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Supra N-compact and Supra N-connected in supra Topological spaces

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Abstract

The Notion of this paper is to introduce the new concept called supra N-compactness and supra N-connectedness in supra topological spaces and study some of the properties.

Keywords: supra N-compact, countably supra N-compact, supra N-Lindelof and supra N-connected.

1. Introduction

The concept of supra topology was introduced by A. S. Mashhour et al [4] in the year 1983. They studied about s-continuous functions and s*-continuous functions. In 2008, R. Devi et al [1] introduced the concept of supra α-open sets and supra α-continuous maps. O. R. Sayed and T. Noiri [6] introduced the concept of supra b-irresoluteness and supra b-compactness on Topological space. Jamal. M. Mustafa [2] studied about supra b-compact and supra b-Lindelof spaces. The Author [7] introduced the new set called supra N-closed set on supra Topological space.
In this paper, the author introduces the concept of supra N-compact, countably supra N-compact, supra N-Lindelof and supra N-connectedness and investigate about their relationships using the concept of continuity.

2. Preliminaries

Definition 2. 1 [4]

A subfamily μ of X is said to be supra topology on X if

i) X, φ ∈ μ

ii) If $A_i \in \mu$, $\forall i \in j$ then $\cup A_i \in \mu$

(X, μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c.

Definition 2. 2 [4]

The supra closure of a set A is denoted by cl^μ (A), and is defined as supra cl(A) = ∩ {B : B is supra closed and A ⊆ B}.

The supra interior of a set A is denoted by int^μ (A), and is defined as supra int(A) = ∪ {B : B is supra open and A ⊇ B}.

Definition 2. 3 [4]

Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if τ ⊆ μ.

Definition 2. 4

A subset A of a space X is called

(i) supra semi-open set [3], if $A \subseteq cl^\mu (int^\mu (A))$.

(ii) supra α-open set [1], if $A \subseteq int^\mu (cl^\mu(int^\mu(A)))$.

(iii) supra Ω closed set [5], if $scl^\mu(A) \subseteq int^\mu(U)$, whenever A ⊆ U, U is supra open set.

(iv) supra N-closed set [7] if $\Omega cl^\mu (A) \subseteq U$, whenever A ⊆ U, U is supra α open set.

The complement of the above mentioned sets are their respective open and closed sets and vice-versa.

Definition 2. 5

A map f:(X, τ) → (Y, σ) is said to be

(i) supra N-continuous [7] if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).

(ii) supra N-irresolute [7] if f^{-1} (V) is supra N-closed in (X, τ) for every supra N-closed set V of (Y, σ).

(iii) perfectly supra N-continuous [9] if f^{-1} (V) is supra clopen in (X, τ) for every supra N-closed set V of (Y, σ).

(iv) Strongly supra N-continuous [9] if f^{-1}(V) is supra closed in (X, τ) for every supra N-closed set V of (Y, σ).

(v) totally supra N-continuous [10] if f^{-1} (V) is supra N-clopen in (X, τ) for every supra open set V of (Y, σ).

(vi) totally supra N-closed map [10] if f(V) is supra clopen in (Y, σ) for every supra N-closed set V of (X, τ).

Definition 2. 6 [8]

A supra topological space (X, τ) is supra T_N-space if every supra N-closed set is supra closed in (X, τ).

3. Supra N-compact space

Definition 3. 1

A collection $\{A_i : i \in I\}$ of supra N-open sets in a supra topological space (X, τ) is called supra N-open cover of a subset B of X if $B \subseteq \cup \{A_i : i \in I\}$.

Definition 3. 2

A supra topological space (X, τ) is called supra N-compact if every supra N-open cover of X has a finite sub cover.

Remark 3. 3

Every finite supra topological space is supra compact.

Definition 3. 4

A subset B of a supra topological space (X, τ) is said to be supra N-compact relative to (X, τ) if, for every collection $\{A_i : i \in I\}$ of supra N-open subsets of (X, τ) such that $B \subseteq \cup \{A_i : i \in I\}$, there exist a finite subset I_0 of I such that $B \subseteq \cup \{A_i : i \in I_0\}$.

Definition 3. 5

A subset B of a supra topological space (X, τ) is said to be supra N-compact if B is supra N-compact as a supra subspace of X.

Theorem 3. 6

Every supra N-compact space is supra compact.

