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An Entropic EOQ Inventory Control Using Dynamic Programming

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Abstract:

There are a variety of efficient approaches to solve crisp inventory models in operations Research. This paper presents an entropic version of an EOQ model with imperfect quality items. A model that uses Bellman & Zadeh's approach to fuzzy dynamic programming is used. As a result, an entropy cost term is added to the classical inventory cost to form an entropic total inventory cost function. This provides an estimation of the hidden or difficult to estimate cost inventory systems that usually are the result of disorder (or entropy). A mathematical model is developed with numerical results presented and discussed.

Keywords: Fuzzy dynamic Programming (FDA), inventory control, EOQ, imperfect quality, Entropy price/quantity.

1. Introduction:

The earliest inventory control models were developed in the stochastic environment such as economic order quantity model, which is applicable when the demand of an item has a constant or nearly constant rate. The aim in any inventory model is to find theamount that should be ordered each period so that it would minimize the total cost, consisting of ordering and holding costs. However, like many other systems, inventory control includes the amount of uncertainty and as such can be modelled more efficiently by using Fuzzy logic modelling techniques. The first approach of fuzzy inventory control was introduced by zadeh in 1965. In literature, various types of fuzzy inventory models were introduced and discussed by many researchers. [1-7]

Recent works assumed that items of imperfect quality (defective) are not reworked but rather salvaged in a secondaryg market at a discounted price. Another critique of the EOQ model is that many of its cost parameters are difficult to estimate, which gives rise to some hidden costs. Jaber et al postulated that an improvement in the performance of production systems can be attained by modelling them as inventory systems. Using this concept, they were able to develop an entropy cost function that they added as a third term to the EOQ inventory cost function(the sum of the order and holding costs). This allows measuring the cost of pushing one unit of commodity from the system to its environment(market). [8-10] Disorder (or entropy) in the system can appear in the form of "queues of materials, waiting to be processed, machines waiting for service, finished products waiting for customers.

This paper will present fuzzy dynamic programming techniques for modelling inventory, as a new and challenging approach. The drawbacks of fuzzy dynamic programing are that this is a method of solving problems exhibiting the properties of overlapping subproblems that takes much less time than some naive methods. It is organised as follows. The next section provides a background of thermodynamic version of the model of Salameh and Jaber et al and develops an entropy price-quality of the model of Maddah and Jaber. This section is followed by section 3, which gives numerical examples and discussion of results. The paper summarizes and concludes in section 4.

2. Traditional Dynamic Programming:

Traditional dynamic programming is a technique introduced first by Bellman in 1957. This technique is very known technique for solving large optimization problem that can be break up into small; Once all the sub-problems have been, we are left with an optimal solution to the large problem. [11-16] Each of the smaller problems is identified with a stage of the dynamic progrmming solution procedure. Basically the problem is formulated interms of state variables $x_{n,}$ representing the amount of inventory on hand at the beginning of stage n=1, 2,.... N: decision variables C_{s_n} , representing the production quantity for stage n =1, 2,.... N; stage rewards, R_n:a reward function $R_n(d_n, \ldots\ldots d_{n-1}, x_N)$; and a transformation function $t_n(p_n, C_{s_n})$. The problem is solved by solving recursively the following[17-20]

$$\text{Max } C_{s_n} \; R_n(p_n, C_{s_n}) = \text{Max } C_{s_n} \; R_n(p_n, C_{s_n}) * R_{n+1}(p_{n+1})$$

Such that

$$P_{n+1} = t_n(p_n, C_{s_n}) \quad n= 1, 2,\ldots. N-1$$

3. Fuzzy Dynamic Programming:

Fuzzy dynamic programming was suggested first by Bellman & Zadeh in 1970. They based their considerations on the symmetrical model of a decision (Zimmermann, 2001:348)[21-25]

Let $\overline{x_n} \in \overline{X}$ $\quad n = 1,2,\ldots..N-1$ be defined as state variable where $\overline{X} = (\tau_1, \tau_2, \tau_3 \ldots\ldots\ldots \tau_N)$ be the set of values permitted for the state variables;

$C_{s_n} \in \overline{C}$, $S_n = 0,1\ldots\ldots\ldots\ldots N$ be defined as decision variable where $\overline{C} = \{\alpha_1, \alpha_2, \alpha_3, \ldots\ldots\ldots\alpha_N\}$ be the set possible decisions.

