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AFuzzy Interval Weights Approach in Fuzzy Goal Programming for a Multi-Criteria Problem

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Abstract

Goal Programming (GP) is an effective method to solve linear multi-objective problems. The weights play an important role for achieving the solution of the multi-objective programming problem according to the needs and desires of the decision makers (DMs), particularly in uncertain environments. To tackle such uncertain matter on the issue of weights, the proposed approach has taken the interval weights associated to the unwanted deviational variable in goal achievement function as triangular fuzzy numbers. Hence, this study presents a new insight into interval weights to solve linear multi-objective fuzzy GP problems by introducing a defuzzification method based on the Data Envelopment Analysis (DEA) model to defuzzify groups of fuzzy interval weights. This method partitions these fuzzy numbers which cover all possible results in this interval, able to give us best and optimal weights. In the solution process, the interval weights which were derived from several pairwise interval judgment matrices associated with unwanted deviational variables are introduced to the goal achievement function with the objective of minimisation of those deviations, and thus, realise the aspired goal levels of the problem. To illustrate the proposed approach, numerical examples are solved with the resulted real or crisp weights. Animproved optimal solution is achieved when the interval weight is represented as fuzzy numbers with their middle points are geometric means as compared to normal interval weights associated with the maximum and minimum interval under same matrix for each standard GP model and fuzzy GP model.

Keywords: Data Envelopment Analysis, Defuzzification, fuzzy interval weights, fuzzy goal programming, multi-objective problem, uncertain weight.

1. Introduction

The majority of real-world problems are characterized by their multiple objectives, whichmay contradicts one another. In order to determine solutions to these problems, goal programming (GP) was proposed byCharnes and Cooper [1]as an effective method. Nevertheless, from the onset it became a challenge to accurately define the goal values for different objectives in the majority of cases. Hence, to tackle the inaccuracy(uncertainty), Zimmermann [2] proposed the fuzzy programming (FP) approach in the multi-objective linear optimisation problems based on the fuzzy set theory proposed by Zadeh [3]. In FP, a membership function is described on the basis of levels of aspiration and limits of tolerance, with the max-min method utilised to bring about the desired outcome. However, in certain cases, tolerance limits could not be described in dynamic decision situations.To resolve these issues, the GP method in a fuzzy environment (FGP) was introduced by Narasimhan [4]. Since then, FGP has been studied extensively by Hannan [5], Pal et al .[6], and Tiwari et al. [7] and many others. It has been expansively employed in various real-life problems as well(e.g. Biswas and Pal [8]; Pal and Sen [9]).

The relative importance of one objective over another in the context of multi-objective optimisation is defined as the first objective's weight. In this regard, weights are significant in the determination of a solution to a problem of multi-objective programming based on the diverse decision makers' (DMs) subjective requirements.Weights related to unwanted deviational variables in GP gauge the relative significance of the relative objective. Various approaches deriving weights or priorities have been explored in early studies (e.g Chen and Tsai [10]; Srinivasan [11]).. Clearly, fuzzyweights have been used to determine solutions to multi-objective FP problems (Pal et al.[6]). Prior methodologies of GP or FGP defined weights of relative importance as crisp values. While,Bijay and Bhola [12]believed that weights are more realistic when viewed in interval forms as opposed to crisp values.

Owing to the involvement of interval uncertainty in multi-objective programming problems, the interval programming approach is the most ideal method to be applied. Specifically, interval programming based on interval arithmetic was proposed by Inuiguchi and Kume [13] after which it has been examined along with other various methodologies (Oliveira and Antunes [14]). Similarly, the uncertainty in weight structure concept was examined (Saaty and Vargas [15]). Along the same line of study, Sugihara et al.[16] suggested a technique in determining the priorities from a pairwise interval comparison matrix. Contrastingly, interval weights were determined from the interval comparison matrix (Wang and Elhag [17]). However, studies dedicated to examining interval weights related with unwanted deviational variables for GP/FGP situations remain few and far between.

