Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 11, Number 4 (2015), pp. 1767-1774

http://www.ripublication.com

# A New Notions of Cordial Labeling Graphs

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**Abstract**

Let $G = (V, E)$ be a simple graph and $f : V \to \{1, 2, 3 \ldots . |V|\}$ be a bijection. For each edge uv, assign the label 1 if $f(u) + f(v)$ is even and the label 0 otherwise. f is called an even sum cordial labeling if $\left|e_f(0) - e_f(1)\right| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. In this paper,the concepts of even sum cordial labeling deeds of paths, cycle, double fan $P_n + 2K_1$, Jewel graph and $C_{n-2} + K_2$ are introduced.

**AMS subject classification:** 05C78.
**Keywords:** Even sum cordial labeling, even sum cordial graph, $P_n + 2K_1$, Jewel graph, $C_{n-2} + K_2$.

## 1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [5]. For standard terminology and notations related to number theory we refer to Burton [3] and graph labeling [6], we refer to Gallian [4]. In 1987, Cahit [1, 2] introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling. The brief summaries of definition which are necessary for the present investigation are provided below. Let $G = (V, E)$ be a simple graph and $f : V \to \{1, 2, 3 \ldots . |V|\}$ be a bijection. For each edge $uv$, assign the label 1 if $f(u) + f(v)$ is even and the label 0 otherwise. $f$ is called an even sum cordial labeling if $\left|e_f(0) - e_f(1)\right| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. In this paper,the concepts of even sum cordial labeling deeds of paths, cycle, double fan $P_n+2K_1$, Jewel graph and $C_{n-2}+ K_2$ are introduced.

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## 2. Preliminaries

**Definition 2.1.** A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

**Definition 2.2.** A mapping $f : V(G) \to \{0, 1\}$ is called binary vertex labelling of G and $f(v)$ is called the label of the vertex v of G under $f$. If for an edge $e = uv$, the induced edge labelling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Then $v_f(i)$ = number of vertices of having label $i$ under $f$ and $e_f(i)$ = number of edges of having label $i$ under $f*$.

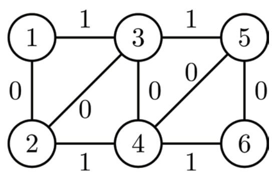
**Definition 2.3.** A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$. A graph G is cordial if it admits cordial labeling.

**Definition 2.4.** The joint $G_1 + G_2$ of $G_1$ and $G_2$ consists of $G_1 \cup G_2$ and all lines joining $V_1$ with $V_2$. The graph $P_n + 2K_1$ is called the double fan.

## 3. Even Sum Cordial Graph

**Definition 3.1.** Let $G = (V, E)$ be a simple graph and $f : V \to \{1, 2, 3 \ldots . |V|\}$ be a bijection. For each edge uv, assign the label 1 if $f(u) + f(v)$ is even and the label 0 otherwise. f is called an even sum cordial labeling if $\left|e_f (0) - e_f (1)\right| \leq 1$, where $e_f (1)$ and $e_f$ (0)denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with an even sum cordial labeling is called an even sum cordial graph.

**Example 3.2.**




Here we see that $e_f (0) = 5, e_f (1) = 4$. Thus $\left|e_f (0) - e_f (1)\right| \leq 1$ and hence G is an even sum cordial graph.

**Remark 3.3.** Every complete graph need not be even sum cordial graph when $n \geq 4$.

**Proposition 3.4.** Given a positive integer $n$, there is an even sum cordial graph $G$ which has $n$ vertices.

*Proof.*
**Case 1:** Suppose $n$ is odd.
By constructing a path containing $\dfrac{n+5}{2}$ vertices, i.e, $v_1, v_2, v_3, \ldots, v_{\frac{n+5}{2}}$ which are

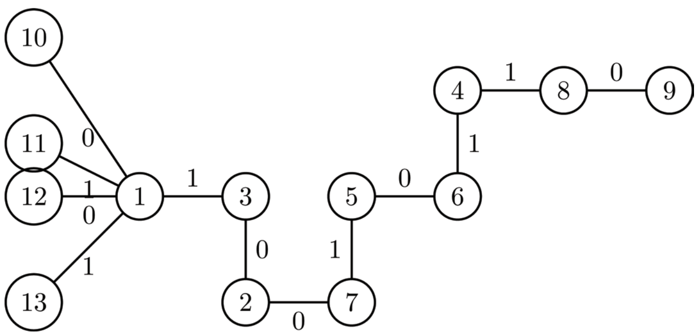
having the label between 1 to $\frac{n+5}{2}$ , $e_f(0) \geq e_f(1)$ and adding $\frac{n-5}{2}$ vertices i.e, $v_{\frac{n+7}{2}}, v_{\frac{n+9}{2}}, \ldots, v_n$ which are labeled as $\frac{n+7}{2}, \frac{n+9}{2}, \ldots, n$ respectively to the vertex $v_1$. we see that $e_f(0) = e_f(1)$.