$P_{n+1} = t_n(p_n, C_{s_n})$ be the transformation function.

For stage t, t=1, 2,..... N we define [26-30]

- A fuzzy constraint $\overline{C_{t_1}}$ limiting the decision space and characterized by its membership function $\mu_{\overline{C_{t_1}}}(C_{s_n})$
- A fuzzy goal $\overline{G_{s_n}}$ characterized by the membership function $\mu_{\overline{Gn}}(C_{s_n})$

The problem is to determine the maximizing decision $\overline{C^{\circ}} = \{\overline{C_{s_n}{}^{\circ}}\}$

N = 1, 2, 3.... N for a given P_0.

A Fuzzy set decision is the confluence of the constraints and goals and its membership function is defined by minimum operator

$\overline{C} = \bigcap_{t=0}^{N-1} \overline{C_{t_1}} \cap \overline{G_N}$ e optimal decision

$\mu_{\bar{c}}(C_{s0}, C_{N-1}) = \min\{\mu_{\bar{c0}}(C_{s0}),......\mu_{\overline{C_{N-1}}}(C_{N-1}), \mu_{\bar{G_N}}(P_n)\}$

The membership function of the maximizing decision is then

$\mu_{\overline{C0}}(C_{S_0}{}^{\circ},.......C_{S_{N-1}}{}^{\circ}) = [Min\{\mu_{\overline{C0}}(C_{S_0}.....\mu_{d_{N-1}} \max_{C_{S0},....C_{SN-2}} \max C_{s_{N-1}}(C_{s_{N-1}}), \mu_{\overline{GN}}(t_N(P_{N-1}, C_{s_{N-1}}))\}]$

Where C_{s_N} represents the optimal decision on stage n.

Mathematical model for fuzzy dynamic programming applied in inventory control:

Let us assume that we have the following problem. A company needs to close down a certain plant within a definite time intervl. The constraint is that the production level should be decreased as steaily as possible over this period. Therefore, the goal and constraints can be expressed as a fuzzy numbers, characterized by its membership function. In this case the demand is assumed to be crisp. The problem is set as follows. Let $C_{s_n} \in \overline{C}$, $n = 0,1,2,......N$ be the decision variable representing the production level in period n. [31, 32] $\overline{C} = \{\alpha_1, \alpha_2,......\alpha_n\}$ be the set of the decisions allowed, a fuzzy set $P_n \in \overline{P}$, $n = 0,1,2,.....N$ be the state variable representing the stock level at the beginning of period n, $\overline{P} = \{\tau_1, \tau_2, \tau_3,......\tau_m\}$ be the set of state possible value a fuzzy set $S_n = 1,2,........N$ be the crisp demand in period n $P_{n+1} = P_n + C_{s_n} - S_n$ be the crisp transformation function. [33, 34] $\overline{D_n}(C_{1s_n}) = \{(C_{s_n}, \mu_{\overline{D_n}}(C_{s_n})\}$ be the fuzzy constraints representing 'Production should decrease as smoothly as possible' $\overline{G_n}(x_{N+1}) = \{x_{N+1}, \mu_{\bar{G}}(x_{N+1})\}$ be the fuzzy goal representing 'to have as low inventory level as possible'[35]

In this case the demand is assumed to be crisp. However, objective function as well as constraints can be non-crisp and therefore they are defined by their membership functions. The aim is to maximize the goal within the ranges specified for the constraints according to mathematial expression.

