Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 4 (2015), pp. 1787-1791

http://www.ripublication.com

Some Results on Fuzzy Dual Modular Lattice

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Abstract

In this paper, Fuzzy dual Modular Pair in Fuzzy Lattice-Definition of Fuzzy Dual Modular-Characterization Theorem-Fuzzy Dual Distributive Lattice.

Keywords: Fuzzy Lattice, Fuzzy Dual Modular Lattice, Fuzzy Dual Distributive Lattice.

Introduction

The concept of fuzzy lattice was already introduced by Ajmal, N[1], D. D. Anderson [2] explained the existence of Dual Modules, S. Nanda[5] and Wilcox, L. R[6][7] explained modularity in the theory of lattices, G. Gratzer [3], M. Mullai and B. Chellappa[4] explained Fuzzy L-ideal, Yuan, B and W. Wu[8] explained Fuzzy ideals on a distributive lattice. A few definitions and results are listed that the fuzzy dual Modular lattice using in this paper, we explain Fuzzy Dually Modular Pair in Fuzzy Dual Distributive Lattice, Definition of Fuzzy Dually Modular Pair and Definition of Fuzzy Dual Distributive Lattice characterization theorem of Fuzzy Dual Distributive Lattice, Fuzzy Dually Modular Pair in Fuzzy Dual Distributive Lattice.

If L is a fuzzy Dual Distributive lattice then $\mu(a)\wedge\mu(b\vee c) = \mu(a\wedge b)\vee\mu(a\wedge c)$, for all $\mu(a),\mu(b),\mu(c)$ in L.

Theorem: 1.1
In any Fuzzy lattice L the following are equivalent.

1. $\mu(a\vee b)\wedge\mu(a\vee c) = \mu(a)\vee[\mu(b)\wedge\mu(a\vee c)]$
2. $\mu(a\wedge b)\vee\mu(a\wedge c) = \mu(a)\wedge[\mu(b)\vee\mu(a\wedge c)]$, for all $\mu(a),\mu(b),\mu(c)$ in L.

Proof:

(i) $\Rightarrow$ (ii)
Let $\mu(a)$, $\mu(b)$, $\mu(c)$ in L be arbitrary. Then

$\mu(a\wedge b)\vee\mu(a\wedge c) \geq \min\{\mu(a),\mu(b)\}\vee\min\{\mu(a),\mu`(c)\}$

$\geq \min\{\mu(a),\mu(c)\}\vee\min\{\mu(a),\mu(b)\}$, by commutative law

$\geq \min\{\mu(a),\mu(c)\}\vee\min\{\mu(b),\mu(a)\}$, by commutative law

$\geq \min\{\mu(a\wedge c),\mu(b)\}\wedge\mu(a)$, by (i) since $\mu(a\wedge c) \leq \mu(a)$

$\geq \mu(a)\wedge\min\{\mu(a\wedge c),\mu(b)\}$, by commutative law

$\geq \mu(a)\wedge\min\{\mu(b),\mu(a\wedge c)\}$, by commutative law

$= \mu(a)\wedge[\mu(b)\vee\mu(a\wedge c)]$

Thus $\mu(a\wedge b)\vee\mu(a\wedge c) = \mu(a)\wedge[\mu(b)\vee\mu(a\wedge c)]$, for all $\mu(a),\mu(b),\mu(c)$ in L

(ii) $\Rightarrow$ (i)

Let μ(a), μ(b), μ(c) in L be arbitrary. Then

$\mu(a\vee b)\wedge\mu(a\vee c) \geq \min\{\mu(a),\mu(b)\}\wedge\min\{\mu(a),\mu(c)\}$

$\geq \min\{\mu(a),\mu(c)\}\wedge\min\{\mu(a),\mu(b)\}$, by commutative law

$\geq \min\{\mu(a),\mu(c)\}\wedge\min\{\mu(b),\mu(a)\}$, by commutative law

$\geq \min\{\mu(a\vee c),\mu(b)\}\vee\mu(a)$, by (ii) since $\mu(a\vee c)\geq \mu(a)$

$\geq \mu(a)\vee\min\{\mu(a\vee c),\mu(b)\}$, by commutative law

$\geq \mu(a)\vee\min\{\mu(b),\mu(a\vee c)\}$, by commutative law

$=\mu(a)\vee[\mu(a\vee c)\wedge\mu(b)]$

Thus $\mu(a\vee b)\wedge\mu(a\vee c) = \mu(a)\vee[\mu(b)\wedge\mu(a\vee c)]$, for all $\mu(a),\mu(b),\mu(c)$ in L.