Therefore, in order to shed light on uncertain weight structure, weights related with unwanted deviational variables in the goal achievement function have been deemed as a fuzzy number form in the proposed method and to this end, the fuzzy Interval Goal Programming (FIGP) method is ideally designed to solve this problem. Moreover, the target achievement function is presented as the unwanted deviational variables weighted summation, where weights are regulated through the use of a pairwise interval judgment matrix in GP method. At this point, the issue is in the form of interval programming, interval goals are transformed into standard ones through IGP (Wang and Elhag [17]).The sum of unwanted deviations in relation to respective goals is believed to achieve the desired goal values within a particular range after which the regret function of the final executable model is developed. In other words, the problem is resolved via a standard GP methodology.

The primary advantage of this method is that the suitable weights for achieving goals can be apportioned in the approximate decision environment on the basis of their significance. On the other hand the existed methods focused on interval as (min, max) which includes just two extreme values from all responses. Such as,Sen and Pal [18]proposed a new alternative method that employs interval weights to provide a solution to multi-objective GP problems. Since the weights given by DMs with different opinions, background, and experience, these responses should be treated under fuzzy environment.

Therefore, the need arises to find a method covering all weights coming from DMs. To solve such cases, the interval weight is represented as a triangular fuzzy number with geometric mean as a middle point of this fuzzy number to provide a solution to multi-objective GP problems in this study. Hence, this paper contributes by proposing fuzzy interval weights to determine solutions for multi-objective decision-making (MODM) problems, and this study specifically presents a method to defuzzify these groups of fuzzy numbers representing interval weights with the Data Envelopment Analysis (DEA) tool.

Subsequently, the organization of the rest of the paper is as follows: Section 2 continues with thebackground on defuzzification and Data Envelopment Analysis model, followed with related theoretical framework of GP, and then the proposed model formulation is presented in Section 4.Numerical examples and their resultsin Section 5.Section 6 concludes the paper along with suggested future work.

2. Background

This section is highlight on the concept of defuzzification and DEA model;

2.1 Defuzzification

Defuzzification is considered an important and continuously very active field of research since the early 1990s. Defuzzification converts a fuzzy set to a crisp value Runkler [19]. This process occurs by reducing all fuzzy numbers to a single number (Leekwijck and Kerre [20]; Lee [21]; Sivanandam et al. [22]). Unfortunately, we have no systematic procedure for choosing a good defuzzification strategy, and thus we have to select one considering the properties of application case (Lee [23]). Methods of defuzzification methods are different in solving approaches. Some methods consider all elements and respective weights to produce an integral output. Whereas other methods examinesonly the pick points of the resulting membership functions. The proposed method is suitable when the observed data have some properties that needed to be retained in crisp result.

2.2 Data Envelopment Analysis

DEA is a recognized modern approach that stems from a linear programming (LP) model to evaluate the relative efficiencies of decision making units (DMUs) with multiple inputs and outputs. DEA is a non-parametric technique and was initially proposed by Charnes et al. [24]as a (CCR) model. This model was improved by other scholars, particularly in the form of the Banker et al.'s [25] (BCC) model.Assuming theinputs$x_{ij}(i = 1,2,..,m)$and outputs y_{rj} $(r = 1,2,..,s)$for DMU$_j$ *(j=1, 2... n)*, the programming statement for the CCR model is formulated as follows:

Model (1):

$$\theta_p^* = min\,\theta_p$$

$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_p x_{ip}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp}$$

$$\lambda_j \ge 0 \;\; j = 1, \dots, n$$

$$\theta \; free$$

Where $\lambda_j$is a non-negative value related to the j^{th}DMU. The vector $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^t$constructs a hull that covers all of the data points.Model (1) is divided into three parts, namely, the left- and right-hand sides of the constraints and the objective function. The left-hand side generates the Production Possibility Set (PPS), and retouching this set changes the space. In such case, the DEA model used not to evaluate the DMUs or find the efficient DMU, but the idea is to use all possible DMUs generated by PPS. The right-hand side and the objective function lead DMU*s* to the frontier.