5. Application:

Let the constraint be represented by the following membership function [Zimmerman (2001, 428)] [36]

$$\mu_{D_n}(C_{s_n}) = \begin{cases} 0 \le C_{s_n} \le 50-10 \\ -4+0.5n+\dfrac{C_{s_n}}{20} & if\ 50-10n \le C_{s_n} \le 70-10n \\ 5-0.5n-\dfrac{C_{s_n}}{20} & if\ 70-10n \le C_{s_n} \le 90-10n \\ 0 & if\ 90-10n \le C_{s_n} \end{cases}$$

For n=1, 2, 3, 4 the membership functions are represented in the figure 1

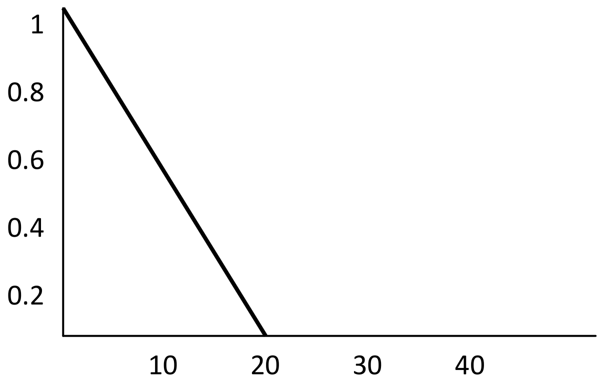

Figure 1 – the membership function of the goal

Let the goal be represented by the following membership function

$$\mu_{\overline{D_n}}(C_{s_n}) = \begin{cases} 1-\dfrac{x_{N+1}}{20} & if \quad 0 \le x_{N+1} \le 20 \\ 0 & if \quad else \end{cases}$$

Let the number of stages be N=4 and the most crisp demands for each stage be $a_1 = 40, a_2 = 45, a_3 = 40, a_4 = 55$

The sets of the values permitted for the decisions and state values are respectively:

C= {0, 5, 10,....... } x={0, 5, 10,....... }

Assume that the stock level at the beginning is $P_0 = 0$ and $0 \le x_5 \le 20$

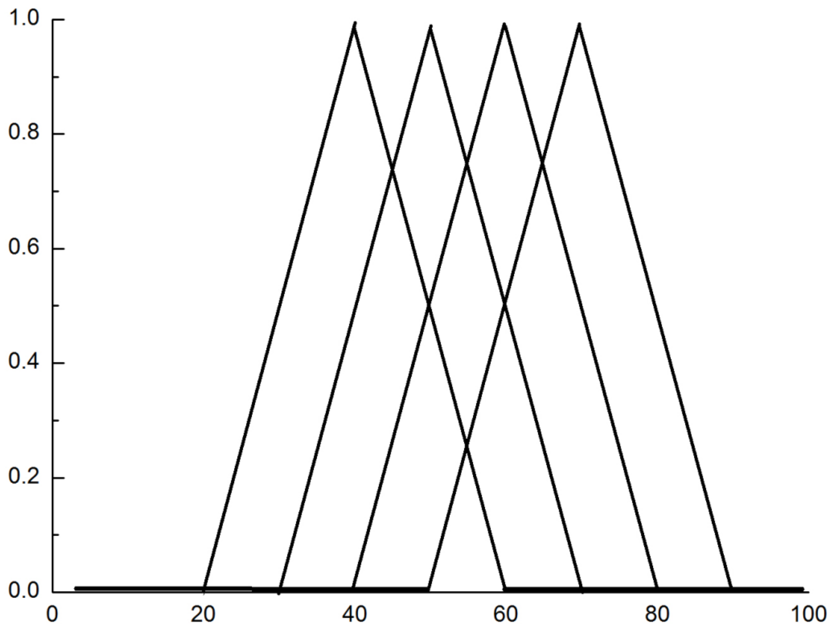

Figure 2 – the membership function of the constraints

Solution:
Since we are interested in the values of C_{s_n} *for which* $\mu_{\overline{D_n}}(C_{s_n}) \geq 0$ the lower and upper bounds for the decision variables at different stages are found from the constraints membership functions as shown in the table [37-39]

