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Sensitivity Analysis of an Interval Assignment Problem

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Abstract

In this paper, we focus the Type II sensitivity analysis (SA) in the interval assignment problem (IAP).We attempt to perform Type II sensitivity analysis which determines the range of perturbation to keep the current optimal assignment remaining optimal. The proposedmethod can help the decision makers to determine what level of exactness is necessary for a parameter to make the model adequately useful and valid when they are handling distribution problem having imprecise parameters. Numerical examples are provided to demonstrate the potentiality of the proposed method.

Keywords: Sensitivity analysis, Interval cost, Assignment problem.

1. Introduction

The assignment problem (AP) is a special type of a transportation problem and a linear zero-one programming problem [1]. It is a one of the well-studied optimization problems in Management Science and has been widely applied in both manufacturing and service systems. The main object of the AP is to find an assignment schedule in a jobs assignment problem where n jobs are allocated to n workers and each worker receives exactly just one job such that the total assignment cost is minimum.

The AP led by Votaw and Orden[2] can be solved using the linear programming technique, the transportation algorithm or the Hungarian method established by Kuhn [3]. The Hungarian method is predictable to be the first practical method for solving the standard assignment problem. Balinskiand Gomory [4]presented a labeling algorithm for solving the transportation and assignment problems. Ford and Fulkerson [5] introduced the models and algorithms in Flows in Networks which are used widely today in the fields of transportation systems, manufacturing, inventory planning, image processing, and Internet traffic. Aggarwal et al. [6]established two algorithms for solving bottleneck assignment problems. Salehi [7] discussed an approach for solving multi-objective assignment problem with interval parameters.

Sensitivity analysis has generally been recognized as an important issue after obtaining the optimum and it provides further useful information for management. The implementation of sensitivity analysis is based primarily on basic optimal solutions. Adlakha and Arsham [8]projected a pivotal algorithm for dealing with sensitivity analysis of anassignment problem without using any extra variables. Chi-Jen Lin and Ue-Pyng Wen [9, 10]deliberate sensitivity analysis of an assignment problem.

In this paper, the technique of the proposed algorithm is divided into two parts: one is when the unassigned cell is perturbed, and the other is when the assigned cell is perturbed.Since Type II sensitivity of the IAP is helped to increase the efficiency and effectiveness of the correct decision making process and provide more precise and exact information.

2. Preliminaries

The classical AP can also be written as a 0–1 integer programming problem as follows:

(P) Minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1; \quad i = 1,2,\ldots,n \;;$$

$$\sum_{i=1}^{n} x_{ij} = 1; \quad j = 1,2,\ldots,n \;;$$

$$x_{ij} = \{0,1\}\,; \; i = 1,2,\ldots,n \;\; j = 1,2,\ldots,n$$

where x_{ij} is the decision variable, that is, ith worker in jth job and c_{ij} is the cost of the ith worker who are doing the jth job.

3. Type II Sensitivity

Three types of sensitivity analysis for a linear programming problem werecharacterizedand summarized by Koltai and Terlaky and Hadigheh and Terlaky[11]. If the optimal solution of a linear programming model is non-degenerate, type I sensitivity delivers complete information. They showed that the type I sensitivity does not provide adequately information when the solution of the linear programming model is degenerate. The assignment problem is completely a degenerate linear programming model. Therefore, for obtaining complete information regarding the costs sensitivity ranges in the assignment problem, Type II sensitivity must be considered.

Now, we need the following theorems which can be found in [12].

Theorem 3.1:
Let $(i, j)^{th}$ cell be a unassigned cell corresponding to an optimal solution of the IAP with $\delta_{ij} = c_{ij} - u_i - v_j (\geq 0)$. If $c_{ij} + \Delta c_{ij}$ is the perturbed cost of c_{ij}, then the range of $\Delta c_{ij} = [-\delta_{ij}, \infty)$.

Theorem 3.2:
Let $(i, j)^{th}$ cell be assigned cell corresponding to an optimal solution of the IAP with $\delta_{ij} = c_{ij} - u_i - v_j (= 0)$. If $c_{ij} + \Delta c_{ij}$ is the perturbed value of c_{ij} and U_i is the minimum value of δ_{ij} for all unassigned cells in the i th origin and V_j is the minimumvalue of δ_{ij} for all unassigned cells in the j^{th} destination, then the range of $\Delta c_{ij} = (-\infty, M_{ij}]$ where $M_{ij} =$ maximum $\{ U_i, V_j \}$.

