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Sensitivity Analysis of Supply, Demand and Conveyance in a Solid Transportation Problem

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Abstract

In this paper, sensitivity analysis (SA) ofsupply, demand and conveyance parameters of solid transportation problem (STP) is presented. A new solution method namely, zero point bound method is proposed to determine the ranges of supply, demand and conveyanceparameters in solid transportation problem such that its optimal basis is invariant. The procedure and efficiency of this approach are shown with numerical example.The SA of STP by the zero point bound method can support the decision makers to determine what level of accuracy is essential for a parameter to make the model sufficiently useful and valid when they are handling distribution problem having crisp parameters.

Keywords: Sensitivity analysis, Solid transportation problem, Zero point bound method.

1.Introduction

The transportation problem is one of the earliest applications of linearprogramming problems. Transportation models play an important role in logistics and supply-chain management for reducing cost and refining service.In the traditional transportation problem(TP), two kinds of constraints namely, source constraint and destination constraint, are taken into consideration generally. But in the real system, we always deal with other constraints besides of source constraint and destination constraint, such as product type constraint or transportation mode constraint. The STP arises when bounds are given on three item properties namely, supply, demand and conveyance. As a generalization of traditional TP, the STP was introduced by Haley [1].In many industrial problems, a homogeneous product is delivered fromsource to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships, etc. In recent years, the STP received much attention and many models and algorithms under both crisp environment and uncertain environment have been investigated.Pandian and Kavitha [2] proposed a cost sensitivity analysis in STP.

Sensitivity analysis is one of the most interesting and preoccupying areas in optimization. Many attempts are made to investigate the problem's behavior when the input data changes. Usually, variation occurs in the right hand side of the constraints and/or the objective function coefficients. Sensitivity analysis is to analyze the effect of the changes of the objective function coefficients and the effect of changes of the right hand side constraints on the optimal value of the objective function as well as the validity ranges of these effects.Most of sensitivity analysis of a transportation problem is based on the assumption of optimal solution of a transportation problem.Doustdargholi et al. [3] studied the SA of right-hand-side parameter in a TP. Kang-Ting Ma and Ue-Pyng Wen [4] presented support set invariant SA in a degenerate TP.Kavitha and Pandian [5] have studied SA of supply and demand in a fully interval TP.Badra [6] introduced sensitivity analysis of multi-objective transportation problems.Dipankar Chakraborty et al. [7] discussed multi-objective multi-item solid transportation problem with fuzzy inequality constraints.

In this paper, we propose a new method namely, Zero point bound method to improve the sensitivity analysis of supply, demand and conveyance parameters insolid transportation problem. So, we will show that the basis to the solid transportation problem remains optimal when the supply, demand and conveyance vary between the limits.

2. Solid Transportation Problem

Consider m origins, which contain various amounts of a commodity that has to be shipped to n destinations by means of l conveyances. Let a_i be the amount of the commodity available at the origin i; b_j be the amount required at the destination j and e_k be the amount transported by conveyance k. Let c_{ijk} be the cost of shipping a unit amount from the origin i to the destination j by means of the conveyance k. Let x_{ijk} be the amount shipped from the origin i to the destination j by means of the kth conveyance. Our aim is to determine the transportation schedule to minimize the transportation cost satisfying supply, demand and conveyance constraints.

Now, the linear programming model of the above STP is given below:

$$\text{(P1) Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk}$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = a_i,\ i = 1,2,\ldots,m \qquad (1)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = b_j,\ j = 1,2,\ldots,n \qquad (2)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k,\ k = 1,2,\ldots,l \qquad (3)$$

$$x_{ijk} \geq 0, \text{ for all i, j and k} \qquad (4)$$

Any set of non-negative allocations to the problem (P1) which satisfies the equations (1), (2), (3) and (4) is called a feasible solution to the problem (P1).A feasible solution to the problem (P1) which minimizes the total shipping cost $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}x_{ijk}$ is called an optimal solution to the problem (P1). If one of basic variable in the optimal solution of the problem (P1) is zero, that is, the number of non-zero basic variables is less than $m+n+l-2$, then the problem (P1) is called degenerate.

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k$,

the problem (P1) is said to be balanced. Otherwise, it is called unbalanced. The balanced condition is the necessary and sufficient condition for the existence of a feasible solution to the problem (P1).If l = 1, the number of conveyances is only one, the problem (P1) reduces to a classical transportation problem.