3. Formulation ofGP and Fuzzy Concept

In GP scenarios, the goals are transformed into objectives by incorporating aspiration levels and inserting under-and over-deviational variables into each of them. In the proposed problem, the objective's goals can be constructed from the expression as in formulation (1).

3.1 Goal programming model

In most practical situations, targets are more precisely defined than goals, which are usually fuzzily defined. The generic form of the FGP problem is presented as follows:

$$Z_k(X) \gtrsim \; b_k \qquad , k = 1,2, \dots, K_1 \qquad (1)$$

$Z_k(X) \lesssim b_k \quad ,k = (K_1+1),(K_1+2),..,K$

$s.to$

$X \in S = \left\{X \in R^n \middle| \begin{Bmatrix} AX \geq C \\ AX \leq C \end{Bmatrix}, X \geq 0, C \in R^m \right\}$

3.2 Fuzzy Concept

In this section FP approaches of membership classes are considered, whichdepends on the specified tolerance value offeredin the decision making context.

Let$b_k$be the imprecise aspiration level of the k^{th}objective $Z_k(X)$ $(k = 1,2,\dots,K)$. The fuzzy goals take the form of either $Z_k(X) \gtrsim b_k$ or$Z_k(X) \lesssim b_k$, depending on whether the objectives should be maximised or minimised, where X is the vector of the decision variablesand $\gtrsim$ and $\lesssim$represent the fuzziness of$\geq$and $\leq$ restrictions, respectively Zimmermann [2]. In a decision-making situation, fuzzy goals are characterised by their respective membership functions. The membership functionis $Z_k(X) \gtrsim b_k$; thus, the following membership function, which correspond to each objective function are introduced:

$$\mu_k(x) = \begin{cases} 1 & Z_k(x) \geq b_k \\ \frac{Z_k(x) - L_k}{b_k - L_k} & L_k \leq Z_k(x) \leq b_k \qquad k = 1,2,\dots,K_1 \\ 0 & Z_k(x) < L_k \end{cases} \tag{2}$$

For the restriction, $Z_k(X) \lesssim b_k$, the membership function is obtained from

$$\mu_k(x) = \begin{cases} 1 & (x) \leq b_k \\ \frac{U_k - Z_k(x)}{U_k - b_k} & b_k < Z_k(x) \leq U_k \; k = (K_1+1),(K_1+2),..,K \\ 0 & Z_k(x) > U_k \end{cases} \tag{3}$$

Consequently, the membership goals of the defined membership functions with the highest membership value (unity) are presented as follows:

$$\frac{Z_k(x) - L_k}{b_k - L_k} + \rho_k^- - \rho_k^+ = 1 \qquad k = 1,2,\dots,K_1 \tag{4}$$

$$\frac{U_k - Z_k(x)}{U_k - b_k} + \rho_k^- - \rho_k^+ = 1 \quad ,k = (K_1+1),(K_1+2),..,K \tag{5}$$

where$\rho_k^-, \rho_k^+ \geq 0$ are under and over-deviational variables concerned with achieving the aspired level of the k^{th}membership goal.