Hence, $|e_f(0) - e_f(1)| = 1$. Thus the resultant graph G is an even sum cordial graph.

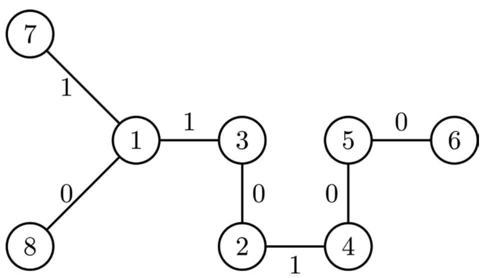
**Case 2:** Suppose $n$ is even.
By constructing a path containing $\frac{n+4}{2}$ vertices, i.e, $v_1, v_2, v_3, \ldots, v_{\frac{n+4}{2}}$ which are having the label between 1 to $\frac{n+4}{2}$, $e_f(0) = e_f(1)$ and adding $\frac{n-4}{2}$ vertices i.e., $v_{\frac{n+6}{2}}, v_{\frac{n+8}{2}}, \ldots, v_n$ which are labeled as $\frac{n+6}{2}, \frac{n+8}{2}, \ldots, n$ respectively to the vertex $v_1$.

We see that $e_f(0) = e_f(1) + 1$. Hence $|e_f(0) - e_f(1)| \leq 1$. Thus the resultant graph G is an even sum cordial graph. ■

**Example 3.5.** When $n = 13$



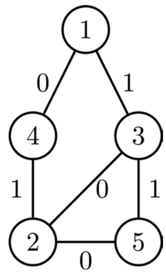

When $n = 8$



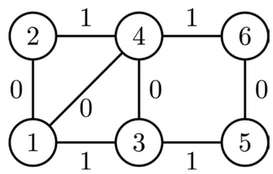

**Proposition 3.6.** If $G$ is an even sum cordial graph then $G - e$ is also an even sum cordial graph.

*Proof.* Let G be an even sum cordial graph of $n$ vertices. Suppose $n$ is even, then we construct the graph having $e_f(0) = e_f(1) = \frac{n}{2}$. Let $e$ be an edge in $G$ which is labeled as either 0 or 1. Then in $G - e$ we have either $e_f(0) = e_f(1) + 1$. (or) $e_f(1) = e_f(0) + 1$ and hence $|e_f(0) - e_f(1)| \leq 1$. Thus the graph $G - e$ is an even sum cordial graph. Similarly we can prove the above Proposition for when $n$ is odd. ∎

**Example 3.7.** When $n = 5$




When $n = 6$




**Proposition 3.8.** Any path is an even sum cordial graph.

*Proof.* Let $P_n$ be the path. Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of the path $P_n$.

The following table gives the even sum cordial labeling of $P_n$, $n \leq 8$.

| n | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ | $u_6$ | $u_7$ | $u_8$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | | | | | | | |
| 2 | 1 | 2 | | | | | | |
| 3 | 1 | 3 | 2 | | | | | |
| 4 | 1 | 3 | 2 | 4 | | | | |
| 5 | 1 | 3 | 2 | 4 | 5 | | | |
| 6 | 1 | 3 | 2 | 4 | 5 | 6 | | |
| 7 | 1 | 3 | 2 | 4 | 5 | 7 | 6 | |
| 8 | 1 | 3 | 2 | 4 | 5 | 6 | 7 | 8 |

Assume $n > 8$.

We construct the even sum cordial path $P_n$ in the following manner:

Define a map $f : V(P_n) \to \{1, 2, \ldots, n\}$ as follows.

**Case 1:** If $n \equiv 0 \pmod 4$
We assigning the label values in the following way. Define for $i = 1, 5, 9 \ldots n-3$.
$f(u_i) = i$
$f(u_{i+1}) = i + 2$
$f(u_{i+2}) = i + 1$
$f(u_{i+3}) = i + 3$

**Case 2:** If $n \equiv 1 \pmod 4$
We assigning the label values in the following way. Define for $i = 1, 5, 9 \ldots n-4$.
$f(u_i) = i$
$f(u_{i+1}) = i + 2$
$f(u_{i+2}) = i + 1$
$f(u_{i+3}) = i + 3$ and $f(u_n) = n$.

