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A New Method To Work Out Exact Solutions Of The (2+1) Dimensional Asymmetrical Nizhnik-Novikov-Veselov System

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Abstract

In this paper, we will employ a generalized ($\frac{G'}{G}$)-expansion method to seek more general exact solutions of the (2+1) dimensional asymmetrical Nizhnik-Novikov-Veselov system. As a result, hyperbolic function solutions, trigonometric function solutions, and rational solutions with parameters are to be obtained. The proposed method will be more effective and can be applied to many other nonlinear evolution equations in mathematical physics.

Keywords: (2+1) dimensional ANNV system; Generalized ($\frac{G'}{G}$)-expansion method; Homogeneous balance; Non-travelling wave solutions; Hyperbolic function solutions; Trigonometric function solutions; Rational solutions.

1. Introduction

As what we have knew, the investigation of exact solutions of nonlinear evolution equation (NLEEs) plays a significant role in the study of nonlinear physical

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phenomena. Recent years, we have obtained many powerful methods to find exact solution of NLEEs. For example, Hirota's bilinear method [1], the inverse scattering method [2], tanhfunction method [3–5], Jacobi elliptic function expansion method [6–8], algebraic method [9–11], asymptotic methods [12], Backlund transformation [13], Painlevé expansion [14], sine–cosine method [15], non-perturbative methods [16],homogenous balance method [17], homotopy perturb-bation method [18-20], variational method [21–24], Adomian decomposition method [25], F-expansion method [26–28], and auxiliary equation method [29–31].

Although it is difficult for us to solve nonlinear evolution equation, while we will introduce generalized ($\frac{G'}{G}$)-expansion method [32] to make a new exploration for a nonlinear high-order evolution equation, that is (2+1) dimensional ANNV system.

Recently, a new method called the ($\frac{G'}{G}$)-expansion method was put forward by Wang et al. [33] to seek travelling wave solutions of NLEEs. The ($\frac{G'}{G}$)-expansion method is based on the assumptions that the travelling wave solutions can be expressed by a polynomial in ($\frac{G'}{G}$), and that $G = G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$G'' + \lambda G' + \mu G = 0, \tag{1}$$

where $G' = \frac{dG(\xi)}{d\xi}, G'' = \frac{d^2G(\xi)}{d\xi^2}, \xi = x - Ct,$ and C is a constant.

But there exists many shortcomings in this method. In most cases, we can apply this method to solve some lower dimensional nonlinear evolution equations. Besides, we also can't analysis solutions' properties under some disturbances. Therefore we want to use the generalized ($\frac{G'}{G}$)-expansion method to get a powerful solution of nonlinear evolution equations. In [34], Li and Ma have studied ($\frac{G'}{G}$)-expansion method and new exact solutions for (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov system and obtained three different solutions: hyperbolic function solutions, trigonometric function solutions, and rational solutions with parameters. The present paper is motivated by the desire to get more general solutions within the generalized ($\frac{G'}{G}$)-expansion method of (2+1) dimensional ANNV system.

$$u_t + u_{xxx} - 3v_x u - 3v u_x = 0, \tag{2}$$

$$u_x = v_{y.} \tag{3}$$

Eq (1) is the earliest derived by Boiti. [35] discussed literature series of a group of doubleperodic ANNV system; Literature [36] discussed the interaction between solitons in various forms of ANNV syst-em; Literature [37] obtained ANNV new exact solutions. After using the generalized ($\frac{G'}{G}$)-expansion method, we get non-travelling solutions with three arbitrary functions, including hyperbolic function sol-utions, trigonometric function solutions, and rational solutions.

In this paper, we will give the description of the generalized ($\frac{G'}{G}$)-expansion method in section 2,and apply this method to Eqs. (2) and (3) in section 3. Finally, some conclusions are given in section4.