A heuristic algorithm for finding the SA of IAP is proposed below:
Step 1: Construct two individual problems of the given IAP namely, upper bound (UB) problem and lower bound (LB) problem.
Step 2: Compute an optimal solution to the (UB) problem by the Hungarian method.
Step3: Create the MODIindex matrix for the solution obtained in Step2:
Step 4:Compute the relation between each pair of MODI indices θ_i and θ_j $(i \neq j)\ i, j = 1,2,......,n$ and their limit values using the MODI index matrix table and the optimality condition $c_{ij} - (u_i + v_j) \geq 0$ for all unassigned cells (i, j)
Step 5: (a) For all assigned cells, we can find the cost range of c_{ij}, by using the optimality conditions

(i) $(c_{ij} + \Delta c_{ij}) - ((u_i + \Delta c_{ij}) + v_j) \geq 0$, for all j(j ≠ i)

(ii) $(c_{ij} + \Delta c_{ij}) - (u_i + (v_j + \Delta c_{ij})) \geq 0$, for all i(i ≠ j) and Theorem 2 to the (UB)problem.

(b) For all unassigned cells, we can find the cost range by using the Theorem 1 to the (UB) problem.

Step 6: Repeat the steps from 2 to 5, we can find the cost ranges for the (LB) problem.

4. Numerical Example

Consider the followingintervalassignment problem (IAP) whose cost matrix are given below:

	1	2	3
1	[3,12]	[2,9]	[1,5]
2	[3,12]	[5,13]	[6,13]
3	[6,8]	[3,7]	[5,11]

Now, the (UB) problem of the given IAP is given below:

	1	2	3
1	12	9	5
2	12	13	13
3	8	7	11

Now, using step 2.the (UB) problem is solved with the Hungarian method and then, the optimal solution is $1 \to 3, 2 \to 1,$ and $3 \to 2$.

Now, Using step 3.construct MODI indices table for theoptimal solution to the (UB)problem.

	$v_1 = 12 - \theta_2$	$v_2 = 7 - \theta_3$	$v_3 = 5 - \theta_1$
$u_1 = \theta_1$	12	9	5
$u_2 = \theta_2$	12	13	13
$u_3 = \theta_3$	8	7	11

Now, we choose $u_1 = \theta_1, u_2 = \theta_2$ and $u_3 = \theta_3$ such that $c_{ij} - (u_i + v_j) = 0$ for all assigned cells (i,j) and $c_{ij} - (u_i + v_j) \geq 0$ for all unassigned cells (i,j).Now, using step 3.we obtain the following results:

$$-8 \leq \theta_1 - \theta_2 \leq 0; \quad -6 \leq \theta_1 - \theta_3 \leq 2; 4 \leq \theta_2 - \theta_3 \leq 6 \qquad (1)$$

Now, we consider the SA of the given problem in the cell (1, 1) which is anunassigned cell.

Now, since $c_{ij} - (u_i + v_j) \geq 0,$ we have, $(12 + \Delta c_{11}) - (\theta_1 + 12 - \theta_2) \geq 0. \Rightarrow$ $12 + \Delta c_{11} \geq 12 + \theta_1 - \theta_2 \geq 12 - 8 = 4.$By using equation (1). We get the Type II sensitivity range of c_{11} is $[4, \infty)$.By using step 5(b).Proceeding in this manner, we can determine the Type II sensitivity ranges of other unassigned cells.

Now, we consider the SA of the given problem in the cell (1, 3) which is a assigned cell.