Proof

Let (X, τ) be a supra N-compact space. Let $\{A_i : i \in I\}$ be a supra open cover of (X, τ). Hence $\{A_i : i \in I\}$ is a supra N-open cover of (X, τ), since every supra open set is supra N-open set. Since (X, τ) is supra N-compact, supra N-open cover $\{A_i : i \in I\}$ has a finite sub cover $\{A_i : i = 1, 2,... n\}$ of (X, τ). Hence (X, τ) is supra compact.

Theorem 3. 7

A supra N-closed subset of supra N-compact space (X, τ) is supra N-compact relative to (X, τ).

Proof

Let A be a supra N-closed subset of (X, τ). Then A^C is supra N-open in (X, τ). Let $S=\{A_i : i \in I\}$ be a supra N-open cover of A, by supra N-open subsets of (X, τ). Then $S* = S \cup A^c$ is a supra N-open cover of (X, τ). ie., $X = \bigcup_{i\in I} A_i \cup A^c$. By Hypothesis (X, τ) is supra N-compact and hence S* has a finite subcover say $X=(\bigcup_{i=1}^{n} A_i) \cup A^c$. But A and A^c are disjoint. Hence $A \subseteq A_1 \cup A_2 \cup .... \cup A_n$, $Ai \in S$. Thus a supra N-open cover S of A contains a finite subcover. Hence A is supra N-compact relative to (X, τ).

Theorem 3. 8

If a map f:(X, τ) → (Y, σ) be a surjective, supra N-continuous map and if X is supra N-compact, then Y is supra compact.

Proof

Let f:(X, τ) → (Y, σ) be a supra N-continuous map from a supra N-compact space (X, τ) onto a supra topological space (Y, σ). Let $\{A_i : i \in I\}$ be a supra open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is supra N-open cover of (X, τ), since f is supra N-continuous. Since (X, τ) is supra N-compact, then $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2,... n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X)= \bigcup_{i=1}^{n}(A_i)$. Then $Y=\bigcup_{i=1}^{n}(A_i)$ is finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is supra compact.

Theorem 3. 9

If a map f:(X, τ) → (Y, σ) be a surjective, supra N-irresolute map and if X is supra N-compact, then Y is supra N-compact.

Proof

Let f:(X, τ) → (Y, σ) be a supra N-irresolute map from a supra N-compact space (X, τ) onto a supra topological space (Y, σ). Let $\{A_i : i \in I\}$ be a supra N-open cover of (Y, σ). Then $\{f^{-1} (A_i) : i \in I\}$ is supra N-open cover of (X, τ), since f is supra N-irresolute. Since (X, τ) is supra N-compact, then $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2,... n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X)= \bigcup_{i=1}^{n}(A_i)$. Then $Y=\bigcup_{i=1}^{n}(A_i)$ is finite sub cover of $\{Ai : i \in I\}$ for (Y, σ). Hence (Y, σ) is supra N-compact.

Theorem 3. 10

If a map f:(X, τ) → (Y, σ) be a surjective, strongly supra N-continuous map and if X is supra compact, then Y is supra N-compact.

Proof

Let f:(X, τ) → (Y, σ) be a strongly supra N-continuous map from a supra N-compact space (X, τ) onto a supra topological space (Y, σ). Let $\{A_i : i \in I\}$be a supra N-open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is supra open cover of (X, τ), since f is strongly supra N-continuous. Since (X, τ) is supra compact, then $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2,... n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X)= \bigcup_{i=1}^{n}(A_i)$. Then $Y=\bigcup_{i=1}^{n}(A_i)$is finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is supra N-compact.

Theorem 3. 11

If a map f:(X, τ) → (Y, σ) be a surjective, perfectly supra N-continuous map and if X is supra compact, then Y is supra N-compact.

Proof

Let f:(X, τ) → (Y, σ) be a perfectly supra N-continuous map from a supra N-compact space (X, τ) onto a supra topological space (Y, σ). Let $\{A_i : i \in I\}$ be a supra N-open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is supra clopen cover of (X, τ), since f is perfectly supra N-continuous. Since (X, τ) is supra compact, then $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2,... n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X)= \bigcup_{i=1}^{n}(A_i)$. Then $Y=\bigcup_{i=1}^{n}(A_i)$ is finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is supra N-compact.

Theorem 3. 12

Let A be a supra N-compact set relative to a supra topological space X and B be a supra N-closed subset of X. Then A ∩ B is supra N-compact relative to X.

