl}(V)$.

This shows that f is almost semi-I- continuous at x. (3)$\Leftrightarrow$(4): Clear.

Theorem 3.6. *For a function $f\colon (X, \tau, I) \to (Y, \sigma)$, the following statements are equivalent:*

(1) *f is almost semi-I-continuous;*

(2) $f^{-1}(\mathrm{Int}(\mathrm{Cl}(V(\in SIO(X)$ *for every open set V of Y;*

(3) $f^{-1}(\mathrm{Cl}(\mathrm{Int}(V(\in SIC(X)$ *for every closed set V of Y;*

(4) $f^{-1}(V) \in SIO(X)$ *for every $V \in RO(Y)$;*

(5) $f^{-1}(F) \in SIC(X)$ *for every $F \in RC(Y)$;*

(6) *for each $x \in X$ and each open set V of Y containing $f(x)$ there exists $U \in SIO(X, x)$ such that $f(U) \subset s\,\mathrm{Cl}(V)$;*

(7) $sI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(F() \subset f^{-1}(F)$ *for every closed set F of Y;*

(8) $sI\,\mathrm{Cl}(f^{-1}(A)) \subset f^{-1}(\mathrm{Cl}(A))$ *for every $A \in BO(Y)$;*

(9) $sI\,\mathrm{Cl}(f^{-1}(A)) \subset f^{-1}(\mathrm{Cl}(A))$ *for every $A \in SO(Y)$;*

(10) $f^{-1}(V) \subset sI\,\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V())$ *for every open set $V \in PO(Y)$;*

(11) $f(sI\,\mathrm{Cl}(A)) \subset \mathrm{Cl}_{\delta}(f(A))$ *for every subset A of X;*

(12) $sI\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_{\delta}(B))$ *for every subset B of Y;*

(13) $f^{-1}(F) \in SIC(X)$ *for every $F \in \delta C(Y)$;*

(14) $f^{-1}(V) \in SIO(X)$ *for every $V \in \delta O(Y)$.*

Proof. (4)$\Rightarrow$(5): Let $F \in RC(Y)$. Then $Y \setminus F \in RO(Y)$. Take $x \in f^{-1}(Y \setminus F)$, then $f(x) \in Y \setminus F$ and since f is almost semi-I-continuous, there exists $W_X \in SIO(X, x)$ such that $x \in W_X$ and $f(W_X) \subset Y \setminus F$

Then $x \in W_X \subset f^{-1}(Y \setminus F)$ so that $f^{-1}(Y \setminus F)=\cup W_X$. Since $x\in f^{-1}(Y \setminus F)$ any union of semi-I-open sets is semi-I-open [3], $f^{-1}(Y \setminus F)$ is semi-I- open in X and hence $f^{-1}(F) \in SIC(X)$. (5)$\Rightarrow$ (11): Let A be a subset of X. Since $\mathrm{Cl}_{\delta}(f(A))$ if δ-