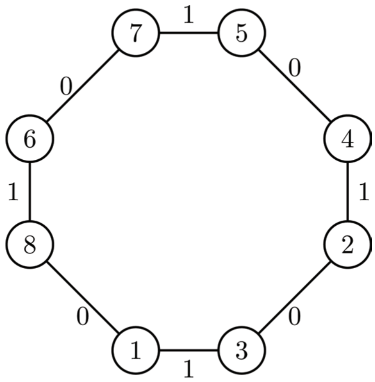
**Case 3:** If $n \equiv 2 \pmod 4$
We assigning the label values in the following way. Define for $i = 1, 5, 9 \ldots n-5$.
$f(u_i) = i$
$f(u_{i+1}) = i + 2$
$f(u_{i+2}) = i + 1$
$f(u_{i+3}) = i + 3$ and $\{f(u_{n-1}) = n-1,\ f(u_n) = n$ (or) $f(u_{n-1}) = n,\ f(u_n) = n-1\}$

**Case 4:** If $n \equiv 3 \pmod 4$
We assigning the label values in the following way. Define for $i = 1, 5, 9 \ldots n-6$.
$f(u_i) = i$
$f(u_{i+1}) = i + 2$
$f(u_{i+2}) = i + 1$
$f(u_{i+3}) = i+3$ and $\{f(u_{n-2}) = n-2,\ f(u_{n-1}) = n,\ f(u_n) = n-1$ (or) $f(u_{n-2}) = n,$ $f(u_{n-1}) = n-2,\ f(u_n) = n-1\}$
Hence we proved that any path is an even sum cordial graph. ■

**Proposition 3.9.** Any cycle $C_n$ is an even sum cordial graph except $n = 6, 6+d, 6+2d, \ldots$.

*Proof.* Let G be the graph $C_n$. First we Construct the path $P_n$ by Proposition 3.8 and then join the first and last vertex by an edge, we form the cycle $C_n$, we see that $|e_f(0) - e_f(1)| \leq 1$. Hence $C_n$ is an even sum cordial graph. ■

**Example 3.10.** Cycle graph $C_8$




**Proposition 3.11.** The graph $P_n + K_1$ is an even sum cordial graph.

*Proof.* Let x be the vertex of $K_1$. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of even sum cordial path $P_n$.

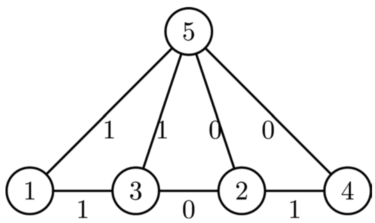
Let G be the graph $P_n + K_1$. $V(G) = \{v_i, x, 1 \leq i \leq n\}$

$$E(G) = \{xv_i, P_n, 1 \leq i \leq n\}$$

Define $f : V(P_n) \to \{1, 2, \ldots, n+1\}$ by $f(x) = n + 1$.

First we construct the path $\mathrm{P_n}$ by Proposition 3.8 then we form the graph $P_n + K_1$, we see that $e_f(0) = e_f(1) + 1 \therefore |e_f(0) - e_f(1)| \leq 1$. Hence G is an even sum cordial graph. ■

**Example 3.12.** Graph $\mathrm{P_4+K_1}$




**Proposition 3.13.** The double fan $P_n + 2K_1$ is an even sum cordial graph.

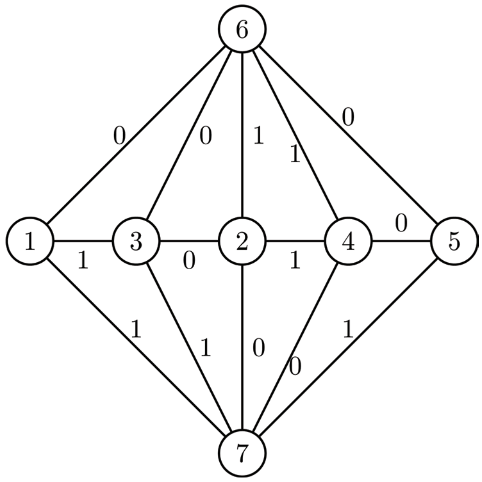
*Proof.* Let $x, y$ be the vertices of $2K_1$. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of even sum cordial path $P_n$.

Let G be the graph $P_n + 2K_1$. $V(G) = \{v_i, x, y, 1 \leq i \leq n\}$. $E(G) = \{xv_i, yv_i, P_n, 1 \leq i \leq n\}$.