Table 1- Lower and upper bounds for the decision variables

n	$C_{s_n}{}^{\ell}$	$C_{s_n}{}^{u}$
1	50	80
2	40	70
3	30	60
4	10	40

In order to find lower and upper bounds for the state variables, first forward calculation is performed and corresponding $p_{n,f}{}^{u}$, $p_{n.f}{}^{l}$ are respectively found using the equations below and the results are showqn in table 2

$$p_{n,f}{}^{u} = P_{n-1,f}{}^{U} + C_{s_{n-1}}{}^{u} - a_{n-1} \quad ,n = 1,2,3,4,5$$

Table 2 –Lower and upper bounds for the state variables by forward calculation

n	$p_{n.f}^{\ell}$	$p_{n,f}^{u}$
1	0	0
2	5	35
3	10	60
4	0	80

Second step is to perform backward calculation recursively and the following results are obtained:

Table 3 –Lower and upper bounds for the state variables by backward calculation

n	$p_{n.b}^{\ell}$	$p_{n,b}^{u}$
1	0	0
2	0	60
3	0	55
4	10	45
5	0	0

The final bounds for the state variables are obtained by the following equations and shown in the table 4:

$$p_n^{l} = \max\{ p_{n,b}^{l}, p_{n,b}^{l} \}$$

$$p_n^{u} = \min\{ p_{n,b}^{u}, p_{n,b}^{u} \}$$

Table 4: final lower and upper bounds for the state variables

n	$p_{n.b}^{\ell}$	$p_{n,b}^{u}$
1	0	0
2	5	30
3	10	40
4	10	50
5	0	0

At this point, it is possible to apply dynamic programming with fuzzy decision and state variables. The aim is to find the maximum value of the goal membership function for each state variable (whose range is given above in table 4) and this is performed by applying the equation

Stage 1 is obtained as follows:

$\mu_{\overline{G_4}}$ is obtained from the following equationresults are shown in table 5 and plotted in the figure 3:

$$\mu_{\overline{G_4}} = \max_{d_4} \min [\mu_c(C_{s_4}), \mu_G(p_4 + C_{s_4} - a_4)$$

Table 5-Stage I

	C_{s_4}							$\mu_{G_4}(x_4)$
P_4	25	30	35	40	45	50	55	
5	0	0	0	0	0	0	1/4	1/4
10	0	0	0	0	0	1/2	1/4	1/2
15	0	0	0	0	3/4	1/2	1/4	3/4
20	0	0	0	1	3/4	1/2	1/2	1/4
25	0	0	3/4	3/4	1/2	1/4	0	3/4
30	0	1/2	3/4	1/2	1/4	0	0	3/4
35	1/4	1/2	1/2	1/4	0	0	0	1/2
40	1/4	1/2	1/4	0	0	0	0	1/2
45	1/4	1/4	0	0	0	0	0	1/2
50	1/4	0	0	0	0	0	0	1/4