2. Description of the generalized ($\frac{G'}{G}$)-expansion method

Suppose that a nonlinear evolution equation with independent variables $X = (x, y, z, ..., t)$ and depend-ent variable u :

$$F \; u, u_t, u_x, u_y, u_z, \ldots, u_{xt}, u_{yt}, u_{zt}, \ldots, u_{tt}, u_{xx}, u_{yy}, u_{zz} \ldots = 0 \tag{4}$$

We suppose that the NLEE (4) has the following solution:

$$u = \sum_{i=0}^{m} \alpha_i(X) \left(\frac{G'}{G} \right)^i, \quad \alpha_m(X) \neq 0 \tag{5}$$

Where $\alpha_0(X), \alpha_1(X)(i = 1, 2, ..., m)$ and $\xi = \xi(X)$ are all functions of X to be determined later, $G = G(\xi)$ satisfies Eq. (1). In the following we give the main steps of the generalized ($\frac{G'}{G}$)-expansion method:

Step1. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (4)

Step2. By substituting (5) along with Eq. (1) into Eq. (4) and using second order LODE (1), collecting all terms with the same order of ($\frac{G'}{G}$) together, the left-hand side of Eq.(4) is converted into another polynomial in ($\frac{G'}{G}$). Equating each coefficient of this polynomial to zero, yields a set of algebraic equation about $\alpha_0(X), \alpha_i(X)$ and ξ.

Step3. Solve the system of over-determined partial differential equations obtained in Step 2 for $\alpha_0(X), \alpha_i(X)$ and ξ by use of Mathematica.

Step4. It is easy for us to obtain the solutions of Eq.(1), and use results obtained in above steps to derive a series of fundamental solutions of Eq.(4)depending on($\frac{G'}{G}$).Then we can obtain exact solutions of Eq.(4).

3. Application to the (2 + 1)-dimensional asymmetrical Nizhnik-Novikov-Veselov system

Let us consider the (2+1) dimensional ANNV system Eqs. (2) and (3), we get $m_1 = 2$ for u and $m_2 = 2$ for v by according to Step 1.In order to seek explicit solutions, we suppose that Eq. (2) and (3) have the following formal solutions:

$$u = \alpha_2(y,t)\left(\frac{G'}{G}\right)^2 + \alpha_1(y,t)\left(\frac{G'}{G}\right) + \alpha_0(y,t), \qquad \alpha_2(y,t) \neq 0, \tag{6}$$

$$v = \beta_2(y,t)\left(\frac{G'}{G}\right)^2 + \beta_1(y,t)\left(\frac{G'}{G}\right) + \beta_0(y,t), \qquad \beta_2(y,t) \neq 0, \tag{7}$$

where $G = G(\xi)$ satisfies Eq. (1), $\xi = kx + \eta(y,t), (k \neq 0)$, k is a constant.

$\alpha_2(y,t), \alpha_1(y,t), \alpha_0(y,t), \beta_2(y,t), \beta_1(y,t), \beta_0(y,t), \eta(y,t)$ are functions of y and t to be determined.

Substituting (6) and (7) into Eqs. (2) and (3) and collecting all terms with the same order of ($\frac{G'}{G}$) together, the left-hand sides of Eqs. (2) and (3) are converted into two polynomials in ($\frac{G'}{G}$).Setting each coefficient of each polynomial to zero, we derive a set of over-determined differential equations for $\alpha_2(y,t), \alpha_1(y,t), \alpha_0(y,t), \beta_2(y,t), \beta_1(y,t), \beta_0(y,t)$ and $\eta(y,t)$ as follows:

$$\left(\frac{G'}{G}\right)^5 : -24k^3\alpha_2 - 3(-2k\alpha_2\beta_2) - 3(-2k\alpha_2\beta_2) = 0,$$

$$\left(\frac{G'}{G}\right)^4 : -(24k^3\lambda\alpha_2 + 30k^3\lambda\alpha_2 + 6k^3\alpha_1) + 3(2k\alpha_2\beta_1 + 2k\lambda\alpha_2\beta_2 + k\alpha_1\beta_2) + 3(2k\alpha_1\beta_2 + 2k\lambda\alpha_2\beta_2 + k\alpha_2\beta_1) = 0,$$

$$\left(\frac{G'}{G}\right)^3:(-2\alpha_2\eta_t)-(24k^3\mu\alpha_2+30k^3\lambda^2\alpha_2+6k^3\lambda\alpha_1+16k^3\mu\alpha_2+8k^3\lambda^2\alpha_2+6k^3\lambda\ \alpha_1)$$
$$+3(2k\alpha_0\beta_2+2k\lambda\alpha_1\beta_2+k\alpha_1\beta_1+2ku\alpha_2\beta_2+k\lambda\alpha_2\beta_1)+3(2k\alpha_2\beta_0+2k\lambda\alpha_2\beta_1+k\alpha_1\beta_1+2k\mu\alpha_2\beta_2+k\lambda\alpha_1\beta_2)=0,$$

