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Graceful Labeling of A Path Related Graphs

J.Sebastian Lawrence
Department of Mathematics
Sathyabama University, Chennai-600119, India.
Email: lawrence.seba@gmail.com
Dr.P.Selvaraju
Department of Mathematics
Veltech Multitech Engineering College
Avadi, Chennai.

Abstract

Paths and many path related graphs like $M(P_n, P_n^2)$ had been proved to admit different types of labeling. In this paper a new class of path related graph $P_A\left(n, \frac{n-1}{2}, 1\right)$ is defined and it is shown to be graceful graph.

Keywords: Graceful labeling, Paths.

Introduction

The gracefulness of a graph *G* without isolated vertices is the smallest positive integer *k* for which it is possible to label the vertices of *G* with distinct elements from the set $\{0,1,2, \ldots\ldots, k\}$ in such a way that the induced labeling of edges are distinct values. This parameter is well defined as there exists a vertex labeling of *G* that assigns the integers $2^0, 2^1, \ldots, 2^{(p-1)}$ to the *m* vertices of *G*. Thus, for every graph *G* of order *p* and size *q* (without isolated vertices), $q \leq$ grac (G) $\leq 2^{m-1}$. If *G* is a graph of size *q* with $grac(G) = q$,then *G* is graceful. Thus a graceful labeling of a graph *G* with *p* vertices and *q* edges, is a one-to-one mapping *f*: $V(G) \rightarrow \{0,1,2, \ldots\ldots, q\}$such that for every edge *xy* of *G*, *f* induces a weight defined by $|f(x) - f(y)|$ and the set of values is $\{0,1,2, \ldots\ldots, q\}$.

Graceful graph labeling was first introduced by Alexander Rosa [4] (around 1967) as mean of attacking the problem of cyclically decomposing the complete graph into other graphs. Since Rosa's original article, literally hundreds of papers have been written on graph labeling. A well-known conjecture due to Ringel and Kotzig is that all trees are graceful. Rosa [4]showed that all caterpillars are graceful. Using deterministic back-tracking algorithm Fang [2] proved that all trees withat most 35 vertices are graceful.

It is known that not every graph is graceful, for instance we can consider the complete graph k_n when n≥5 and the cycle c_n when n≡ $1 or\ 2 (mod 4)$. The smallest graph, in order and size, that is not graceful is $C_3 \cup K_{1,1}$. These examples represent three reasons why a graph fails to be graceful. The graph has too many edges (k_n , $n \geq 5$), the graph has not the right parity (C_n,n≡ $1 or\ 2\ (mod 4)$), or the graph has too many vertices and not enough edges ($C_3 \cup K_{1,1}$).

A path on n vertices is denoted by P_n. P_n^2 is graceful was proved by Kang, Liang, Gao, and Yang [3].Acharya and Gill [1] proved that the planar grids $P_m \times P_n$ are graceful. The prisms $P_m \times C_n$ were also proved to be graceful.Sethuraman and Selvaraju [6] and [5] have shownthat $P_n + K_2$ is harmonious.

Definition 1.1

A graph $P_A\left(n, \frac{n-1}{2}, 1\right)$ is a path P_n in n vertices [$n \equiv 1 (\text{mod } 2),\ n \geq 3$] $v_0, v_1, \cdots, v_{n-1}$ with arcs joining the vertices v_i, v_{n-1-i} and a pendant edge at $v_{\frac{n-1}{2}}$

Graceful labeling of $P_A\left(n, \frac{n-1}{2}, 1\right)$:

Theorem 2.1

The Graph $P_A\left(m, \frac{m-1}{2}, 1\right)$ is graceful for $m \equiv 1 (\text{mod } 2),\ m \geq 3$.

Vertex labeling:

We define the function as f:V(G) → {0,1,2,…….,3k-2}, n = 3k-2.

Case: i

If the edges are even then

$f(v_0) = 0$

$$f(v_i) = \begin{cases} n - \left(\frac{i-1}{2}\right) & if\, i = 1,3,\dots,2k-3 \\ \frac{i}{2} & if\, i = 2,4,\dots,2k-2 \\ \frac{n}{2} & if\, i = 2k-1 \end{cases}$$

Case :ii

If the edges are odd then

$$f(v_i) = \begin{cases} n - \left(\frac{i-1}{2}\right) \ if i = 1,3, \dots ,2k-3 \\ \frac{i}{2} \ if i = 2,4, \dots ,2k-2 \\ \left(\frac{n+1}{2}\right) \ if i = 2k-1 \end{cases}$$

Table of $P_A\left(m, \frac{m-1}{2}, 1\right)$ **graph:**

K	V(G)= 2k	E(G) = 3k-2	Arc's = k-1
2	4	4	1
3	6	7	2
4	8	10	3
5	10	13	4
6	12	16	5
7	14	19	6

Edge labeling:

We define the function as f:E(G) → {0,1,2,…….,3k-2}

$$f(E_i) = \begin{cases} n - (i-1) \ if \ 1 \le i \le 2k-2 \\ n - i \ if \ 2k-1 \le i \le 3k-3 \\ \frac{i-(k-2)}{2} \ if i = 3k-2 \end{cases}$$

for k = 2,3,4,….

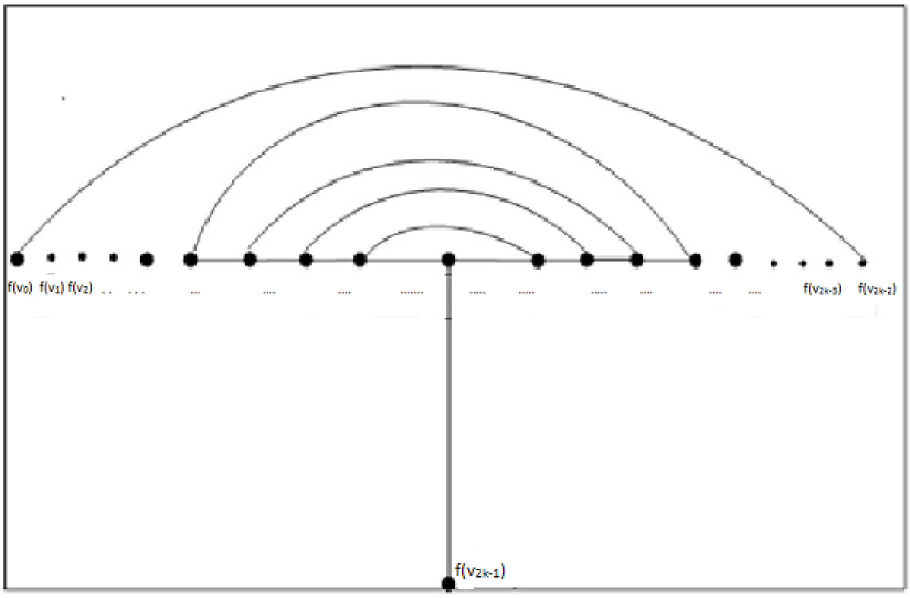

Figure 2.1: $P_A\left(m, \frac{m-1}{2}, 1\right)$ graph.

Conclusion

In this paper, we have shown that $P_A\left(m, \frac{m-1}{2}, 1\right)$ graph is graceful and further study can extendedto prove gracefulness of copies of $P_A\left(m, \frac{m-1}{2}, 1\right)$ graph.

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