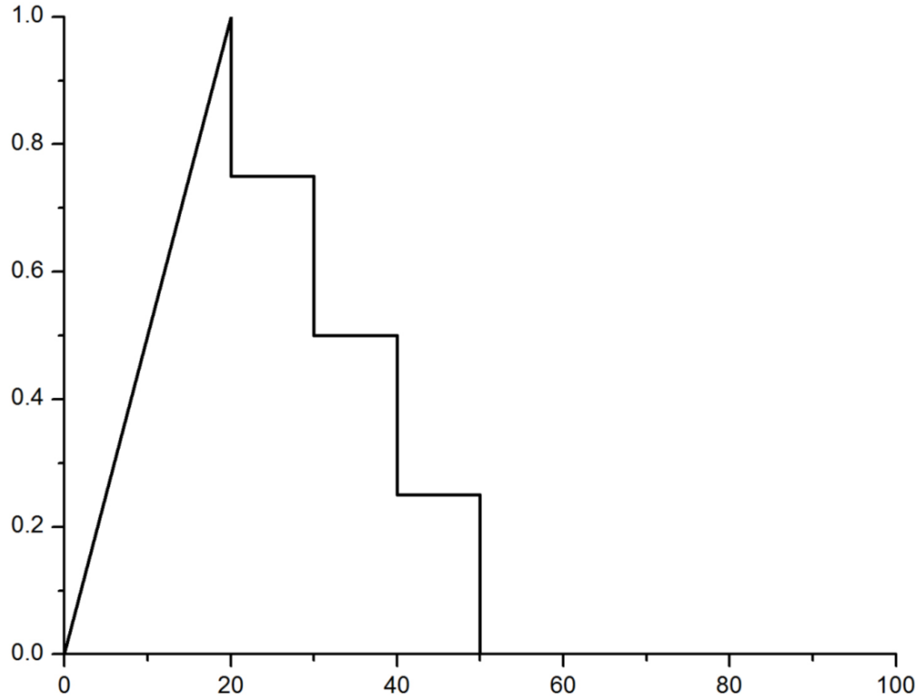

Figure 3 – Membership function μ_{G_4}

Stage2 The corresponding equation for the second stage is:

$$\mu_{\overline{Dn}} = \max_{d_3} \min [\mu_c(C_{s_3}), \mu_G(p_3 + C_{s_3} - a_3)$$

Function μ_G is obtained first by calculating the value of the transformation function $p_4 = p_3 + C_{s_3} - a_3$, then taking the corresponding value from the abscissa vector of the function $\mu_{G_4}(x_4)$. The results are shown in the table6, and function $\mu_{G3}(x_3)$. is shown in figure 4.

Table 6 –Stage 2

	C_{s3}							$\mu_{G3}(x_3)$
P_3	35	40	45	50	55	60	65	
5	0	0	1/4	1/2	3/4	1/2	1/4	3/4
10	0	1/4	1/2	3/4	3/4	1/2	1/4	3/4
15	1/4	1/2	3/4	1	3/4	1/2	1/4	1
20	1/4	1/2	3/4	3/4	3/4	1/2	1/4	3/4
25	1/4	1/2	3/4	3/4	1/2	1/2	1/4	3/4
30	1/4	1/2	3/4	1/2	1/2	1/4	1/4	3/4
35	1/4	1/2	1/2	1/2	1/4	1/4	0	1/2
40	1/4	1/2	1/2	1/4	1/4	0	0	1/2
45	1/4	1/2	1/4	1/4	0	0	0	1/2
50	1/4	1/4	1/4	0	0	0	0	1/4
55	1/4	1/4	0	0	0	0	0	1/4
60	1/4	0	0	0	0	0	0	1/4