4. The Proposed Defuzzification Method Formulation

The formulation of the proposed method to defuzzify groups of fuzzy numbers (fuzzy interval weight)operatesas follows:

4.*1 Determination of Interval Weights as fuzzy number*

Weights of importance of unwanted deviational variables are used to represent the relative importance of the respective criteria. It is more realistic to measure the relative importance in interval form rather than in deterministic values**.**

1. We consider the interval weight $[W^L, W^U]$ as a fuzzy number by fuzzification $W = (W^L, W^{mid}, W^U)$where the lower value,(W^L) is the minimum weight, the upper value, (W^U) is the maximum weight, and (W^{mid}) is the geometric mean of all weights.
2. The interval weight $[W^L, W^U]$is divided into m partitions, namely, with equal width, with each subinterval being of width $\Delta w = \frac{w^u - w^l}{m}$. We label each element in these subintervals as shown $w_k = w^l + k * \Delta w \quad k = 0,1,..,m$. Then the corresponding subintervals are;

$$\{[w_i^l = w_{i0}, w_{i1}],[w_{i1}, w_{i2}], \dots [w_{i\,(m-1)}, w_{im} = w_i^u]\} \; (i = 1,2,..n).$$

where, i is the number of objective.

This method displays the following membership functions for w_{ik} in each interval;

$$\mu_{\tilde{w}}(w) = \begin{cases} L\left(\frac{w - w^l}{w^{mid} - w^l}\right) & w^l \le w \le w^{mid} \\ R\left(\frac{w^u - w}{w^u - w^{mid}}\right) & w^{mid} \le w \le w^u \\ 0 & otherwise \end{cases} \tag{6}$$

3. With these subintervals, *mDMUs* are created. The PPS of these *DMUs* generate all of the possible solutions in the fuzzy interval. In other words, if $i = 1 \; then \; w_1^l = w_{10}, w_{11}, .., w_{1\,(m-1)}, w_{1m} = w_1^u$represents the input values of *DMU_j (j=1, 2… m)* that are used to produce PPS. The single output corresponding to *DMU_j* is assumed to be one. The inputs of each *DMUs*are illustrated in the following Table 1.

Table (1): Illustration of inputs and outputs of DMUs

i j	I_1	I_2	.	.	I_{n-1}	I_n	O_1
DMU_0	$w_1^L = w_{10}$	$w_2^L = w_{20}$	.	.	$w_{n-1}^L = w_{(n-1)0}$	$w_n^L = w_{n0}$	1
DMU_1	w_{11}	w_{21}	.	.	$w_{(n-1)1}$	w_{n1}	1
DMU_2	w_{12}	w_{22}	.	.	$w_{(n-1)2}$	w_{n2}	1
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
DMU_{m-1}	$w_{1(m-1)}$	$w_{2(m-1)}$	.	.	$w_{(n-1)(m-1)}$	$w_{n(m-1)}$	1
DMU_m	$w_1^u = w_{1m}$	$w_2^u = w_{2m}$	.	.	$w_{n-1}^u = w_{(n-1)m}$	$w_n^u = w_{nm}$	1

4. In this step, we propose the following non-linear programming model based on the CCR model (1), but we replace the main objective with n objectives. The number of these objectives depends on the number of fuzzy numbers. Each objective gives an optimal solution $\overline{w}_i$ which has a minimum distance to all values in the fuzzy interval.

Model (2):$min \frac{\sum_{k=0}^{m} \mu_{\tilde{w}_i}(w_{ik})|\overline{w}_i - w_{ik}|}{\sum_{k=0}^{m} \mu_{\tilde{w}_i}(w_{ik})} \quad i = 1,2,\dots,n$

$$\text{s.t.} \sum_{k=0}^{m} \lambda_k w_{ik} \le \overline{w}_i \quad i = 1,2,..,n$$

$$\sum_{k=0}^{m} \lambda_k \ge 1$$

$$\sum_{i=1}^{n} \overline{w}_i = 1$$

$w_i^l \le \overline{w}_i \le w_i^u i = 1,2,\dots,n$

$\lambda_k \ge 0 \qquad\qquad k = 0,1,\dots,m$

Theconstraint $\sum_{i=0}^{n} \overline{w}_i = 1$ presents the relationship among these groups of fuzzy numbers (interval weight). Here, the contribution of propose method which keep this relationship in the crisp result. The fourth constraint includes all intervals of the weight.