Proof

Let A be supra N-compact set relative to X. Suppose that $\{A_i : i \in I\}$ is a supra cover of A ∩ B by supra N-open sets in (X, τ). Then $\{A_i : i \in I\} \cup \{X - B\}$ is a cover of A by supra N-open sets in X, but A is supra N-compact relative to X, so there exist $\{(A_i) : i = 1, 2,... n\}$ such that $A \subseteq U\{A_i : i = 1, 2,... n\} \cup (X - B)$. Then $A \cap B \subseteq (U \{A_i : i = 1, 2,... n\} \cap B)$. ie., $A \cap B \subseteq U \{A_i \cap B : i = 1, 2,... n\} \subseteq \cup \{A_i : i = 1, 2,... n\}$. Hence A ∩B is supra N-compact relative to X.

4. Countably supra N-compact

Definition 4. 1

A supra topological space (X, τ) is said to be countably supra compact if every countable supra open cover of (X, τ) has a finite subcover.

Definition 4. 2

A supra topological space (X, τ) is said to be countably supra N-compact if every countable supra N-open cover of (X, τ) has a finite subcover.

Theorem 4. 3

If (X, τ) is a countably supra N-compact space, then (X, τ) is countably supra compact.

Proof

Let $\{A_i : i \in I\}$ be a countable supra open cover of (X, τ). Hence $\{A_i : i \in I\}$ is a supra N-open cover of (X, τ). Since (X, τ) is countably supra N-compact, then countable supra N-open cover $\{A_i : i \in I\}$ has a finite sub cover $\{A_i : i = 1, 2... n\}$ of (X, τ). Hence (X, τ) is countably supra N-compact.

Theorem 4. 4

If (X, τ) is a countably supra compact space and supra T_N-space, then (X, τ) is countably supra N-compact.

Proof

Let $\{A_i : i \in I\}$ be a countable supra N-open cover of (X, τ). Since (X, τ) is supra T_N-space, then $\{A_i : i \in I\}$ is a countable supra open cover of (X, τ). Since (X, τ) is countably supra compact, then countable supra open cover $\{A_i : i \in I\}$ has a finite sub cover $\{A_i : i = 1, 2,... n\}$ of (X, τ). Hence (X, τ) is countably supra N-compact.

Theorem 4. 5

Every supra N-compact space is countably supra N-compact.

Proof

Let $\{A_i : i \in I\}$ be a countable supra N-open cover of (X, τ). Since every countable supra N-open cover is supra N-open cover, $\{A_i : i \in I\}$ is supra N-open cover of (X, τ). Since (X, τ) is supra N-compact, then supra N-open cover $\{A_i : i \in I\}$ has a finite sub cover $\{A_i : i = 1, 2,... n\}$ of (X, τ). Hence (X, τ) is countably supra N-compact.

Theorem 4. 6

If the mapping f:(X, τ) → (Y, σ) is supra N-continuous map from a countably supra N-compact space (X, τ) on to a supra topological space (Y, σ), then (Y, σ) is countably supra compact.

Proof

Let {A_i : i ∈ I} be a countable supra open cover of (Y, σ). Since f is supra N-continuous map, {$f^{-1}(A_i)$: i ∈ I} is countable supra N-open cover of (X, τ). Again since (X, τ) is countably supra N-compact space, then {$f^{-1}(A_i)$: i ∈ I} of (X, τ) has a finite subcover {$f^{-1}(A_i)$: i = 1, 2,... n}. Therefore X $=\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X)= $\bigcup_{i=1}^{n}(A_i)$. Then Y=$\bigcup_{i=1}^{n}(A_i)$ is finite sub cover of {A_i : i ∈ I} for (Y, σ). Hence (Y, σ) is countably supra compact.

Theorem 4. 7

If the mapping f:(X, τ) → (Y, σ) is strongly supra N-continuous map from a countably supra compact space (X, τ) on to a supra topological space (Y, σ), then (Y, σ) is countably supra N-compact.

Proof

Let {A_i : i ∈ I} be a countable supra N-open cover of (Y, σ). Since f is strongly supra N-continuous map, {$f^{-1}(A_i)$: i ∈ I} is countable supra open cover of (X, τ). Again since (X, τ) is countably supra compact space, the countable supra open cover {$f^{-1}(A_i)$: i ∈ I} of (X, τ) has a finite subcover {$f^{-1}(A_i)$: i = 1, 2,... n. Therefore X $=\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X)= $\bigcup_{i=1}^{n}(A_i)$. Then Y=$\bigcup_{i=1}^{n}(A_i)$ is finite sub cover of {A_i : i ∈ I} for (Y, σ). Hence (Y, σ) is countably supra N-compact.