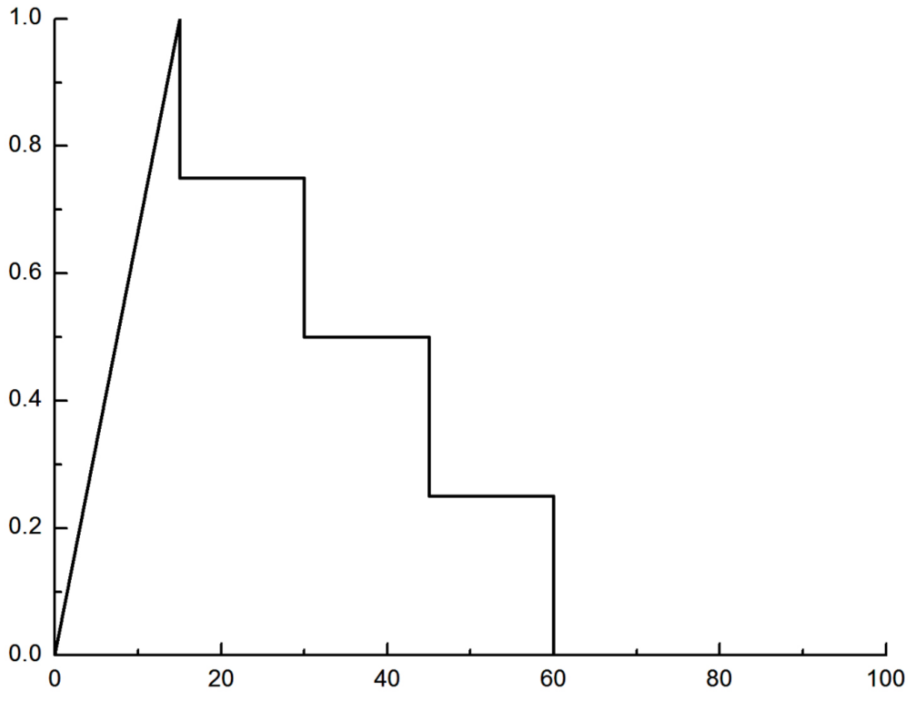

Figure 4 Membership function $\mu_{\overline{G3}}$

Stage3: correspondind equation for decision membership function wil be:
$\mu_{\overline{Dn}} = \max_{d_2} \min \{ [\mu_c(C_{s_2}), \mu_G(p_2 + C_{s_3} - a_2)\}$ by using same procedure described in stage 2, the results for stage3 are shown in the table 7 and figure 5

Table 7-stage 3

	C_{s2}							$\mu_{G2}(x_2)$
P_2	45	50	55	60	65	70	75	
10	1/4	1/2	3/4	1	3/4	1/2	1/4	1
15	1/4	1/2	3/4	3/4	3/4	1/2	1/4	3/4
20	1/4	1/2	3/4	3/4	1/2	1/2	1/4	3/4
25	1/4	1/2	3/4	1/2	1/2	1/2	1/4	3/4
30	1/4	1/2	1/2	1/2	1/2	1/4	1/4	1/2
35	1/4	1/2	1/2	1/2	1/4	1/4	1/4	1/2
40	1/4	1/2	1/2	1/2	1/4	1/4	0	1/2

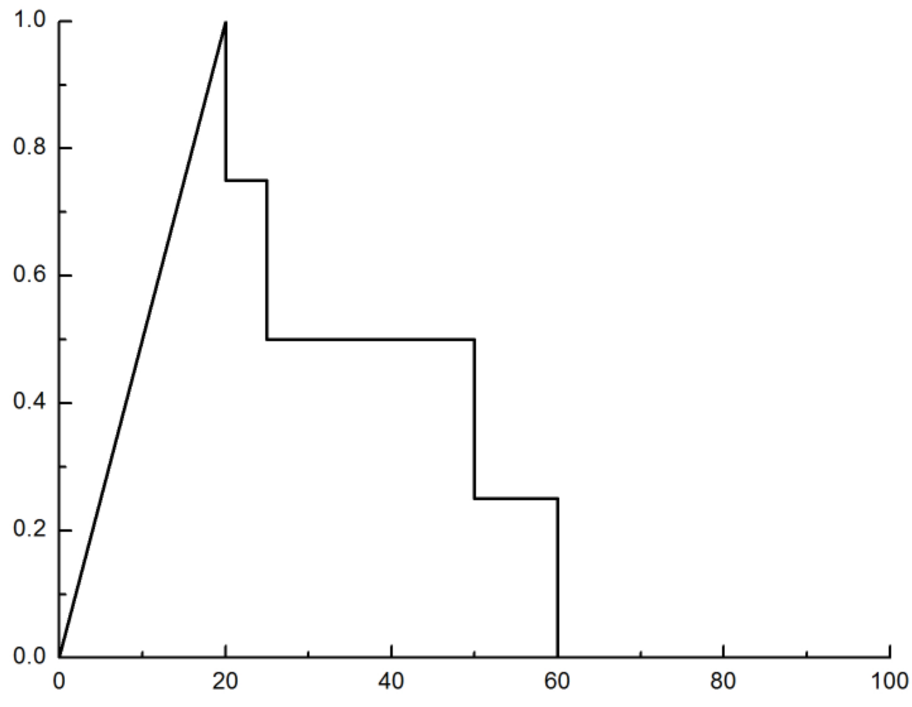

Figure 5 – Membership function $\mu_{\overline{G2}}$

Stage 4: Corresponding equation for decision membership function will be:

$$\mu_{\overline{Dn}} = \max_{d1} \{ \min [\mu_c(C_{s_1}), \mu_G(p_1 + C_{s_1} - a_1)] \}.$$