closed in Y, it is equal to $\cap \{F_\alpha: F_\alpha$ is regular closed in Y, $\alpha \in \Lambda\}$, where Λ is an index set. From (5), we have $A \subset f^{-1}(\mathrm{Cl}_\delta\ (f\ (A)))=\cap\{f^{-1}(F_\alpha): \alpha \in \Lambda\} \in SIC(X)$ and hence $sI\ \mathrm{Cl}(A) \subset f^{-1}(\mathrm{Cl}_\delta\ (f\ (A)))$. Therefore, we obtain $f\ (sI\ \mathrm{Cl}(A)) \subset \mathrm{Cl}_\delta\ (f\ (A))$. (11)$\Rightarrow$ (12): Set $A=f^{-1}(B)$ in (11), then $f\ (sI\ \mathrm{Cl}(f^{-1}(B))) \subset \mathrm{Cl}_\delta\ (f\ (f^{-1}(B)))$ $\subset \mathrm{Cl}_\delta\ (B)$ and hence $sI\ \mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_\delta\ (B))$.(12)$\Rightarrow$ (13): Let F be δ-closed set of Y, then $sI\ \mathrm{Cl}(f^{-1}(F(\subset f^{-1}(F)$ so $f^{-1}(F) \in SIC(X)$. (13)$\Rightarrow$ (14): Let V be δ-open set of Y, then $Y \setminus V$ is δ-closed set in Y. This gives $f^{-1}(Y \setminus V) \in SIC(X)$ and hence $f^{-1}(V) \in SIO(X)$.(14)$\Rightarrow$ (1): Let V be any regular open set of Y. Since V is δ-open in Y, $f^{-1}(V) \in SIO(X)$ and hence from $f\ (f^{-1}(V(\subset V=\mathrm{Int}(\mathrm{Cl}(V($. Then f is almost semi-I-continuous.(5)$\Rightarrow$ (8): Let A be any b-open set in Y. Since $\mathrm{Cl}(A)$ is regular closed, $f^{-1}(\mathrm{Cl}(A))$ is δ-closed and $f^{-1}(A) \subset f^{-1}(\mathrm{Cl}(A))$. Hence, $sI\ \mathrm{Cl}(f^{-1}(A)) \subset f^{-1}(\mathrm{Cl}(A))$.(8)$\Rightarrow$ (9): obvious.(9)$\Rightarrow$(10): Let V be a preopen set. Then we have $V \subset \mathrm{Int}(\mathrm{Cl}(V($ and $\mathrm{Cl}(\mathrm{Int}(Y \setminus V(\subset Y \setminus V$. Moreover, since the set $\mathrm{Cl}(\mathrm{Int}(Y \setminus V($ is semiopen, it follows that $X \setminus sI\ \mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V())=sI$ $\mathrm{Cl}(X \setminus f^{-1}(\mathrm{Int}(\mathrm{Cl}(V()) =sI\ \mathrm{Cl}(f^{-1}(Y \setminus \mathrm{Int}(\mathrm{Cl}(V())=sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(Y \setminus V()) \subset$ $f^{-1}(\mathrm{Cl}(\mathrm{Int}(Y \setminus V() \subset f^{-1}(Y \setminus V) \subset X \setminus f^{-1}(V)$. Hence, we obtain $f^{-1}(V) \subset sI$ $\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V())$. (10)$\Rightarrow$(4): Let V be a regular open set. Since V is preopen, we get $f^{-1}(V) \subset sI\ \mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V())=sI\ \mathrm{Int}(f^{-1}(V($. Hence $f^{-1}(V) \in SIO(X)$.
The other implications are obvious.

Theorem 3.7. *For a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$, the following statements are equivalent:*

(1) *f is almost semi-I-continuous;*

(2) $sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))))) \subset f^{-1}(\mathrm{Cl}(B)))$ *for every open sub-set B of Y;*

(3) $sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(F()) \subset f^{-1}(F($ *for every closed subset F of Y;*

(4) $sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(V()) \subset f^{-1}(\mathrm{Cl}(V($ *for every open subset V of Y;*

(5) $f^{-1}(V) \subset sI\ \mathrm{Int}(f^{-1}(s\ \mathrm{Cl}(V()$ *for every open subset V of Y;*

(6) $f^{-1}(V) \subset \mathrm{Cl}^\star(\mathrm{Int}(f^{-1}(s\ \mathrm{Cl}(V())$ *for every open subset V of Y;*

(7) $f^{-1}(V) \subset \mathrm{Cl}^\star(\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V()))$ *for every open subset V of Y.*

Proof. (1)$\Rightarrow$(2): Let B be any subset of Y. Assume that $x \in X \setminus f^{-1}(\mathrm{Cl}(B))$. Then $f\ (x) \in Y \setminus \mathrm{Cl}(B)$ and there exists an open set V containing $f\ (x)$ such that $V \cap B=\varnothing$; hence $\mathrm{Int}(\mathrm{Cl}(V(\cap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))=\varnothing$. Since f is almost semi-I-continuous, there exists $U \in SIO(X, x)$ such that $f(U) \subset \mathrm{Int}(\mathrm{Cl}(V($. Therefore, we have $U \cap f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B))))=\emptyset$ and hence $x \in X \backslash sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))))$. Thus, we obtain $sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))))) \subset f^{-1}(\mathrm{Cl}(B)))$. (2)⇒(3): Let F be any closed subset of Y. Then we have $sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F())))) = sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(F()) \subset f^{-1}(\mathrm{Cl}(\mathrm{Int}(F() \subset f^{-1}(F)$. (3)⇒(4): For any open set V of Y, $\mathrm{Cl}(V)$ is regular closed in Y and we have $sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(V()=sI\ \mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(V())) \subset f^{-1}(\mathrm{Cl}(V($. (4)⇒(5): Let V be any open set of Y. Then $Y \backslash \mathrm{Cl}(V)$ is open in Y and we have $X \backslash sI\ \mathrm{Int}(f^{-1}(s\ \mathrm{Cl}(V()=sI\ \mathrm{Cl}(f^{-1}(Y \backslash(s\ \mathrm{Cl}(V() \subset f^{-1}(\mathrm{Cl}(Y \backslash \mathrm{Cl}(V() \subset X \backslash f^{-1}(V)$. Therefore, we obtain $f^{-1}(V) \subset sI\ \mathrm{Int}(f^{-1}(s\ \mathrm{Cl}(V()$. (5)⇒(6): Let V be any open set of Y. Then we obtain $f^{-1}(V) \subset sI\ \mathrm{Int}(f^{-1}(s\ \mathrm{Cl}(V() \subset \mathrm{Cl}^{\star}(\mathrm{Int}(f^{-1}(s\ \mathrm{Cl}(V())$. (6)⇒(1): Let x be any point of X and V any open set of Y containing $f(x)$. Then $x \in f^{-1}(V) \subset \mathrm{Cl}^{\star}(\mathrm{Int}(f^{-1}(s\ \mathrm{Cl}(V())$. It follows from Theorem 3.5 that f is almost semi-I-continuous at any point x of X. Therefore, f is almost semi-I-continuous at any point x of X. (6)⇔(7): Clear.