Case(i) :Attach Δc_{13} to v_3

Now, since $c_{i3}-(u_i+v_3)\geq 0, i=1, 2$ we have,

$$13-[\theta_2+(5-\theta_1+\Delta c_{13})]\geq 0 \quad \Rightarrow \Delta c_{13}\leq 8-\theta_2+\theta_1\leq 8-0=8 \ ;$$

$$11-[\theta_3+(5-\theta_1+\Delta c_{13})]\geq 0 \quad \Rightarrow \Delta c_{13}\leq 6-\theta_3+\theta_1\leq 6+2=8 \ ;$$

Therefore, U_{13} = minimum {8,8}= 8

Case(ii) :Attach Δc_{13} to u_1

Now, since $c_{1j}-(u_1+v_j)\geq 0, \ j=2, 3.$ we have,

$$12-[(\theta_1+\Delta c_{13})+12-\theta_2]\geq 0 \ \Rightarrow \Delta c_{13}\leq 12-12-\theta_1+\theta_2\leq 0+8=8;$$

$$9-[(\theta_1+\Delta c_{13})+7-\theta_3]\geq 0 \Rightarrow \Delta c_{13}\leq 2-\theta_1+\theta_3\leq 2+6=8;$$

Therefore, V_{13} = minimum {8, 8}= 8.

Hence, by using step 5(a), the sensitivity range of an allocated cell (1,3) is $(-\infty,13]$.

Proceeding in this manner, we can determine the Type II sensitivity ranges of other assigned cells.

Similarly, we can find the Type II sensitivity ranges of other costs in the given (LB) problem.

Now, the type II sensitivity ranges of all cost coefficients in the given interval assignment problem such that its optimal solution is invariant are given in the following table:

$[-2,\infty)$ $[4,\infty)$	$[-1,\infty)$ $[1,\infty)$	$(-\infty,4]$ $(-\infty,13]$
$(-\infty,8]$ $(-\infty,14]$	$[0,\infty)$ $[11,\infty)$	$[1,\infty)$ $[5,\infty)$
$[1,\infty)$ $[6,\infty)$	$(-\infty,6]$ $(-\infty,9]$	$[2,\infty)$ $[3,\infty)$

5. Conclusion

Assignment problem is one of the most important problem in decision-making. In this paper, we investigate the Type II sensitivity range of the IAP. Commercially available software can only help to determine Type I sensitivity ranges, which is impractical in degenerate problems. Type I sensitivity range is always a subset of Type II sensitivity range.There are many application areas of this study, such as human resource management, job assignment planning, transportation planning, etc.

References

1. T. Koltai and T. Terlaky, 2000, "The difference between the managerial and mathematical interpretation of sensitivity analysis results in linear programming", International Journal of Production Economics, 65, pp. 257-274.
2. D.F. Votaw and A.Orden,1952, "The personal assignment problem", Symposium on linear inequalities and programming, Scoop 10 , US Airforce, pp. 155-163.
3. H.W. Kuhn, 1955, "The Hungarian method for the assignment and transportation problems", Naval Research Logistics Quarterly, 2, pp. 83–97.
4. M.L. Balinski and R.E. Gomory,1967, "A primal method for the assignment and transportation problems", Management science, 10, pp. 578-593.
5. V. Aggarwal, V.G. Tikekar and L-F. Hsu,1986, "Bottleneck assignment problems under Categorization", Computers and Operations Research, 13, pp. 11–26.
6. L.R. Ford Jr. and D.R. Fulkerson, 1966, "Flows in Networks", Princeton University Press, Princeton, NJ.
7. K.Salehi,2014, "An approach for solving multi-objective assignment problem with interval parameters", Management Science Letters, 4, pp.2155-2160.
8. V. Adlakha and H. Arsham, 1998, "Managing cost uncertainty in transportation and assignment problems", Journal of Applied Mathematics and Decision Science, 2, pp. 65-104.
9. Chi-Jen Lin and Ue-Pyng Wen, 2007, "Sensitivity analysis of objective functioncoefficientsof the assignment problem", Asia-Pacific Journal of OperationalResearch, 24, pp. 203-221.
10. Chi-Jen Lin,2011,"A labeling algorithm for the sensitivity ranges of the assignment problem", Applied Mathematical Modeling, 35, pp. 4852-4864.
11. A.G. Hadigheh and T. Terlaky, 2007, "Active constraint set invariancy sensitivity analysis in linear optimization", Journal of Optimization Theory and Applications, 133, pp.303-315.
12. K.kavitha and D.Anuradha, 2015, "Heuristic Algorithm for finding sensitivity anslysis of a more for less solution to transportation problems", Global Journal of Pure and Applied Mathematics, 11, pp. 479-485.