$$\left(\frac{G'}{G}\right)^2:\alpha_{2t}-2\lambda\alpha_2\eta_t-\alpha_1\eta_t-52k^3\lambda\ \mu\alpha_2-8k^3\mu\alpha_1-7k^3\lambda^2\alpha_1-8k^3\lambda^3\alpha_2+6k\lambda\alpha_0\beta_2+3k\beta_1\alpha_0+9k\mu\alpha_1\beta_2$$
$$+6k\lambda\alpha_1\beta_1+6k\lambda\alpha_2\beta_0+3k\alpha_1\beta_0+3k\mu\alpha_2\beta_1=0,$$

$$\left(\frac{G'}{G}\right):-2\mu\alpha_2\eta_t+\alpha_{1t}-\lambda\alpha_1\eta_t-16k^3\mu^2\alpha_2-14k^3\lambda^2\mu\alpha_2-8k^3\lambda\mu\alpha_1-k^3\lambda^3\alpha_1+6k\mu\beta_2\alpha_0+3k\lambda\alpha_0\beta_1+6k\mu\alpha_1\beta_1$$
$$+6k\mu\alpha_2\beta_0+3k\lambda\alpha_1\beta_0=0,$$

$$\left(\frac{G'}{G}\right)^0:\alpha_{0t}-\mu\alpha_1\eta_t-6k^3\mu^2\lambda\alpha_2-2k^3\mu^2\alpha_1-k^3\lambda^2\mu\alpha_1+3k\mu\alpha_0\beta_1+3k\mu\alpha_1\beta_0=0,$$

$$\left(\frac{G'}{G}\right)^3:-2k\alpha_2+2\beta_2\eta_y=0,$$

$$\left(\frac{G'}{G}\right)^2:-(2k\lambda\alpha_2+k\alpha_1)-(\beta_{2y}-2\lambda\beta_2\eta_y-\beta_1\eta_y)=0,$$

$$\left(\frac{G'}{G}\right):-(2k\mu\alpha_2+k\lambda\alpha_1)-(-2\mu\beta_2\eta_y+\beta_{1y}-\lambda\beta_1\eta_y)=0,$$

$$\left(\frac{G'}{G}\right)^0:-k\mu\alpha_1-(-\mu\beta_1\eta_y+\beta_{0y})=0.$$

Solving the set of over-determined differential equations by use of Mathematica, we have

$$\alpha_0(y,t)=m,\quad \alpha_1(y,t)=2k\lambda,\quad \alpha_2(y,t)=2k,\quad \eta(y,t)=y+f(t), \tag{8}$$

$$\beta_0(y,t)=\frac{f'(t)+k^3\lambda^2-3k^2m}{3k},\quad \beta_1(y,t)=2\lambda k^2,\quad \beta_2(y,t)=2k^2. \tag{9}$$

Where $f(t)$ are arbitrary functions of the indicated variables, $f'(t)=\frac{df(t)}{dt}$, and m is an arbitrary constant.

Substituting (8) and (9) into (6) and (7), we have the fundamental solutions of Eqs. (2) and (3):

$$u=2k\left(\frac{G'}{G}\right)^2+2k\lambda\left(\frac{G'}{G}\right)+m, \tag{10}$$

$$v=2k^2\left(\frac{G'}{G}\right)^2+2\lambda k^2\left(\frac{G'}{G}\right)+\frac{f'(t)+k^3\lambda^2-3k^2m}{3k}. \tag{11}$$

Where $G=G(\xi)$, $\xi=kx+y+f(t)$ .We solve a second order linear ordinary differential equation (LODE): $G''+\lambda G'+\mu G=0$, and we have:

When $\lambda^2-4\mu>0$,

$$\frac{G'}{G}=-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{c_1\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{c_1\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right). \tag{12}$$

When $\lambda^2-4\mu<0$,

$$\frac{G'}{G}=-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-c_1\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{c_1\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right). \tag{13}$$

When $\lambda^2-4\mu=0$,

$$\frac{G'}{G}=-\frac{\lambda}{2}+\frac{c_2}{c_1+c_2\xi}. \tag{14}$$

Substituting the general solutions of Eq. (12-14) into (10) and (11), we have three types of exact solutions of Eqs. (2) and (3) as follows:

When $\lambda^2-4\mu>0$, we obtain hyperbolic function solutions:

$$u=2k\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{c_1\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{c_1\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right)^2$$

$$+2k\lambda\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{c_1\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{c_1\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right)+m, \tag{15}$$

$$v=2k^2\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{c_1\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{c_1\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right)^2$$

$$+2\lambda k^2\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{c_1\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}{c_1\cosh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)+c_2\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi)}\right)+\frac{f'(t)+k^3\lambda^2-3k^2m}{3k}, \tag{16}$$

Where $\xi=kx+y+f(t)$, c_1 and c_2 are arbitrary constants.