3. Sensitivity Analysis

Sensitivity analysis is used to determine how “sensitive” a model is to change in the value of the parameters of the model and to change in the structure of the model. Parameter sensitivity allows decision makers to determine what level of precision is necessary for a parameter to make the model sufficiently useful and valid. By showing how the model behavior responds to changes in parameter values, sensitivity analysis is a useful tool in model building as well as in model evaluation and it can also specify which parameter values are reasonable to use in the model.

3.1Zero Point Bound Method

We, now introduce a new method namely, Zero Point Bound method to study the sensitivity analysis of supply, demand and conveyance in STP.

The zero point bound method proceeds as follows.

Step 1. Compute an optimal solution to the given STP using the Pandian-Anuradhamethod (PA method) [8].

Step 2. Now, to analyze the sensitivity in solid transportation problem, after determining the optimal solution. In the allotment table of the solid transportation problem, we replace a_i by $a_i + \alpha_i$, i =1,2,..., m; $i \neq t$ and a_t by $a_t - \sum_{\substack{i=1 \\ i \neq t}}^{m} \alpha_i$, we replace b_j by $b_j + \mu_j$, j =1,2,...,n; $j \neq s$ and b_s by $b_s - \sum_{\substack{j=1 \\ j \neq s}}^{n} \mu_j$ and alsowe replace e_k by $e_k + \beta_k$, k =1,2,..., l; $k \neq r$ and e_r by $e_r - \sum_{\substack{k=1 \\ k \neq r}}^{l} \beta_k$.

Step 3. Compute the minimum and maximum values of α_i , μ_j *and* β_k, for all i, j *and* k , using the allotment conditions of zero point method[9] to maintain the current optimal basis. Then, find the ranges of supply, demand and conveyancevalues in the solid transportation problem.

Step 4. From Step 1. to Step 4., we obtain the ranges of the supply, demand and conveyance in the solid transportation problem to maintain the current optimal basis.

The zero point bound method is illustrated with help of the following numerical example.

4. Numerical Example

Consider the following solid transportation problem:

										Demand
Conveyance	D_1			D_1			D_1			7
		D_2			D_2			D_2		15
			D_3			D_3			D_3	12
	E_1			E_2			E_3			Supply
O_1	4	3	6	7	9	7	8	7	2	11
O_2	4	1	8	2	3	4	6	8	5	13
O_3	8	4	5	1	7	6	3	3	4	10
Capacity	11			14			9			

Now, by step 1. the optimal solution to the given STP is $x^{\circ}_{121} = 2$, $x^{\circ}_{133} = 9$, $x^{\circ}_{221} = 9$, $x^{\circ}_{222} = 4$, $x^{\circ}_{312} = 7$ and $x^{\circ}_{332} = 3$. Since the number of basic cells $= 6 < 7 = m + n + l - 2$, the given STP is degenerate.

Now, Using step 3. replace the parameters for theoptimal solution to the STP.

										Demand
Destination	D1			D1			D1			$7+\mu_1$
		D2			D2			D2		$15+\mu_2$
			D3			D3			D3	$12-\mu_1-\mu_2$
	E1			E2			E3			Supply
O1	5	0	2	6	4	1	11	6	0	$11+\alpha_1$
O2	7	0	6	3	0	0	11	9	5	$13+\alpha_2$
O3	9	1	1	0	2	0	6	2	2	$10-\alpha_1-\alpha_2$
Capacity	$11+\beta_1$			$14+\beta_2$			$9-\beta_1-\beta_2$			

Now, Using Step 3. the allotment conditions of the zero point method related to supply, we have

Supply-Demand conditions:

$$11+\alpha_1 \le 15+\mu_2+12-\mu_1-\mu_2 \Rightarrow \alpha_1 \le 16-\mu_1$$
$$13+\alpha_2 \le 15+\mu_2+12-\mu_1-\mu_2 \Rightarrow \alpha_2 \le 14-\mu_1$$
$$10-\alpha_1-\alpha_2 \le 7+\mu_1+12-\mu_1-\mu_2 \Rightarrow -\alpha_1-\alpha_2 \le 9-\mu_2$$

Supply-Conveyance conditions:

$$11+\alpha_1 \le 11+\beta_1+9-\beta_1-\beta_2 \Rightarrow \alpha_1 \le 9-\beta_2$$
$$13+\alpha_2 \le 11+\beta_1+14+\beta_2 \Rightarrow \alpha_2 \le 12+\beta_1+\beta_2$$
$$10-\alpha_1-\alpha_2 \le 14+\beta_2 \Rightarrow -\alpha_1-\alpha_2 \le 4+\beta_2$$

(5)

Now, Using Step 3. the allotment conditions of the zero point method related to demand, we have