Table 8 and stage 4

	C_{s1}							$\mu_{G1}(x1)$
P_1	55	60	65	70	75	80	85	
10	0	0	0	0	0	0	1/4	1/4
15	0	0	0	0	0	1/4	1/4	1/4
20	1/4	1/2	3/4	3/4	1	1	1	1
25	1/4	1/2	3/4	1/2	1/2	1/2	1/4	3/4
30	1/4	1/2	1/2	1/2	1/2	1/4	1/4	1/2
35	1/4	1/2	1/2	1/2	1/4	1/4	1/4	1/2
40	1/4	1/2	1/2	1/2	1/4	1/4	0	1/2

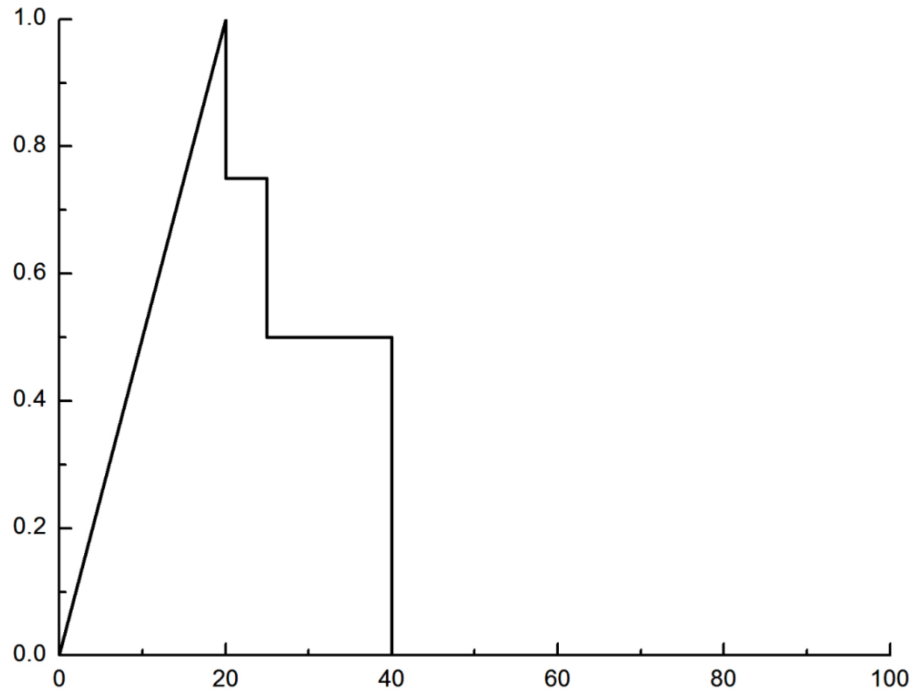

Figure 5 – Membership function $\mu_{\overline{G1}}$

6. Conclusion:

The idea is to develop an entropy cost function that is added to the classical inventory cost (order, holding cost). Entropy costs are intended to capture the hiden costs of inventory sygstems. Thw result showed thtat accoutnting for entropy cost recommends ordering in larger lots as larger lots are cheaper to control. The results also showed that accounting for entropy cost is more relevant when the items are expensive or the system suffers from disturbance due to high percentage of imperfect quality items. In this work a new approach to inventory control is shown. The method described uses fuzzy dynamic programming, whichg has been proved as a powerful tool for optimization when non deterministic information exists. The complex problem can be subdivided into smallerproblems and the state spaces were reduced by the introduction of a bound on the basis of heuristic considerations. Using a transformation function, upper and lower bounds for the state variables are found on the several intermediate stages and final solutions are found by fuzzy inference. Further work can be performed by introducing new variables such as non crisp demand.

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