When $\lambda^2-4\mu<0$, we have trigonometric function solutions:

$$u = 2k\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-c_1\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{c_1\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right)^2 + 2k\lambda\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-c_1\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{c_1\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right)+m \quad (17)$$

$$v = 2k^2\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-c_1\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{c_1\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right)^2 + 2\lambda k^2\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{-c_1\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}{c_1\cosh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)+c_2\sinh(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi)}\right)+\frac{f'(t)+k^3\lambda^2-3k^2m}{3k}, \quad (18)$$

Where $\xi = kx + y + f(t)$, c_1 and c_2 are arbitrary constants.

When $\lambda^2 - 4\mu = 0$, we get rational solutions:

$$u = 2k\left(-\frac{\lambda}{2}+\frac{c_2}{c_1+c_2\xi}\right)^2 + 2k\lambda\left(-\frac{\lambda}{2}+\frac{c_2}{c_1+c_2\xi}\right)+m, \quad (19)$$

$$v = 2k^2\left(-\frac{\lambda}{2}+\frac{c_2}{c_1+c_2\xi}\right)^2 + 2\lambda k^2\left(-\frac{\lambda}{2}+\frac{c_2}{c_1+c_2\xi}\right)+\frac{f'(t)+k^3\lambda^2-3k^2m}{3k}, \quad (20)$$

Where $\xi = kx + y + f(t)$, c_1 and c_2 are arbitrary constants.

4. Conclusion

In this paper, the generalized ($\frac{G'}{G}$)-expansion method was successfully employed to obtain more general exact solution of the (2 + 1)-dimensional asymmetrical Nizhnik-Novikov-Veselov system. Through the comparisons between our work and [34], it's easy to find that the method we put forward is much more effective and practical. Therefore, we can obtain more general solutions in this method. Moreover we can also use the method to get exact non-traveling wave solution of some high-dimensional NLEEs.

References

[1] R. Hirota, Exact solution of the Korteweg-de Vries equation for multiple collisions of solitons, Phys. Rev. Lett. 27 (1971) 1192–1194.

[2] M.J. Ablowitz, P.A. Clarkson, Soliton, Nonlinear Evolution Equations and Inverse Scattering, Cambridge Univ. Press, New York, 1991.

[3] E.M.E. Zayed, H.A. Zedan, K.A. Gepreel, Group analysis and modified extended tanh-function to find the invariant solutions and soliton solutions for nonlinear Euler equations, Int. J. Nonlinear Sci. Numer. Simul. 5 (2004) 221–234.

[4] H.A. Abdusalam, On an improved complex tanh-function method, Int. J. Nonlinear Sci. Numer. Simul. 6 (2005) 99–106.

[5] S. Zhang, T.C. Xia, Symbolic computation and new families of exact non-travelling wave solutions of (2 + 1)-dimensional Broer–Kaup equations,Commun. Theor. Phys. 45 (2006) 985–990. Beijing, China.

[6] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A 289(2001) 69–74.

[7] Z.T. Fu, S.K. Liu, S.D. Liu, Q. Zhao, New Jacobi elliptic function expansion and new periodic solutions of nonlinear wave equations, Phys. Lett. A 290(2001) 72–76.

[8] E.J. Parkes, B.R. Duffy, P.C. Abbott, The Jacobi elliptic-function method for finding periodic-wave solutions to nonlinear evolution equations, Phys. Lett. A 295 (2002) 280–286.

[9] J.Q. Hu, An algebraic method exactly solving two high-dimensional nonlinear evolution equations, Chaos Solitons Fract. 23 (2005) 391–398.

[10] E. Yomba, The modified extended Fan sub-equation method and its application to the (2 + 1)-dimensional Broer–Kaup–Kupershmidt equation, Chaos Solitons Fract. 27 (2006) 187–196.

[11] S. Zhang, T.C. Xia, A further improved extended Fan sub-equation method and its application to the (3 + 1)-dimensional Kadomstev–Petviashvili equation, Phys. Lett. A 356 (2006) 119–123.