5. We assume that $z_{ik} = \overline{w}_i - w_{ik}$ and$|z_{ik}| = z_{ik}^+ + z_{ik}^-$ $\forall(i,k)$. The multi-objective nonlinear programming Model (2) is then proposed as follows:

Model (3): $min \frac{\sum_{k=0}^{m} \mu_{\tilde{w}_i}(w_{ik})(z_{ik}^{+}+z_{ik}^{-})}{\sum_{k=0}^{m} \mu_{\tilde{w}_i}(w_{ik})}$ $i = 1,2, \dots, n$

$$s.t. \sum_{k=0}^{m} \lambda_k w_{ik} \leq \bar{w}_i \qquad i = 1,2,..,n$$

$$\sum_{k=0}^{m} \lambda_k \geq 1$$

$$w_i^l \leq \bar{w}_i \leq w_i^u \quad i = 1,2, \dots, n$$

$$\sum_{i=1}^{n} \bar{w}_i = 1$$

$$\bar{w}_i - w_{ik} - (z_{ik}^{+} - z_{ik}^{-}) = 0 \qquad i = 1,2, \dots, n \quad k = 0,1, \dots, m$$

$$\lambda_k \geq 0 \qquad k = 0,1, \dots, m$$

In order to determine the solution to Model (3) using weighted goal programming model (WGP) following solution approach is taken. Each objective is assumed equally important and allocates equal weight without losing generality, the MOLP model is computed as single objective *n* times, i.e. by considering each objective individually.

5. NumericalExamples and Results

To illustrate the proposed fuzzyinterval method of finding weightscomparing with interval weight for optimal value, the following examples are considered.

Example (1);

$$\max Z_1 = 70x_1 - 30x_2$$

$$\max Z_2 = 3x_1 + 8x_2$$

$$\max Z_3 = -4x_1 + x_2$$

Subjectedto.

$$2x_1 + x_2 \geq 8, \quad x_1 + x_2 \geq 5, \quad x_1 - 2x_2 \geq -6, \quad 5x_1 - 2x_2 \leq 18,$$

$x_1, x_2 \geq 0$

Where, the target and the low tolerance limits of the three objective goals are taken as follows: $(20 \leq Z_1 \leq 250)$, $(20 \leq Z_2 \leq 66)$, $(-18 \leq Z_3 \leq -4)$

5.1 MembershipGoals

The membership goals associated with the objectives can be expressed as follows:

$$(1/230)(70\, x_1 - 30\, x_2 - 20) + \rho_1^- - \rho_1^+ = 1$$

$$(1/46)(3\, x_1 + 8\, x_2 - 20) \quad + \rho_2^- - \rho_2^+ = 1 \quad (7)$$

$$(1/14)(-4\, x_1 + \; x_2 - 18) + \rho_3^- - \rho_3^+ \quad = 1$$

5.2 Weights from Decision Makers

To explain the approach further, weights are used which were collected through the questionnaires constructedfor a particular multi-criteria decision making problem. The questionnaireswere distributed to 20 DMs, whoprovided their opinions anddecisions regarding the importance of the three alternatives through pairwise comparison matrices. The expert choice software was used in computation and the summary of weights is given in Table 2.

Table (2): Summary of the weights

Number of DMs	Weight 1	Weight 2	Weight 3
5 DMs	0.799	0.096	0.105
4DMs	0.511	0.42	0.069
3DMs	0.451	0.49	0.059
2 DMs	0.435	0.487	0.078
2DMs	0.42	0.511	0.069
2DMs	0.785	0.149	0.066
1 DM	0.692	0.077	0.231
1 DM	0.731	0.081	0.188
Max weight	0.799	0.511	0.231
Min weight	0.42	0.077	0.059
Geometric weight	0.603	0.216	0.085