Theorem 4. 8

If the mapping f:(X, τ) → (Y, σ) is perfectly supra N-continuous map from a countably supra compact space (X, τ) on to a supra topological space (Y, σ), then (Y, σ) is countably supra N-compact.

Proof

Let {A_i : i ∈ I} be a countable supra N-open cover of (Y, σ). Since f is perfectly supra N-continuous map, {$f^{-1}(A_i)$: i ∈ I} is countable supra open cover and countable supra closed cover of (X, τ). Again since (X, τ) is countably supra compact space, the countable supra open cover {$f^{-1}(A_i)$: i ∈ I} of (X, τ) has a finite subcover {$f^{-1}(A_i)$: i = 1, 2,... n}. Therefore X $=\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X)= $\bigcup_{i=1}^{n}(A_i)$. Then Y=$\bigcup_{i=1}^{n}(A_i)$ is finite sub cover of {A_i : i ∈ I} for (Y, σ). Hence (Y, σ) is countably supra N-compact.

Theorem 4. 9

If the mapping f:(X, τ) → (Y, σ) is supra N-irresolute map from a countably supra N-compact space (X, τ) on to a supra topological space (Y, σ) then (Y, σ) is countably supra N-compact.

Proof

Let {A_i : i ∈ I} be a countable supra N-open cover of (Y, σ). Since f is supra N-irresolute map, {f^{-1} (A_i) : i ∈ I} is countable supra N-open cover of (X, τ). Again since (X, τ) is countably supra N-compact space, the countable supra N-open cover {f^{-1} (A_i) : i ∈ I} of (X, τ) has a finite subcover {f^{-1} (A_i) : i = 1, 2,... n}. Therefore X $=\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X)= $\bigcup_{i=1}^{n}(A_i)$. Then Y=$\bigcup_{i=1}^{n}(A_i)$ is finite sub cover of {A_i : i ∈ I} for (Y, σ). Hence (Y, σ) is countably supra N-compact.

Definition 4. 10

A supra topological space (X, τ) is said to be supra Lindelof space if every supra open cover of (X, τ) has a countable subcover.

Definition 4. 11

A supra topological space (X, τ) is said to be supra N-Lindelof space if every supra N-open cover of (X, τ) has a countable subcover.

Theorem 4. 12

Every supra N-compact space is supra N-Lindelof space.

Proof

Let {A_i : i ∈ I} be a supra N-open cover of (X, τ). Since (X, τ) is supra N-compact, supra N-open cover {A_i : i ∈ I} has a finite sub cover {A_i : i = 1, 2,... n} of (X, τ). Since every finite sub cover is always a countable sub cover, we have {A_i : i = 1, 2,... n}is countable sub cover of {A_i : i ∈ I} for (X, τ). Hence (X, τ) is supra N-Lindelof space.

Theorem 4. 13

If the mapping f:(X, τ) → (Y, σ) is supra N-continuous map from a supra N-Lindelof space (X, τ) on to a supra topological space (Y, σ) then (Y, σ) is supra Lindelof space.

Proof

Let {A_i : i ∈ I} be a supra open cover of (Y, σ). Since f is supra N-continuous map, {f^{-1} (A_i) : i ∈ I} is supra N-open cover of (X, τ). Again since (X, τ) is supra N-Lindelof space, {f^{-1}(A_i) : i ∈ I} of (X, τ) has a countable sub cover{f^{-1} (A_i) : i = 1, 2,... n}. Therefore X $=\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X)= $\bigcup_{i=1}^{n}(A_i)$. Then Y=$\bigcup_{i=1}^{n}(A_i)$ is a countable subcover of {A_i : i ∈ I} for (Y, σ). Hence(Y, σ) is supra Lindelof.

Theorem 4. 14

If the mapping f:(X, τ) → (Y, σ) is supra N-irresolute map from a supra N-Lindelof space (X, τ) on to a supra topological space (Y, σ) then (Y, σ) is supra N-Lindelof space.

Proof

Let $\{A_i : i \in I\}$ be a supra N-open cover of (Y, σ). Since f is supra N-irresolute map, $\{f^{-1}(A_i) : i \in I\}$ is supra N-open cover of (X, τ). Again since (X, τ) is supra N-Lindelof space, $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a countable subcover $\{f^{-1}(A_i) : i = 1, 2,... n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X) = \bigcup_{i=1}^{n}(A_i)$. Then $Y = \bigcup_{i=1}^{n}(A_i)$ is a countable subcover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is supra N-Lindelof.