Demand-Supply conditions: $7+\mu_1 \le 10-\alpha_1-\alpha_2 \Rightarrow \mu_1 \le 3-\alpha_1-\alpha_2$ $15+\mu_2 \le 11+\alpha_1+13+\alpha_2 \Rightarrow \mu_2 \le 9+\alpha_1+\alpha_2$ $12-\mu_1-\mu_2 \le 11+\alpha_1+13+\alpha_2+10-\alpha_1-\alpha_2 \Rightarrow -\mu_1-\mu_2 \le 22$
Demand-Conveyance conditions: $7+\mu_1 \le 14+\beta_2 \Rightarrow \mu_1 \le 7+\beta_2$ $15+\mu_2 \le 11+\beta_1+14+\beta_2 \Rightarrow \mu_2 \le 10+\beta_1+\beta_2$ $12-\mu_1-\mu_2 \le 14+\beta_2+9-\beta_1-\beta_2 \Rightarrow -\mu_1-\mu_2 \le 11\text{-}\beta_1$

(6)

Now, Using Step 3. the allotment conditions of the zero point method related to conveyance, we have

Conveyance-Demand conditions: $11+\beta_1 \le 15+\mu_2 \Rightarrow \beta_1 \le 4+\mu_2$ $14+\beta_2 \le 7+\mu_1+15+\mu_2+12-\mu_1-\mu_2 \Rightarrow \beta_2 \le 20$ $9-\beta_1-\beta_2 \le 12-\mu_1-\mu_2 \Rightarrow -\beta_1-\beta_2 \le 3-\mu_1-\mu_2$
Conveyance-Supply conditions: $11+\beta_1 \le 11+\alpha_1+13+\alpha_2 \Rightarrow \beta_1 \le 13+\alpha_1+\alpha_2$ $14+\beta_2 \le 13+\alpha_2+10-\alpha_1-\alpha_2 \Rightarrow \beta_2 \le 9-\alpha_1$ $9-\beta_1-\beta_2 \le 11+\alpha_1 \Rightarrow -\beta_1-\beta_2 \le 2+\alpha_1$

(7)

Now, since availability at each supply point, the requirement at each demand point and mode of transport at each conveyance point are non-negative, we have the following:

$$11+\alpha_1 \ge 0 ; 13+\alpha_2 \ge 0 ; 10\text{-}\alpha_1-\alpha_2 \ge 0 ; 7+\mu_1 \ge 0 ; 15+\mu_2 \ge 0 ;$$

$$12\text{-}\mu_1-\mu_2 \ge 0 ; 11+\beta_1 \ge 0 ; 14+\beta_2 \ge 0 ; 9\text{-}\beta_1-\beta_2 \ge 0 . \qquad (8)$$

Now, using the equations (5), (6), (7) and (8), we get the minimum and maximum values of α_i , μ_j *and* β_k, for all i, j *and* k .

$$-11 \le \alpha_1 \le 23 ; -13 \le \alpha_2 \le 21 ; -24 \le \alpha_1+\alpha_2 \le 10 ;$$

$-7 \le \mu_1 \le 27 ; -15 \le \mu_2 \le 19 \; ; \; -22 \le \mu_1 + \mu_2 \le 12 \; ;$
$-11 \le \beta_1 \le 23 \; ; \; -14 \le \beta_2 \le 20 ; \; -25 \le \beta_1 + \beta_2 \le 9 .$

Now, using step 4.we obtain the ranges of all supply, demand and conveyance in the given solid transportation problem such that its optimal basis is invariant.

$0 \le S_1 \le 34 \; ; \; 0 \le S_2 \le 34 \; ; \; 0 \le S_3 \le 34 \; ; \; 0 \le D_1 \le 34 \; ;$
$0 \le D_2 \le 34 ; 0 \le D_3 \le 34 \; ; \; 0 \le E_1 \le 34 \; ; \; 0 \le E_2 \le 34 \; ; \; 0 \le E_3 \le 34$

5. Conclusion

We discuss the sensitivity analysis of supply, demand and conveyance in the solid transportation problem in this paper. We propose a method namely, zero pint bound method for finding a critical region of the supply, demand and conveyanceparametersat which any change inside the ranges of the region does not disturb the optimal basis, while, any change outside their ranges will disturb the optimal basis.In general, information of sensitivity analysis in a solid transportation problem is usually more important than the optimal solution itself. The sensitivity analysis of supply, demand and conveyance parameters in solid transportation problem by the proposed method can help the decision makers to know in what range of variation of sources in the market they can keep the installed production lines active, and only production's levels would change when they are handling distribution problems having crisp parameters.

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