Theorem 3.8. *(1) A function $f: (X, \tau, \{\emptyset\}) \to (Y, \sigma)$ is almost semi-I-continuous if and only if it is almost semicontinuous.*

Proof. It follows from Proposition 2.4 of [2].

Definition 3.9. [6] *Let A and B be subsets of an ideal topological space (X, τ, I) such that $A \subset B \subset X$. Then $(B, \tau|_B, I|_B)$ is an ideal topological space with an ideal $I|_B=\{\mathbf{I} \in I | \mathbf{I} \subset B\}=\{\mathbf{I} \cap B | \mathbf{I} \in I\}$.*

Lemma 3.10. [2] Let A and B be subsets of an ideal topological space (X, τ, I). If A ∈ SIO(X) and B is open in (X, τ), then A ∩ B ∈ SIO(B).

Theorem 3.11. *Let $f: (X, \tau, I) \to (Y, \sigma)$ be an almost semi-I- continuous function and $A \subset X$. If $A \in \tau$, then $f|_A: (A, \tau|_A, I|_A) \to (Y, \sigma)$* is almost semi-$I|_A$-continuous.

Proof. It follows from Lemma 3.10.

Theorem 3.12. *Let $f: (X, \tau, I) \to (Y, \sigma)$ be a function and $\Lambda=\{U_i: i \in I\}$ be a family such that $U_i \in \tau$ for each $i \in I$. If $f|_{U_i}$ is almost semi-I-continuous for each $i \in I$, then f is almost semi-I- continuous.*

Proof. Suppose that V is any regular open subset of (Y, σ). Since $f|U_i$ is almost semi-I-continuous for each $i \in I$, it follows that $(f|U_i)^{-1}(V)$ is semi-I-open in U_i. We have $f^{-1}(V)=\cup_{i\in I}(f^{-1}(V)\cap U_i)=_{i\in}(f|U_i)^{-1}(V)$ since any union of semi-I-open sets is semi-I-open, $f^{-1}(V) \in SIO(X)$. Hence f is semi-I-continuous.

Definition 3.*13.* *A filterbase Λ is said to be*

(1) *semi-I-convergent to a point x in X if for any $U \in SIO(X, x)$, there exists $B \in \Lambda$ such that $B \subset U$.*

(2) *r-convergent to a point x in X if for any regular open set U of X containing x, there exists $B \in \Lambda$ such that $B \subset U$.*

Theorem 3.14. If a function f: $(X, \tau, I) \rightarrow (Y, \sigma)$ is almost semi- I-continuous, then for each point $x \in X$ and each filter base A in X semi-I-converging to x, the filter base f (A) is r-convergent to f (x).

Proof. Let $x \in X$ and Λ be any filter base in X semi-I-converging to x. Since f is semi-I-continuous, then for any open set V of (Y, σ) containing $f(x)$, there exists $U \in SIO(X, x)$ such that $f(U) \subset V$. Since Λ is semi-I-converging to x, there exists $B \in \Lambda$ such that $B \subset U$. This means that $f(B) \subset V$ and hence the filter base $f(\Lambda)$ is convergent to $f(x)$.

Definition 3.15. A sequence (x_n) is said to be semi-I-convergent to a point x if for every semi-I-open set V containing x, there exists an index n_0 such that for $n \geq n_0$, $x_n \in V$.