Theorem 4. 15

If the mapping f:(X, τ) → (Y, σ) is strongly supra N-continuous map from a supra N-Lindelof space (X, τ) on to a supra topological space (Y, σ) then (Y, σ) is supra N-Lindelof.

Proof

Let $\{A_i : i \in I\}$ be a supra N-open cover of (Y, σ). Since f is strongly supra N-continuous map, $\{f^{-1}(A_i) : i \in I\}$ is supra open cover of (X, τ), which implies $\{f^{-1}(A_i) : i \in I\}$ is supra N-open cover of (X, τ). Again since (X, τ) is supra N-Lindelof space, the supra N-open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a countable subcover $\{f^{-1}(A_i) : i = 1, 2,... n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies $f(X) = \bigcup_{i=1}^{n}(A_i)$. Then $Y = \bigcup_{i=1}^{n}(A_i)$ is a countable subcover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is supra N-Lindelof.

Theorem 4. 16

If the mapping f:(X, τ) → (Y, σ) is perfectly supra N-continuous map from a supra Lindelof space (X, τ) on to a supra topological space (Y, σ) then (Y, σ) is supra N-Lindelof.

Proof

Let $\{A_i : i \in I\}$ be a supra N-open cover of (Y, σ). Since f is perfectly supra N-continuous map, $\{f^{-1}(A_i) : i \in I\}$ is supra open cover and supra closed cover of (X, τ). Again since (X, τ) is supra Lindelof space, the supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a countable subcover$\{f^{-1}(A_i) : i = 1, 2,... n\}$. Therefore X

$=\bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies f(X)= $\bigcup_{i=1}^{n}(A_i)$. Then Y=$\bigcup_{i=1}^{n}(A_i)$ is a countable subcover of {A_i : i ∈ I} for (Y, σ). Hence (Y, σ) is supra N-Lindelof.

Remark 4. 17
If (X, τ) is a supra N-Lindelof space and countably supra N-compact space then (X, τ) is supra N-compact.

5. Supra N-connected

Definition 5. 1
A supra topological space X is said to be supra N-connected if X cannot be written as a disjoint union of two non-empty supra N-open sets.

Theorem 5. 2
Every supra N-connected space is connected.

Proof
Suppose (X, τ) is not connected, then X=A∪B where A and B are disjoint non-empty open sets. Since every open set is supra N-open, A and B are disjoint non-empty supra N-open sets, which contradicts that X is supra N-connected. Therefore X is supra connected.
The converse of the above theorem need not be true. It is shown by the following example.

Example 5. 3
Let X={a, b, c} and τ = {X, φ, {a}}. The supra N-open sets in (X, τ) are {X, φ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}. Here (X, τ) is connected but not supra N-connected space, since X = {a} U {b, c}, where {a} and {b, c} are disjoint non-empty supra N-open sets.

Theorem 5. 4
For a supra topological space X, the following are equivalent.
i) X is supra N-connected.
ii) X and φ are the only subsets of X which are both supra N-open and supra N-closed.
iii) Each supra N-continuous map of X into a discrete space Y with atleast two points is a constant map.

Proof
i) ⇒ ii) Let X be supra N-connected subset of X. Let A be any supra N-open and supra N-closed subset of X. Then A^c is both supra N-closed and supra N-open. Then X = A ∪ A^c, a disjoint union of two non-empty supra N-open sets, which contradicts the fact that X is supra N-connected. Hence A = φ or A=X.

ii) ⇒ i) Suppose X=AUB, where A and B are two non-empty disjoint supra N-open sets. Since $A=B^C$, A is supra N-closed set. Hence by our assumption A = φ which is a contradiction. Hence X is supra N-connected.
ii) ⇒ iii) Let f:(X, τ) → (Y, σ) be a supra N-continuous map, where Y is a discrete space with atleast two points. Then f^{-1} ({y}) is supra N-open and supra N-closed for each y ∈ Y and X = U {f^{-1} (y) : y ∈ Y}. By assumption, f^{-1} ({y})=φ or X. If f^{-1} ({y}) = φ for all y ∈ Y then f will not be a map. Also there cannot exist more than one y ∈ Y such that f^{-1} ({y}) = X. Hence there exists only one y ∈ Y such that f^{-1} ({y})= X and f^{-1} ({y_1}) = φ, where y = y_1∈ Y. This shows that f is a constant map.
iii)⇒ii) Let S be both supra N-open and supra N-closed in X. Suppose S = φ. Let f:(X, τ) → (Y, σ) be a supra N-continuous map defined by f(S)={a} and f (S^C) = {b}, where a, b ∈ Y and a = b. By assumption f is constant. Therefore S=X.