Definition: 1.1

A Fuzzy lattice L is said to be Fuzzy dual modular, if $\mu(a\wedge b)\vee\mu(a\wedge c) = \mu(a)\wedge[\mu(b)\vee\mu(a\wedge c)]$, for all $\mu(a),\mu(b),\mu(c)$ in L.

Theorem: 1.2

Every Fuzzy modular lattice is a Fuzzy dual modular lattice

Proof:

Follows from the previous theorem.

Theorem: 1.3

If L is a Fuzzy modular lattice, then $(\mu(a),\mu(b))$ is a dually modular pair for all $\mu(a),\mu(b)\in$ L.

Proof:

Given L is a Fuzzy modular lattice.

To prove $(\mu(a),\mu(b))$ is a Fuzzy dually modular pair for all $\mu(a),\mu(b)\in$ L .

L is a Fuzzy modular lattice

$\Rightarrow \mu(x\vee y)\wedge\mu(x\vee z) = \mu(x)\vee[\mu(y)\wedge\mu(x\vee z)]$ for all $\mu(x),\mu(y),\mu(z)\in L$

$\Rightarrow \mu(x\wedge y)\vee\mu(x\wedge z) = \mu(x)\wedge[\mu(y)\vee\mu(x\wedge z)]$ for all $\mu(x),\mu(y),\mu(z)\in L$, by Theorem 1.1

Take $\mu(c) = \mu(x)$, $\mu(a) = \mu(y)$, $\mu(b) = \mu(x\wedge z)$ we get

$\mu(c\wedge a)\vee\mu(b) = \mu(c)\wedge\mu(a\vee b)$ for every $\mu(c) \geq \mu(b)$.

$\Rightarrow$ $(\mu(a),\mu(b))$ is a Fuzzy dually modular pair.

Theorem: 1.4
In any Fuzzy lattice L the following are equivalent.

1. $\mu(a)\vee\mu(b\wedge c) = \mu(a\vee b)\wedge\mu(a\vee c)$
2. $\mu(a)\wedge\mu(b\vee c) = \mu(a\wedge b)\vee\mu(a\wedge c)$, for all $\mu(a),\mu(b),\mu(c)$ in L.

Proof
(i) $\Rightarrow$ (ii)
Let $\mu(a),\mu(b),\mu(c)$ in L be arbitrary.

Then $\mu(a\wedge b)\vee\mu(a\wedge c) \geq \min\{\mu(a),\mu(b)\}\vee\min\{\mu(a),\mu(c)\}$

$\geq \min\{\mu(a),\mu(b)\}\vee\mu(a)\wedge\min\{\mu(a),\mu(b)\}\vee\mu(c)$, by (i)

$\geq [\mu(a)\vee\min\{\mu(a),\mu(b)\}]\wedge[\mu(c)\vee\min\{\mu(a),\mu(b)\}]$, by commutative law

$\geq \mu(a)\wedge[\mu(c)\vee\min\{\mu(a),\mu(b)\}]$, by absorption law

$\geq \mu(a)\wedge[\min\{\mu(c),\mu(a)\}\wedge\min\{\mu(c),\mu(b)\}]$, by (i)

$\geq [\mu(a)\wedge\min\{\mu(c),\mu(a)\}]\wedge\min\{\mu(c),\mu(b)\}$, by associative law

$\geq \mu(a)\wedge\min\{\mu(c),\mu(b)\}$, by absorption law

$\geq \mu(a)\wedge\min\{\mu(b),\mu(c)\}$, by commutative law

$= \mu(a)\wedge\mu(b\vee c)$

Hence $\mu(a\wedge b)\vee\mu(a\wedge c) = \mu(a)\wedge\mu(b\vee c)$,for all $\mu(a),\mu(b),\mu(c)$ in L.

(ii) $\Rightarrow$ (i)
Let for all $\mu(a),\mu(b),\mu(c)$ in L be arbitrary.