Theorem 3.16. If a function f: $(X, \tau, I) \rightarrow (Y, \sigma)$ is almost semi- I-continuous, then for each point $x \in X$ and each net (x_n) which is semi-I-convergt to x, the net $(f(x_n))$ is r-convergent to f (x).

Proof. The proof is similar to that of Theorem 3.14.

Theorem 3.17. *If an injective function f: $(X, \tau, I) \rightarrow (Y, \sigma)$ is almost semi-I-continuous and (Y, σ) is r-T_1, then (X, τ, I) is semi-I-T_1.*

Proof. Suppose that Y is r-T_1. For any distict points x and y in X, there exist regular open sets V and W such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is almost semi-I-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are semi-I-open subsets of (X, τ, I) such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that (X, τ, I) is semi-I-T_1.

Theorem 3.18. *If f: $(X, \tau, I) \rightarrow (Y, \sigma)$ is a almost semi-I-continuous injective function and (Y, σ) is r-T_2, then (X, τ) is semi-I-T_2.*

Proof. For any pair of distinct points x and y in X, there exist disjoint regular open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is almost semi-I-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are semi-I-open sets in X containing x and y, respectively. Therefore, $f^{-1}(U) \cap f^{-1}(V)=\emptyset$ because $U \cap V=\emptyset$. This shows that (X, τ, I) is semi-I-T_2.

Theorem 3.19. If f: $(X, \tau, I) \to (Y, \sigma)$ is a almost continuous function and g: $(X, \tau, I) \to (Y, \sigma)$ is a almost semi-I-continuous function and Y is a r-T_2-space, then the set $E=\{x \in X: f(x)=g(x)\}$ is semi-I-closed set in (X, τ, I).

Proof. If $x \in X \setminus E$, then it follows that $f(x) \neq g(x)$. Since Y is r-T_2, there exist disjoint regular open sets V and W of Y such that $f(x) \in V$ and $g(x) \in W$. Since f is almost continuous and g is almost semi-I- continuous, then $f^{-1}(V)$ is open and $g^{-1}(W)$ is semi-I-open in X with $x \in f^{-1}(V)$ and $x \in g^{-1}(W)$. Put $A=f^{-1}(V) \cap g^{-1}(W)$. By Lemma 3.10, A is semi-I-open in X. Therefore, $f(A) \cap g(A)=\emptyset$ and it follows that $x \notin sI\mathrm{Cl}(E)$. This shows that E is semi-I-closed in X.

Definition 3.20. A function f: $(X, \tau, I) \to (Y, \sigma)$ is said to be faintly semi-I-continuous if for each $x \in X$ and each θ-open set V of Y containing f (x), there exists $U \in SIO(X, x)$ such that $f(U) \subset V$.

Theorem 3.21. *A function f:* $(X, \tau, I) \to (Y, \sigma)$ *is faintly semi- I-continuous if and only if for every θ-closed set V of Y* $f^{-1}(V) \in SIC(X)$.

Theorem 3.22. *The implications* (1) ⇒ (2) ⇒ (3) ⇒ (4) ⇒ (5) *hold for the following properties of a function f:* $(X, \tau, I) \to (Y, \sigma)$*:*

(1) *f is semi-I-continuous.*

(2) $f^{-1}(\mathrm{Cl}_\delta(B))$ *is semi-I-closed in X for every subset B of Y.*

(3) *f is almost semi-I-continuous.*

(4) *f is weakly semi-I-continuous.*

(5) *f is faintly semi-I-continuous.*

If, in addition, Y is regular, then the five properties are equivalent of one another.

Proof. (1) ⇒ (2): Since $\mathrm{Cl}_\delta(B)$ is closed in Y for every subset B of Y, by Theorem 3.6, $f^{-1}(\mathrm{Cl}_\delta(B))$ is semi-I-closed in X.

(2) ⇒(3): For any subset B of Y, $f^{-1}(\mathrm{Cl}_\delta(B))$ is semi-I-closed in X and hence we have $sI\,\mathrm{Cl}(f^{-1}(B)) \subset sI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}_\delta(B))=f^{-1}(\mathrm{Cl}_\delta(B))$. It follows from Theorem 3.6 that f is almost semi-I-continuous.

(3) ⇒(4): This is obvious.

(4) $\Rightarrow$ (5): Let F be any θ-closed set of Y. It follows from 3.21 that $sI\ \mathrm{Cl}(f^{-1}(F(\subset f^{-1}(\mathrm{Cl}_\theta (F(=f^{-1}(F)$. Therefore, $f^{-1}(F)$ is semi-I- closed in X and hence f is faintly semi-I-continuous.