Example 5. 5
Let X = {a, b, c} and τ = {X, φ, {a, b}, {a, c}}. supra N-open sets in (X, τ) are {X, φ, {a}, {a, b}, {a, c}}. The supra N-closed sets in (X, τ) are{X, φ, {b}, {c}, {b, c}}. The only subsets of X which are both supra N-open and supra N-closed are X and φ. Hence by theorem 5. 4, (X, τ) is supra N-connected.

Theorem 5. 6
If f:(X, τ) → (Y, σ) be a surjective and supra N-continuous map and X is supra N-connected, then Y is supra connected.

Proof
Suppose Y is not supra connected. Let Y=AUB, where A and B are disjoint non-empty supra open sets in Y. since f is supra N-continuous surjective map, then X= f^{-1} (A) U f^{-1} (B), where f^{-1} (A) and f^{-1} (B) are disjoint non empty supra N-open sets in X. This contradicts the fact that X is supra N-connected. Hence Y is supra connected.

Theorem 5. 7
If f:(X, τ) → (Y, σ) be a surjective and strongly supra N-continuous map and X is supra connected, then Y is supra N-connected.

Proof
Suppose Y is not supra N-connected. Let Y=AU B, where A and B are disjoint non-empty supra N-open sets in Y. since f is strongly supra N-continuous surjective map, then X= f^{-1} (A) U f^{-1} (B), where f^{-1} (A) and f^{-1} (B) are disjoint non empty supra open sets in X. This contradicts the fact that X is supra connected. Hence Y is supra N-connected.

Theorem 5. 8

If f:(X, τ) → (Y, σ) be a surjective and perfectly supra N-continuous map and X is supra connected, then Y is supra N-connected.

Proof

Suppose Y is not supra N-connected. Let Y=AUB, where A and B are disjoint non-empty supra N-open sets in Y. since f is perfectly supra N-continuous surjective map, then X= f^{-1} (A) U f^{-1} (B), where f^{-1} (A) and f^{-1} (B) are disjoint non empty supra open sets and supra closed in X. This contradicts the fact that X is supra connected. Hence Y is supra N-connected.

Theorem 5. 9

If f:(X, τ) → (Y, σ) be a surjective and supra N-irresolute map and X is supra N-connected, then Y is supra N-connected.

Proof

Suppose Y is not supra N-connected. Let Y=AUB, where A and B are disjoint non-empty supra N-open sets in Y. since f is supra N-irresolute surjective map, then X= f^{-1} (A) U f^{-1} (B), where f^{-1} (A) and f^{-1} (B) are disjoint non empty supra N-open sets in X. This contradicts the fact that X is supra N-connected. Hence Y is supra N-connected.

Theorem 5. 10

Suppose(X, τ) is a supra T_N-space and if X is connected then X is supra N-connected space.

Proof

Suppose (X, τ) is not supra N-connected, then X=AUB where A and B are disjoint non-empty supra N-open sets. Since X is a T_N-space every supra N-open set is supra open set. A and B are disjoint non-empty supra open sets, which contradicts that X is supra connected. Therefore X is supra N-connected.

Theorem 5. 11

If f:(X, τ) → (Y, σ) be a surjective and totally supra N-continuous map and X is supra N-connected, then Y is supra connected.

Proof

Suppose Y is not supra connected. Let Y=AUB, where A and B are disjoint non-empty supra open sets in Y. Since every supra open set is supra N-open set, A and B are disjoint non-empty supra N-open set in Y. Since f is totally supra N-continuous surjective map, X= f^{-1} (A) U f^{-1} (B), where f^{-1} (A) and f^{-1} (B) are disjoint non empty supra N-clopen sets in X. This contradicts the fact that X is supra N-connected. Hence Y is supra connected.

Theorem 5. 12

If f:(X, τ) → (Y, σ) be a injective and totally supra N-closed map and Y is supra N-connected, then X is supra N-connected.

Proof

Suppose X is not supra N-connected. Let X=AUB, where A and B are disjoint non-empty supra N-open sets in X. Since f is totally supra N-closed in-jective map, Y=f(A) U f (B), where f (A) and f (B) are disjoint non empty supra clopen sets in Y. Implies f (A) and f (B) are disjoint non empty supra N-clopen sets in Y. This contradicts the fact that Y is supra N-connected. Hence X is supra N-connected.

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