Then $\mu(a\vee b)\wedge\mu(a\vee c) \geq \min\{\mu(a),\mu(b)\}\wedge\min\{\mu(a),\mu(c)\}$

$\geq \min\{\mu(a),\mu(b)\}\wedge\mu(a)\vee\min\{\mu(a),\mu(b)\}\wedge\mu(c)$, by (i)

$\geq [\mu(a)\wedge\min\{\mu(a),\mu(b)\}]\vee[\mu(c)\wedge\min\{\mu(a),\mu(b)\}]$, by commutative law

$\geq \mu(a)\vee[\mu(c)\wedge\min\{\mu(a),\mu(b)\}]$, absorption law

$\geq \mu(a)\vee[\min\{\mu(c),\mu(a)\}\vee\min\{\mu(c),\mu(b)\}]$, by (i)

$\geq [\mu(a)\vee\min\{\mu(c),\mu(a)\}]\vee\min\{\mu(c),\mu(b)\}$, by associative law

$\geq \mu(a)\vee\min\{\mu(c),\mu(b)\}$, by absorption law

$\geq \mu(a)\vee\min\{\mu(b),\mu(c)\}$, by commutative law

$= \mu(a)\vee\mu(b\wedge c)$

Hence $\mu(a\vee b)\wedge\mu(a\vee c) = \mu(a)\vee\mu(b\wedge c)$,for all $\mu(a),\mu(b),\mu(c)$ in L.

Definition: 1.2

A Fuzzy lattice L is said to be Fuzzy dual distributive if $\mu(a)\wedge\mu(b\vee c) = \mu(a\wedge b)\vee\mu(a\wedge c)$, for all $\mu(a),\mu(b),\mu(c)$ in L.

Theorem: 1.5

Every Fuzzy distributive lattice is a Fuzzy dual distributive lattice.

Proof:

Follows from the previous theorem.

Theorem: 1.6

If L is a Fuzzy distributive lattice then $(\mu(a),\mu(b))$ is a Fuzzy dually modular pair for all$\mu(a),\mu(b)\in$ L.

Proof:

Given L is a Fuzzy distributive lattice.

To prove $(\mu(a),\mu(b))$ is a Fuzzy dually modular pair for all $\mu(a),\mu(b)\in$ L.

L is a Fuzzy distributive lattice.

$\Rightarrow \mu(x)\vee\mu(y\wedge z) = \mu(x\vee y)\wedge\mu(x\vee z)$, for all $\mu(x),\mu(y),\mu(z)\in$ L

$\Rightarrow \mu(x\vee y)\wedge\mu(x\vee z) = \mu(x)\vee[\mu(y)\wedge\mu(x\vee z)]$,

for all $\mu(x),\mu(y),\mu(z) \in$ L,

$\Rightarrow \mu(x\wedge y)\vee\mu(x\wedge z) = \mu(x)\wedge[\mu(y)\vee\mu(x\wedge z)]$,

for all $\mu(x),\mu(y),\mu(z)\in$ L, by Theorem 1.1

Take $\mu(c) = \mu(x)$, $\mu(a) = \mu(y)$, $\mu(b) = \mu(x\wedge z)$, we get

$\mu(c\wedge a)\vee\mu(b) = \mu(c)\wedge\mu(a\vee b)$ for any$\mu(c) \geq \mu(b)$.

$\Rightarrow$ $(\mu(a),\mu(b))$ is a dually modular pair.

Conclusion

The Paper is proved thatIn any Fuzzy lattice L the following are equivalent $\mu(a\vee b)\wedge\mu(a\vee c) = \mu(a)\vee[\mu(b)\wedge\mu(a\vee c)]$ and $\mu(a\wedge b)\vee\mu(a\wedge c) = \mu(a)\wedge[\mu(b)\vee\mu(a\wedge c)]$ for all $\mu(a),\mu(b),\mu(c)$ in L, Every Fuzzy modular lattice is a Fuzzy dual modular lattice, If L is a Fuzzy modular lattice then $(\mu(a), \mu(b))$ is a dually modular pair for all $\mu(a)$, $\mu(b)\in$ L, In any Fuzzy lattice L the following are equivalent $\mu(a)\vee\mu(b\wedge c) = \mu(a\vee b)\wedge\mu(a\vee c)$ and $\mu(a)\wedge\mu(b\vee c) = \mu(a\wedge b)\vee\mu(a\wedge c)$ for all $\mu(a)$, $\mu(b)$, $\mu(c)$ in L, Every Fuzzy distributive lattice is a Fuzzy dual distributive lattice, If

L is a Fuzzy distributive lattice then (μ(a),μ(b)) is a Fuzzy dually modular pair for allμ(a),μ(b)∈ L.

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