Define $f : V(P_n) \to \{1, 2, \ldots, n+2\}$ by $f(x) = n + 1$, $f(y) = n + 2$. First we construct the path $P_n$ by Proposition 3.8 then we form the graph $P_n + 2K_1$.

Thus when $n$ is odd, $e_f(0) = e_f(1) = \dfrac{3n-1}{2}$ and when $n$ is even, $e_f(0) = \dfrac{3n-2}{2}$, $e_f(1) = \dfrac{3n}{2}$. $\therefore |e_f(0) - e_f(1)| \leq 1$. Hence G is an even sum cordial graph. ■

**Example 3.14.** Graph double fan $P_5 + 2K_1$



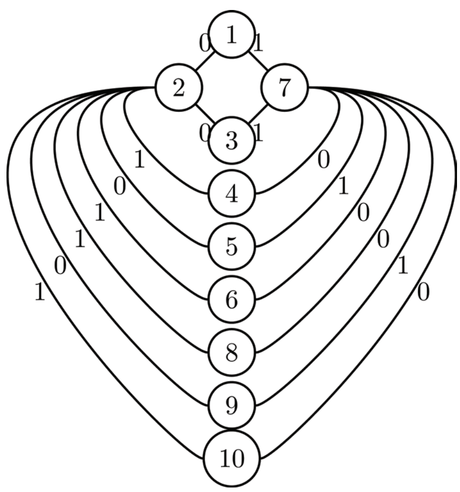

**Proposition 3.15.** The Jewel graph is an even sum cordial graph.

*Proof.* Let $G$ be a jewel graph. Let $V(G) = \{v, w, x, y, v_i, 1 \leq i \leq n\}$. $E(G) = \{vx, xy, vw, wy, xv_i, wv_i, 1 \leq i \leq n\}$. Then $|V(G)| = n + 4$ and $|E(G)| = 2n + 4$.

Define $f : V(G) \to \{1, 2, \ldots, n + 4\}$ as follows. $f(x) = 2$, $f(y) = 3$, $f(v) = 1$, $f(w) = p$ where $p$ is the largest prime number and $p \leq n + 4$ and the label the vertices $v_1, v_2, v_3, \ldots, v_n$ with the numbers $4, 5, 6 \ldots n + 4$ other than $p$. Thus we get $e_f(0) = e_f(1)$

$\therefore |e_f(0) - e_f(1)| \leq 1$. Hence G is an even sum cordial graph. ■

**Example 3.16.** Jewel graph, When $n = 6$

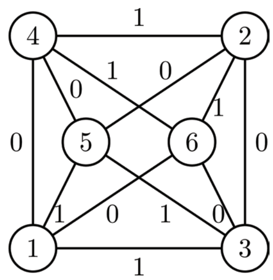



**Proposition 3.17.** The graph $C_{n-2} + K_2$ is an even sum cordial graph.

*Proof.* Let $v_1, v_2, v_3, \ldots, v_{n-2}$ be the vertices of $C_{n-2}$. Let $v_{n-1}, v_n$ be the vertices of $K_2$. Define $f : V(S_n) \to \{1, 2, \ldots, n\}$ as follows. $f(v_{n-1}) = n-1$, $f(v_n) = n$.

First we construct the cycle $C_{n-2}$ by Proposition 3.9. Then by using joint operation form the graph $C_{n-2} + K_2$. We see that, $\left|e_f(0) - e_f(1)\right| \leq 1$. Hence $C_{n-2} + K_2$ is an even sum cordial graph. ■

**Example 3.18.** Graph $C_4 + K_2$




## 4. Conclusions

In this paper, we established the even sum cordial labeling deeds of paths, cycle, double fan $P_n + 2K_1$, Jewel graph and $C_{n-2} + K_2$.

## References

[1] I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, *Ars combinatoria*, 23(1987), 201–207.

[2] I. Cahit, On cordial and 3-equitable labelings of graph, *Utilitas Math*, 370(1990), 189–198.

[3] David M. Burton, Elementary Number Theory, Second Edition, Wm. C. Brown Company Publishers, 1980.

[4] J. A. Gallian, A dynamic survey of graph labeling, *Electronic Journal of Combinatorics*, 17 (2010), DS6.

[5] F. Harary, Graph Theory, Addition-Wesley, Reading, Mass, 1972.

[6] P. Lawrence Rozario Rajand, R. Lawrence Joseph Manohara, Some Results on Divisor Cordial Labeling of Graphs, *International Journal of Innovative Science, Engineering & Technology*, Vol. 1, Issue 10, (2014), 226–231.