Suppose that Y is regular. We prove that (5) $\Rightarrow$ (1). Let V be any open set of Y. Since Y is regular, V is θ-open in Y. By the faint b-continuity of f, f^{-1} is semi-I-open in X. Therefore, f is semi-I-continuous.

Definition 3.23. *A function f: $(X, \tau, I) \rightarrow (Y, \sigma)$ is said to be semi-I-preopen if $f(U) \in PO(Y)$ for every semi-I-open set U of X.*

Theorem 3.24. If a function f: (X, τ, I) → (Y, σ) is semi-I-preopen and weakly semi-I-continuous, then f is almost semi-I-continuous.

Proof. Let $x \in X$ and let V be an open set of Y containing $f(x)$. Since f is weakly semi-I-continuous, there exists $U \in SIO(X, x)$ such that $f(U) \subset \mathrm{Cl}(V)$. Since f is semi-I-preopen, $f(U) \subset \mathrm{Int}(\mathrm{Cl}(f(U() \subset \mathrm{Int}(\mathrm{Cl}(V($; hence f is almost semi-I-continuous.

Theorem 3.25. Let f: (X, τ, I) → (Y, σ) be a function and g: X → X × Y the graph function defined by g(x)=(x, f (x)) for every x ∈ X. Then g is almost semi-I-continuous if and only if f is almost semi-I- continuous.

Proof. Let x be any point of X and V any regular open set of Y containing $f(x)$. Then we have $g(x)=(x, f(x)) \in X \times V$ is regular open in $X \times Y$. Since g is almost semi-I-continuous, there exists $U \in SIO(X, x)$ such that $g(U) \subset X \times V$. Therefore, we obtain $f(U) \subset V$; hence f is almost semi-I-continuous. Conversely, let $x \in X$ and W be a regular open set of $X \times Y$ containing $g(x)$. There exist a regular open set U_1 in X and a regular open set V in Y such that $U_1 \times V \subset W$. Since f is almost semi-I-continuous, there exist $U_2 \in SIO(X, x)$ such that $f(U_2) \subset V$. Put $U=U_1 \cap U_2$, then we obtain $x \in U \in SIO(X)$ and $g(U) \subset U \times V \subset W$. This shows that g is almost semi-I-continuous.

Theorem 3.26. Let f: (X, τ, I) → (Y, σ, I) and g: (Y, σ, I) → (Z, η) be functions. Then the composition g ∘ f: (X, τ, I) → (Z, η) is almost semi-I-continuous if f and g satisfy one of the following conditions:

(1) *f is almost semi-I-continuous and g is R-map.*

(2) *f is semi-I-irresolute and g is almost semi-I-continuous.*

(3) *f is semi-I-continuous and g is almost continuous*

Proof. Clear.

Definition 3.27. *A topological space (X, τ) is said to be:*

(1) *almost regular* [18] *i f for any regular closed set F of X and any point* $x \in X \backslash F$ *there exist disjoint open sets U and V such that* $x \in U$ *and* $F \subset V$.

(2) *semi-regular if for any open set U of X and each point* $x \in U$ *there exists a regular open set V of X such that* $x \in V \subset U$.

Theorem 3.28. If f: (X, τ, I) → (Y, σ) is a weakly semi-I-continuous function and Y is almost regular, then f is almost semi-I-continuous.

Proof. Let $x \in X$ and let V be any open set of Y containing $f(x)$. By the almost regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset \mathrm{Cl}(G) \subset \mathrm{Int}(\mathrm{Cl}(V($ [[18], Theorem 2.2]. Since f is weakly semi-I-continuous, there exists $U \in SIO(X, x)$ such that $f(U) \subset \mathrm{Cl}(G) \subset \mathrm{Int}(\mathrm{Cl}(V($. Therefore, f is almost semi-I-continuous.

Theorem 3.29. If f: (X, τ, I) → (Y, σ) is an almost semi-I-continuous function and Y is semi-regular, then f is semi-I-continuous.

Proof. Let $x \in X$ and let V be any open set of Y containing $f(x)$. By the semi-regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset V$. Since f is almost semi-I-continuous, there exists $U \in SIO(X, x)$ such that $f(U) \subset \mathrm{Int}(\mathrm{Cl}(G))=G \subset V$ and hence f is semi-I-continuous.