Assuming uncertainty in pairwise judgments, the sum of weights of the obtained intervals can be regarded as the degree of inconsistency included in the given data(Elliott [26]). The imprecise pairwise comparison matrix could be presented by using the formulations defined in Sen and Pal [18] as follows:

$$A^L = \begin{pmatrix} 1 & 0.822 & 1.818 \\ 0.096 & 1 & 0.333 \\ 0.074 & 0.115 & 1 \end{pmatrix} A^U = \begin{pmatrix} 1 & 10.377 & 13.542 \\ 1.217 & 1 & 8.661 \\ 0.550 & 3.0 & 1 \end{pmatrix} \quad (8)$$

5.3 Obtaining Weights via the GP Model

In this section,the weightis obtained as a first step to solve the examples.

5.3.1Obtaining Weights using proposed method

Applying proposed method using model 3 and 4. The fuzzy numbers $W_i = (W_i^L \ ,W_i^{mid} \ ,W_i^U \ )$whichare the representative of the realweights are $w_1 = (0.42{,}0.603{,}0.799)$; $w_2 = (0.077, 0.216 ,0.511)$;$w_3 = (0.059{,}0.085{,}0.231)$

Then we divided each interval to m=100, subintervals until we get a stable weight we will stop the partition process. Ifnot, we increase the number of partitions.The optimal weights are obtained after m=47 partitions, as follows;

$$w_1 = 0.61 \ ; \ w_2 = 0.27; \ w_3 = 0.12 \quad (9)$$

5.3.2 Obtaining Weights using interval weight

By using the pairwise comparison matrix in (8)and the GP model for determination of weights (in interval form) as Sen and Pal [18]presented the weights as intervals using LINGO (ver.14) are as follows:

$$[w_1^L, w_1^U] = [0.54, 0.78] \ ; \ [w_2^L, w_2^U] = [0.10, 0.38] \ ; \ [w_3^L, w_3^U] = [0.08, 0.12] \quad (10)$$

5.4 Solving the FGP

By using the results of weights from (9)and (10), the GP formulation can be expressed as follows:

$$\min G = 0.61\rho_1^- + 0.27\rho_2^- + 0.12\rho_3^- \quad (11)$$

$$\min G = [0.54, 0.78]\rho_1^- + [0.10, 0.38]\rho_2^- + [0.08, 0.12]\rho_3^- \quad (12)$$

to satisfy

$$(1/230)(70\,x_1 - 30\,x_2 - 20) + \rho_1^- - \rho_1^+ = 1$$

$$(1/46)(3\,x_1 + 8\,x_2 - 20) \quad + \rho_2^- - \rho_2^+ = 1 \quad (13)$$

$$(1/14)(-4\,x_1 + \ x_2 - 18) + \rho_3^- - \rho_3^+ \quad = 1$$

$$s.to. \ 2x_1 + x_2 \geq 8, \quad x_1 + x_2 \geq 5, \qquad x_1 - 2x_2 \geq -6, \qquad 5x_1 - 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$

Using interval arithmetic, the objective function (12) can be solved to obtain the target interval as follows: $[t_1^L, t_1^U] = [0.1720497, 0.2567702]$
Using the procedure defined in Sen and Pal [18], the goal expression can be written as follows:

$$(0.54)\rho_1^- + (0.10)\rho_2^- + (0.08)\rho_3^- + \gamma_{1L}^- - \gamma_{1L}^+ \quad = 0.1720497 \tag{14}$$

$$(0.78)\rho_1^- + (0.38)\rho_2^- + (0.12)\rho_3^- + \gamma_{1U}^- - \gamma_{1U}^+ \ = 0.2567702$$

Then, the executable GP model can be expressed as follows:

$$\min G = \gamma_{1L}^- + \gamma_{2U}^+$$

Consequently, the goal relations and set of constraints in (13) and (14) are satisfied.The problem is solved using LINGO (Ver. 14.0) and the solution is obtained as follows:
For proposed method (fuzzy interval weight) the GP and FGP results are $(x_1, x_2) = (6, 6)$, with $(Z_1, Z_2, Z_3) = (250, 66, -18)$.
While the results of interval weight are;
GP the results are $(x_1, x_2) = (3, 2)$, with $(Z_1, Z_2, Z_3) = (150, 25, -10)$.
FGP the results are $(x_1, x_2) = (4, 1)$, with $(Z_1, Z_2, Z_3) = (250, 20, -15)$.