[12] J.H. He, Some asymptotic methods for strongly nonlinear equations, Int. J. Mod. Phys. B 20 (2006) 1141–1199.

[13] M.R. Miurs, Bcklund Transformation, Springer, Berlin, 1978.

[14] J. Weiss, M. Tabor, G. Carnevale, The Painlevé property for partial differential equations, J. Math. Phys. 24 (1983) 522–526.

[15] C.T. Yan, A simple transformation for nonlinear waves, Phys. Lett. A 224 (1996) 77–84

[16] J.H. He, Non-Perturbative Methods for Strongly Nonlinear Problems, Dissertation, de-Verlag im Internet GmbH, Berlin, 2006.

[17] M.L. Wang, Exact solution for a compound KdV–Burgers equations, Phys. Lett. A 213 (1996) 279–287.

[18] M. El-Shahed, Application of He's homotopy perturbation method to Volterra's integro-differential equation, Int. J. Nonlinear Sci. Numer. Simul. 6 (2005) 163–168.

[19] J.H. He, Homotopy perturbation method for bifurcation of nonlinear problems, Int. J. Nonlinear Sci. Numer. Simul. 6 (2005) 207–208.

[20] J.H. He, Application of homotopy perturbation method to nonlinear wave equations, Chaos Solitons Fract. 26 (2005) 695–700.

[21] J.H. He, Variational iteration method—a kind of nonlinear analytical technique: some examples, Int. J. Nonlinear Mech. 34 (1999) 699–708.

[22] J.H. He, Variational iteration method for autonomous ordinary differential systems, Appl. Math. Comput. 114 (2000) 115–123.

[23] J.H. He, Variational principles for some nonlinear partial differential equations with variable coefficients, Chaos Solitons Fract. 19 (2004) 847–851.

[24] J.H. He, Variational approach to (2 + 1)-dimensional dispersive long water equations, Phys. Lett. A 335 (2005) 182–184.

[25] T.A. Abassy, M.A. El-Tawil, H.K. Saleh, The solution of KdV and mKdV equations using adomian pade approximation, Int. J. Nonlinear Sci. Numer. Simul.5 (2004) 327–340.

[26] J.B. Liu, K.Q. Yang, The extended F-expansion method and exact solutions of nonlinear PDEs, Chaos Solitons Fract. 22 (2004) 111–121.

[27] S. Zhang, New exact solutions of the KdV–Burgers–Kuramoto equation, Phys. Lett. A 358 (2006) 414–420.

[28] S. Zhang, The periodic wave solutions for the (2 + 1)-dimensional Konopelchenko–Dubrovsky equations, Chaos Solitons Fract. 30 (2006) 1213–1220.

[29] Sirendaoreji, J. Song, Auxiliary equation method for solving nonlinear partial differential equations, Phys. Lett. A 309 (2003) 387–396.

[30] S. Zhang, New exact non-traveling wave and coefficient function solutions of the (2 + 1)-dimensional breaking soliton equations, Phys. Lett. A 368(2007) 470–475.

[31] S. Zhang, T.C. Xia, A generalized new auxiliary equation method and its applications to nonlinear partial differential equations, Phys. Lett. A 363 (2007)356–360.

[32] Sheng Zhang *, Wei Wang, Jing-Lin TongA generalized ($\frac{G'}{G}$)-expansion method and its application to the (2 +1)-dimensional Broer–Kaup equationsApplied Mathematics and Computation 209 (2009) 399–404

[33] M.L. Wang, X.Z. Li, J.L. Zhang, The ($\frac{G'}{G}$)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, Phys. Lett. A 372 (2008) 417–423.

[34] Li Bang-Qing,MaYu-Lan ($\frac{G'}{G}$)-expansion method and new exact solutions for (2+1) dimensional asymmetrical Nizhnik-Novikov-Veselov system

[35] S. Zhang, Symbolic computation and new families of exact non-travelling wave solutions of (2 + 1)-dimensional Konopelchenko–Dubrovsky equations, Chaos Solitons Fract. 31 (2007) 951–959.

[36] A.M. Wazwaz, The tanh method and a variable separated ODE method for solving double sine-Gordon equation, Phys. Lett. A 350 (2006) 367–370.

[37] J.Q.Hu, An algebraic method exactly solving two high-dimensional nonlinear evolution equations, Chaos Solitons Fract. 23 (2005) 391–398.