Definition 3.30. *A semi-I-frontier of a subset A of* (X, τ, I), *denoted by* $sIFr(A)$, *is defined by* $sI\,\mathrm{Cl}(A) \cap sI\,\mathrm{Cl}(X\backslash A)$.

Theorem 3.31. The set of all points x ∈ X in which a function f: (X, τ, I) → (Y, σ) is not almost semi-I-continuous is identical with the union of semi-I-frontier of the inverse images of regular open sets containing f (x).

Proof. Suppose that f is not almost semi-I-continuous at $x \in X$. Then there exists a regular open set V of Y containing $f(x)$ such that $U \cap (X\backslash f^{-1}(V(\square = \varnothing$ for every $U \in SIO(X, x)$. Therefore, we have $x \in sI\ \mathrm{Cl}(X\backslash f^{-1}(V(=X\backslash sI\ \mathrm{Int}(f^{-1}(V($ and $x \in f^{-1}(V)$. Thus, we obtain $x \in sIFr(f^{-1}(U($. Conversely, suppose that f is almost semi-I-continuous at $x \in X$ and let V be a regular open set of Y containing $f(x)$.

Then there exists $U \in SIO(X, x)$ such that $U \subset f^{-1}(V)$. That is $x \in sI$ Int$(f^{-1}(V($. Therefore, $x \in X \backslash sIFr(f^{-1}(V($

Theorem 3.32. If g: (X, τ, I) → (Y, σ) is almost semi-I-continuous and S is δ-closed set of X × Y, then p_X (S ∩ G(g)) is semi-I-closed in X, where p_X represents the projection of X × Y onto X and G(g) denotes the graph of g.

Proof. Let S be any δ-closed set of $X \times Y$ and $x \in bI$ Cl$(p_X (S \cap G(g))$. Let U be any open set of X containing x and V any open set of Y containing $g(x)$. Since g is almost semi-I-continuous, we have $x \in g^{-1}(V) \subset bI$ Int$(g^{-1}$(Int(Cl$(V()$) and $U \cap bI$ Int$(g^{-1}$(Int(Cl$(V()))$ $\in SIO(X, x)$. Since $x \in bI$ Cl$(p_X (S \cap G(g))$, $(U \cap bI$ Int$(g^{-1}$(Int(Cl$(V()))\cap p_X (S \cap G(g))$ contains some point u of X. This implies that $(u, g(u)) \in S$ and $g(u) \in$ Int(Cl$(V($. Thus, we have $\varnothing \neq (U \times$ Int(Cl$(V(\cap S \subset$ Int(Cl$(U \times V(\cap S$ and hence $(x, g(x)) \in \mathrm{Cl}_\delta (S)$. Since S is δ-closed, $(x, g(x)) \in p_X (S \cap G(g))$ and $x \in p_X (S \cap G(g))$. Then $p_X (S \cap G(g))$ is semi-I-closed.

Corollary 3.33. *If f: (X, τ, I) → (Y, σ) has a δ-closed graph and g: (X, τ, I) → (Y, σ) is almost semi-I-continuous, then the set {x ∈ X: f (x)=g(x)} is semi-I-closed in X.*

Proof. Since $G(f)$ is δ-closed and $p_X (G(f) \cap G(g))=\{x \in X: f(x)=g(x)\}$ it follows from Theorem 3.32 that $\{x \in X: f(x)=g(x)\}$ is semi-I-closed in X.

Theorem 3.34. *If for each pair of for each pair of distinct x_1 and x_2 in an ideal topological space (X, τ, I) there exists a function f of X into a Hausdorff space Y such that $f(x_1) \neq f(x_2)$, f is weakly semi-I-continuous and f is almost semi-I-continuous at x_2, then X is semi-I-T_2.*

Proof. Since Y is *Hausdorff*, if for each pair of distinct point x_1 and x_2 there exist disjoint open sets V_1 and V_2 of Y containing $f(x_1)$ and $f(x_2)$, respectively; hence Cl$(V_1) \cap$ Int(Cl$(V_2))=\varnothing$. Since f is weakly semi- I-continuous at x_1, there exists $U_1 \in SIO(X, x_1)$ such that $f(U_1) \subset$ Cl(V_1). Since f is almost semi-I-continuous at x_2, there exists $U_2 \in SIO(X, x_2)$ such that $f(U_2) \subset$ Int(Cl(V_2)). Therefore, we obtain $U_1 \cap U_2=\varnothing$. This shows that X is semi-I-T_2.

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