Table (3): Comparison of the objective values obtained under multi-interval and one interval

Optimal objective	X_1	X_2	Z_1	Z_2	Z_3
Under proposed method for GP&FGP	6	6	250	66	-18
Under oneinterval for GP	3	2	150	25	-10
Under oneinterval for FGP	4	1	250	20	-15

The preceding resultsshow that the proposed method to find optimal weight approach is better in achieving the objective values when compared with the conventional interval weight with standard and fuzzy objectives.

Examples (2) and Results

$$\max Z_1 = 2x_1 + 5x_2 + x_3$$

$$\min Z_2 = 4x_1 - 3x_2 + x_3$$

$$\min Z_3 = x_2 - 2x_3$$

s.to.

$4x_1 + x_3 \leq 10, \quad 5x_1 + x_2 \leq 20$

$x_1, x_2, x_3 \geq 0$

Where the target and the low tolerance limits of the three objective goals are taken as follows:

$(50 \leq Z_1 \leq 110)\,, \quad (-60 \leq Z_2 \leq -30), \qquad (-20 \leq Z_3 \leq 20)$

And the membership goals associated with the objectives can be expressed as follows;

$(1/60)(2x_1 + 5x_2 + x_3 - 50) + \rho_1^- - \rho_1^+ = 1$

$(1/30)(-4x_1 + 3x_2 - x_3 - 30) \quad + \rho_2^- - \rho_2^+ = 1$

$(1/40)(20 - x_2 + 2x_3) + \rho_3^- - \rho_3^+ \quad = 1$

By applying the same steps in section 5.3 above, with the samepairwise comparison matrix of weights, the problem is solved and the solution is obtained as follows:
By using proposed methodthe results obtained as;
GP resultsare$(x_1, x_2, x_3) = (0, 20, 0)$, with $(Z_1, Z_2, Z_3) = (100, -60, 20)$.
FGP results are$(x_1, x_2, x_3) = (0, 20, 10)$, with $(Z_1, Z_2, Z_3) = (110, -50, 0)$
Consequently, the problem is solved under interval weight method and the solution is obtained for GP and FGP as follows:

$(x_1, x_2, x_3) = (0, 0, 0)$, with $(Z_1, Z_2, Z_3) = (0, 0, 0)$.

Table (4): Comparison of the objective values obtained under Fuzzy-interval and interval weight

Optimal objective	X_1	X_2	X_3	Z_1	Z_2	Z_3
Under proposed method for GP	0	20	0	100	-60	20
Under proposed method for FGP	0	20	10	110	-50	0
Under oneinterval for GP&FGP	0	0	0	0	0	0

Under different comparison examples and same matrix weights, we obtained the optimal solution of weight by using the proposed method is better than Sen and Pal [18] interval.

6. Conclusions and Future Work

This paper presents a new alternative method which introduces interval weights as a fuzzy number in solving a multi-objective GP problem in a fuzzy manner. This is a novel means of exploiting weights in the form of intervals, but not as the conventionally interval weights version. Our recommended fuzzy interval weights provided optimal values or solutions under any opinion or decision. The proposed approach is advantageous, with the main advantage being that better results in terms of optimal solution can be obtained when the main interval is divided into sub-intervals to get the optimal solution. The numerical examples are given to illustrate the advantages of the method compared to the existing methods. As a consequence, this benefit can be extended to solve other MODM problems in the future in an uncertain decision environment.

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